Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.3-General/48-1.2.3.4-f-x-^m-d+e-x^n-^q-a+b-x^n+c-x^-2-n-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [156]. This is test number [48].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (156)	0.00 (0)
Mathematica	94.23 (147)	5.77 (9)
Maple	87.82 (137)	12.18 (19)
Mupad	78.21 (122)	21.79 (34)
Fricas	77.56 (121)	22.44 (35)
Giac	70.51 (110)	29.49 (46)
Sympy	49.36 (77)	50.64 (79)
Maxima	44.23 (69)	55.77 (87)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

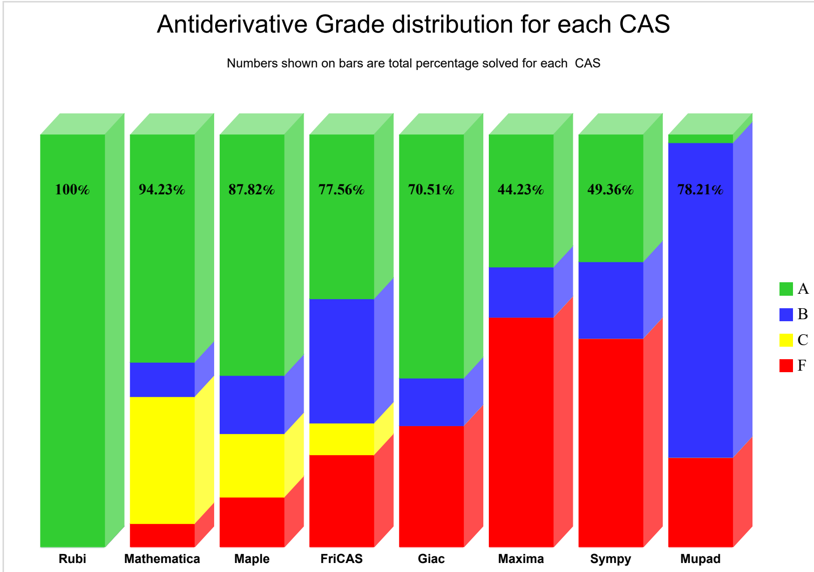
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

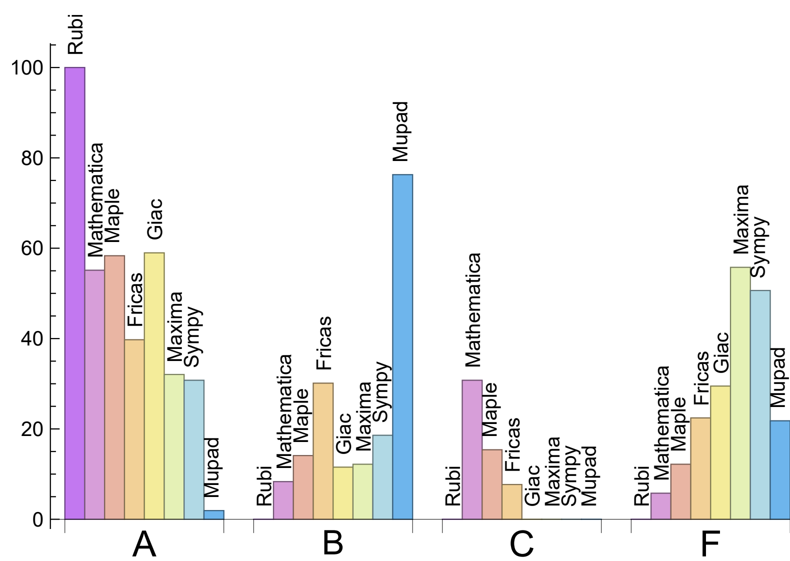
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Giac	58.97	11.54	0.00	29.49
Maple	58.33	14.10	15.38	12.18
Mathematica	55.13	8.33	30.77	5.77
Fricas	39.74	30.13	7.69	22.44
Maxima	32.05	12.18	0.00	55.77
Sympy	30.77	18.59	0.00	50.64
Mupad	N/A	76.28	0.00	21.79

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	9	100.00 %	0.00 %	0.00 %
Maple	19	100.00 %	0.00 %	0.00 %
Fricas	35	54.29 %	45.71 %	0.00 %
Giac	46	78.26 %	13.04 %	8.70 %
Maxima	87	71.26 %	0.00 %	28.74 %
Sympy	79	6.33 %	82.28 %	11.39 %
Mupad	34	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

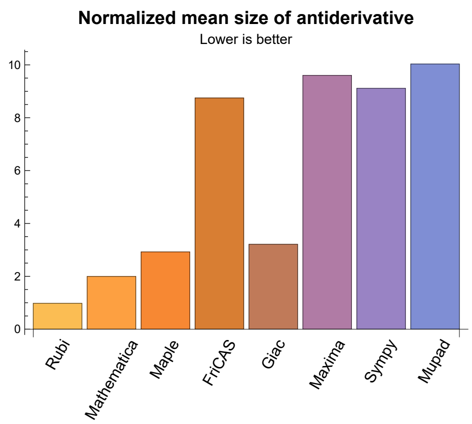
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

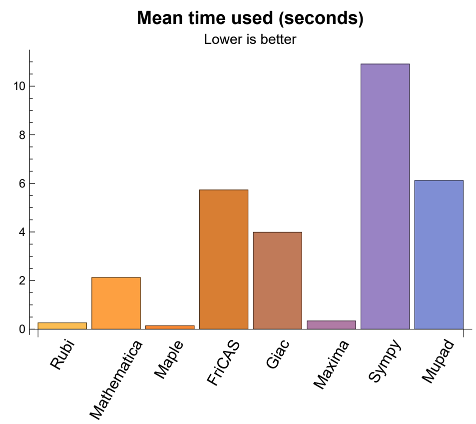
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	208.71	0.98	153.50	1.00
Mathematica	2.12	436.33	2.00	80.00	0.99
Maple	0.14	477.19	2.92	47.00	0.95
Maxima	0.34	212.48	9.60	37.00	1.00
Fricas	5.73	893.31	8.75	220.00	1.68
Sympy	10.91	217.74	9.11	87.00	1.02
Giac	3.99	220.31	3.21	70.00	1.00
Mupad	6.12	854.22	10.03	295.00	2.77

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{86, 155, 156\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {143}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```python
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

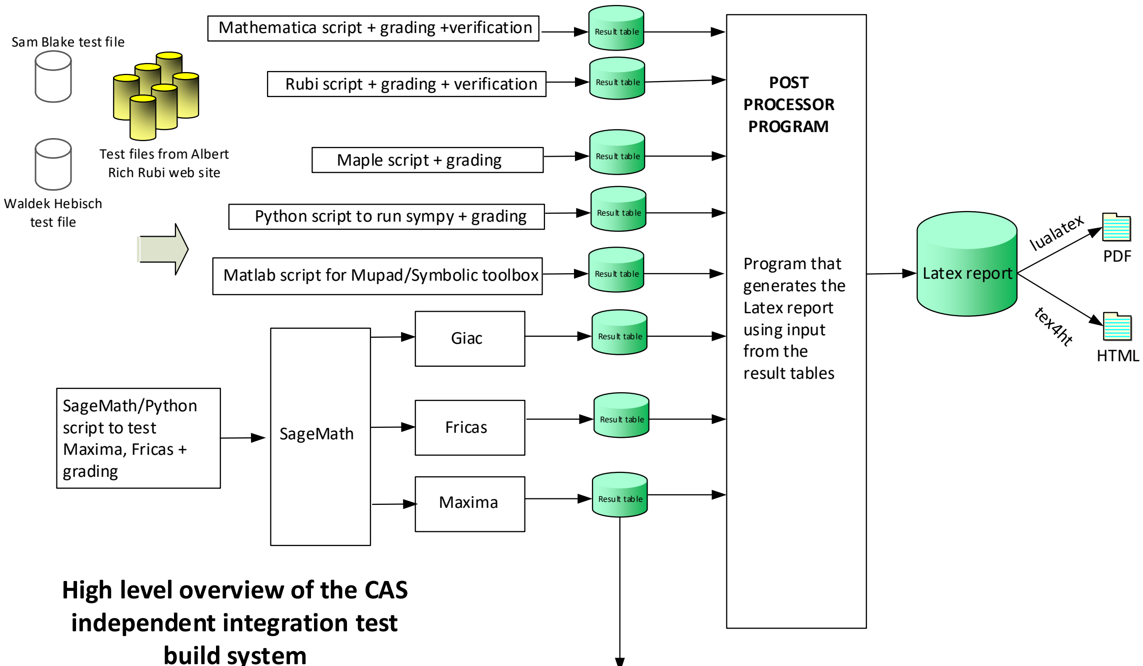
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 20, 21, 22, 32, 44, 46, 53, 55, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 149, 152, 153, 154, 155, 156 }

B grade: { 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 142, 143 }

C grade: { 12, 13, 14, 15, 16, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 79, 80, 81, 82, 83, 84, 85 }

F grade: { 90, 91, 92, 145, 146, 147, 148, 150, 151 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 46, 48, 50, 53, 55, 57, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 93, 94, 95, 97, 98, 99, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 144, 155, 156 }

B grade: { 35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 82, 83, 84, 85, 96, 100, 104, 125, 126, 127, 128 }

C grade: { 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 43, 45, 47, 49, 51, 52, 54, 56, 58, 60, 140 }

F grade: { 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 32, 33, 53, 57, 86, 93, 97, 101, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 121, 122, 123, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 155, 156 }

B grade: { 94, 95, 96, 98, 99, 100, 102, 103, 104, 110, 111, 112, 118, 119, 120, 124, 126, 127, 128 }

C grade: { }

F grade: { 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 25, 26, 27, 28, 29, 30, 31, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 55, 57, 58, 61, 62, 63, 64, 65, 66, 67, 86, 105, 106, 107, 108, 113, 114, 115, 116, 121, 122, 123, 124, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 155, 156 }

B grade: { 8, 25, 26, 27, 28, 29, 30, 31, 43, 45, 46, 47, 50, 54, 56, 59, 60, 70, 71, 72, 73, 74, 75, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 109, 110, 111, 112, 117, 118, 119, 120, 125, 126, 127, 128 }

C grade: { 35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 82 }

F grade: { 14, 15, 16, 17, 18, 19, 49, 51, 68, 69, 76, 77, 78, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 52, 53, 54, 55, 56, 57, 58, 59, 60, 105, 106, 107, 113, 114, 115, 121, 122, 123 }

B grade: { 10, 11, 44, 93, 94, 95, 97, 98, 99, 101, 102, 103, 109, 110, 111, 117, 118, 119, 124, 125, 126, 127, 129, 130, 133, 134, 137, 138, 144 }

C grade: { }

F grade: { 9, 12, 13, 14, 15, 16, 17, 18, 19, 42, 43, 45, 46, 47, 48, 49, 50, 51, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 100, 104, 108, 112, 116, 120, 128, 131, 132, 135, 136, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 21, 22, 23, 24, 32, 33, 34, 44, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 155, 156 }

B grade: { 25, 26, 27, 28, 29, 30, 31, 46, 50, 93, 94, 95, 96, 97, 98, 99, 100, 104 }

C grade: { }

F grade: { 14, 15, 16, 17, 18, 19, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 47, 49, 51, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.1.8 Mupad

A grade: { 86, 155, 156 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144 }

C grade: { }

F grade: { 35, 36, 37, 38, 39, 40, 41, 42, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	164	169	157	170	187	173	158
N.S.	1	1.00	1.01	1.04	0.96	1.04	1.15	1.06	0.97
time (sec)	N/A	0.125	0.036	0.224	0.272	0.321	0.021	3.352	1.599

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	135	136	129	138	151	141	130
N.S.	1	1.00	1.00	1.01	0.96	1.02	1.12	1.04	0.96
time (sec)	N/A	0.084	0.026	0.253	0.296	0.331	0.020	3.879	0.056

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	104	103	99	106	117	109	102
N.S.	1	1.00	1.01	1.00	0.96	1.03	1.14	1.06	0.99
time (sec)	N/A	0.062	0.020	0.278	0.273	0.392	0.015	4.035	0.043

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	74	75	76	70
N.S.	1	1.00	1.00	0.96	0.95	1.01	1.03	1.04	0.96
time (sec)	N/A	0.041	0.015	0.191	0.272	0.350	0.013	4.464	0.035

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	37	39	42	39	43	38
N.S.	1	1.00	1.00	0.88	0.93	1.00	0.93	1.02	0.90
time (sec)	N/A	0.019	0.007	0.056	0.274	0.325	0.007	4.078	0.043

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	176	133	146	199	175	173	165
N.S.	1	1.00	0.94	0.71	0.78	1.06	0.93	0.92	0.88
time (sec)	N/A	0.137	0.108	0.188	0.495	0.372	0.452	4.146	0.269

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	199	156	168	323	206	199	187
N.S.	1	1.00	0.93	0.73	0.79	1.52	0.97	0.93	0.88
time (sec)	N/A	0.146	0.138	0.236	0.500	0.362	0.903	3.946	1.801

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	209	183	203	442	246	224	221
N.S.	1	1.00	0.86	0.76	0.84	1.83	1.02	0.93	0.91
time (sec)	N/A	0.167	0.190	0.230	0.510	0.348	11.252	3.748	0.288

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	126	136	0	440	0	131	2500
N.S.	1	1.00	0.95	1.03	0.00	3.33	0.00	0.99	18.94
time (sec)	N/A	0.148	0.048	0.133	0.000	0.592	0.000	3.075	2.404

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	98	0	311	434	95	2624
N.S.	1	1.00	0.96	1.01	0.00	3.21	4.47	0.98	27.05
time (sec)	N/A	0.090	0.049	0.089	0.000	0.423	96.428	3.242	2.950

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	66	0	220	287	70	1632
N.S.	1	1.00	0.99	0.92	0.00	3.06	3.99	0.97	22.67
time (sec)	N/A	0.052	0.036	0.046	0.000	0.384	9.690	3.162	2.627

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	75	0	242	0	76	2500
N.S.	1	1.00	1.03	0.96	0.00	3.10	0.00	0.97	32.05
time (sec)	N/A	0.089	0.024	0.055	0.000	0.473	0.000	4.464	6.765

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	130	126	0	409	0	128	2500
N.S.	1	1.00	1.16	1.12	0.00	3.65	0.00	1.14	22.32
time (sec)	N/A	0.128	0.035	0.077	0.000	0.718	0.000	3.912	9.569

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	723	723	88	70	0	0	0	0	2500
N.S.	1	1.00	0.12	0.10	0.00	0.00	0.00	0.00	3.46
time (sec)	N/A	1.135	0.034	0.201	0.000	0.000	0.000	0.000	42.007

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	718	718	88	67	0	0	0	0	2500
N.S.	1	1.00	0.12	0.09	0.00	0.00	0.00	0.00	3.48
time (sec)	N/A	0.949	0.034	0.194	0.000	0.000	0.000	0.000	30.152

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	634	59	49	0	0	0	0	2500
N.S.	1	1.00	0.09	0.08	0.00	0.00	0.00	0.00	3.94
time (sec)	N/A	0.422	0.022	0.027	0.000	0.000	0.000	0.000	24.559

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	634	634	61	47	0	0	0	0	2500
N.S.	1	1.00	0.10	0.07	0.00	0.00	0.00	0.00	3.94
time (sec)	N/A	0.439	0.021	0.030	0.000	0.000	0.000	0.000	18.962

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	653	653	85	71	0	0	0	0	2500
N.S.	1	1.00	0.13	0.11	0.00	0.00	0.00	0.00	3.83
time (sec)	N/A	0.705	0.032	0.068	0.000	0.000	0.000	0.000	38.020

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	655	655	89	68	0	0	0	0	2500
N.S.	1	1.00	0.14	0.10	0.00	0.00	0.00	0.00	3.82
time (sec)	N/A	0.735	0.033	0.072	0.000	0.000	0.000	0.000	37.903

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	38	37	37	42	37	39
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.91	0.80	0.85
time (sec)	N/A	0.038	0.011	0.020	0.513	0.396	0.046	3.312	0.056

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	24	24	32	24	26
N.S.	1	1.00	1.00	0.81	0.77	0.77	1.03	0.77	0.84
time (sec)	N/A	0.023	0.006	0.037	0.503	0.406	0.041	3.910	0.038

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.027	0.007	0.016	0.525	0.444	0.046	3.688	0.046

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	38	34	41	35	36
N.S.	1	1.00	1.07	0.85	0.93	0.83	1.00	0.85	0.88
time (sec)	N/A	0.037	0.010	0.023	0.502	0.401	0.053	3.708	1.859

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	45	25	24	28	36	24	26
N.S.	1	1.00	1.45	0.81	0.77	0.90	1.16	0.77	0.84
time (sec)	N/A	0.030	0.009	0.026	0.524	0.407	0.051	3.333	0.045

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	770	31	645	332
N.S.	1	1.00	0.11	0.11	0.00	1.84	0.07	1.54	0.79
time (sec)	N/A	0.361	0.009	0.020	0.000	0.454	0.069	3.734	0.652

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	48	44	0	999	32	820	309
N.S.	1	1.00	0.13	0.12	0.00	2.62	0.08	2.15	0.81
time (sec)	N/A	0.220	0.010	0.020	0.000	0.498	0.070	3.832	2.279

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	378	46	41	0	770	24	635	330
N.S.	1	1.00	0.12	0.11	0.00	2.04	0.06	1.68	0.87
time (sec)	N/A	0.171	0.009	0.018	0.000	0.452	0.067	4.018	2.376

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	55	44	0	992	22	824	281
N.S.	1	1.00	0.13	0.11	0.00	2.41	0.05	2.00	0.68
time (sec)	N/A	0.188	0.010	0.020	0.000	0.466	0.068	5.621	2.264

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	57	44	0	765	26	640	319
N.S.	1	1.00	0.14	0.11	0.00	1.86	0.06	1.56	0.78
time (sec)	N/A	0.181	0.008	0.036	0.000	0.400	0.069	4.713	2.299

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	47	46	0	1009	31	832	313
N.S.	1	1.00	0.11	0.11	0.00	2.43	0.07	2.00	0.75
time (sec)	N/A	0.176	0.009	0.046	0.000	0.435	0.076	4.553	0.398

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	47	46	0	802	32	645	332
N.S.	1	1.00	0.11	0.11	0.00	1.92	0.08	1.54	0.79
time (sec)	N/A	0.232	0.009	0.043	0.000	0.422	0.077	4.187	2.399

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	37	33	32	32	37	32	34
N.S.	1	1.00	1.03	0.92	0.89	0.89	1.03	0.89	0.94
time (sec)	N/A	0.025	0.007	0.036	0.514	0.347	0.045	4.756	1.845

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	38	34	41	35	36
N.S.	1	1.00	1.41	0.90	0.97	0.87	1.05	0.90	0.92
time (sec)	N/A	0.036	0.010	0.023	0.514	0.342	0.053	4.008	1.847

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	55	35	0	34	41	35	36
N.S.	1	1.00	1.41	0.90	0.00	0.87	1.05	0.90	0.92
time (sec)	N/A	0.041	0.007	0.040	0.000	0.331	0.054	4.829	0.039

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	103	1070	0	162	400	0	-1
N.S.	1	1.00	0.26	2.70	0.00	0.41	1.01	0.00	-0.00
time (sec)	N/A	0.269	8.111	0.608	0.000	0.103	4.295	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	101	1010	0	130	257	0	-1
N.S.	1	1.00	0.28	2.84	0.00	0.37	0.72	0.00	-0.00
time (sec)	N/A	0.206	6.960	0.221	0.000	0.085	2.869	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	98	956	0	98	124	0	-1
N.S.	1	1.00	0.31	3.03	0.00	0.31	0.39	0.00	-0.00
time (sec)	N/A	0.165	5.222	0.211	0.000	0.077	1.639	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	98	907	0	69	119	0	-1
N.S.	1	1.00	0.35	3.26	0.00	0.25	0.43	0.00	-0.00
time (sec)	N/A	0.128	10.067	0.204	0.000	0.079	1.387	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	102	934	0	123	119	0	-1
N.S.	1	1.00	0.35	3.23	0.00	0.43	0.41	0.00	-0.00
time (sec)	N/A	0.131	10.080	0.235	0.000	0.088	6.100	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	129	1005	0	187	119	0	-1
N.S.	1	1.00	0.42	3.25	0.00	0.61	0.39	0.00	-0.00
time (sec)	N/A	0.137	10.101	0.205	0.000	0.085	33.555	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	166	1095	0	262	119	0	-1
N.S.	1	1.00	0.48	3.14	0.00	0.75	0.34	0.00	-0.00
time (sec)	N/A	0.215	10.141	0.217	0.000	0.076	152.635	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	200	1182	0	339	0	0	-1
N.S.	1	1.00	0.51	3.04	0.00	0.87	0.00	0.00	-0.00
time (sec)	N/A	0.254	10.176	0.223	0.000	0.095	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	88	67	0	21980	0	0	2500
N.S.	1	1.00	0.20	0.15	0.00	50.76	0.00	0.00	5.77
time (sec)	N/A	0.765	0.055	0.043	0.000	78.736	0.000	0.000	9.632

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	71	66	0	220	287	70	2500
N.S.	1	1.00	0.99	0.92	0.00	3.06	3.99	0.97	34.72
time (sec)	N/A	0.054	0.041	0.068	0.000	0.523	113.148	5.786	4.206

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	59	51	0	20777	0	0	2500
N.S.	1	1.00	0.16	0.14	0.00	55.41	0.00	0.00	6.67
time (sec)	N/A	0.288	0.035	0.046	0.000	195.922	0.000	0.000	9.569

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	179	168	0	1531	0	1404	2500
N.S.	1	1.00	0.97	0.91	0.00	8.32	0.00	7.63	13.59
time (sec)	N/A	0.146	0.106	0.063	0.000	0.535	0.000	7.239	7.053

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	375	375	61	47	0	16274	0	0	2500
N.S.	1	1.00	0.16	0.13	0.00	43.40	0.00	0.00	6.67
time (sec)	N/A	0.225	0.033	0.033	0.000	60.852	0.000	0.000	8.746

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	80	74	0	242	0	78	2500
N.S.	1	1.00	1.03	0.95	0.00	3.10	0.00	1.00	32.05
time (sec)	N/A	0.086	0.024	0.064	0.000	0.799	0.000	6.550	5.271

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	85	73	0	0	0	0	2500
N.S.	1	1.00	0.22	0.19	0.00	0.00	0.00	0.00	6.38
time (sec)	N/A	0.443	0.046	0.061	0.000	0.000	0.000	0.000	9.459

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	89	177	0	2780	0	3007	2500
N.S.	1	1.00	0.45	0.89	0.00	13.97	0.00	15.11	12.56
time (sec)	N/A	0.223	0.035	0.078	0.000	0.878	0.000	11.200	7.620

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	86	68	0	0	0	0	2500
N.S.	1	1.00	0.22	0.17	0.00	0.00	0.00	0.00	6.35
time (sec)	N/A	0.399	0.050	0.069	0.000	0.000	0.000	0.000	10.224

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	46	34	0	223	170	208	56
N.S.	1	1.00	0.17	0.12	0.00	0.80	0.61	0.75	0.20
time (sec)	N/A	0.207	0.012	0.044	0.000	0.414	0.119	3.776	1.920

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	33	32	32	37	32	34
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.95	0.82	0.87
time (sec)	N/A	0.027	0.010	0.019	0.505	0.365	0.078	3.566	0.049

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	55	46	0	719	27	253	248
N.S.	1	1.00	0.15	0.13	0.00	2.03	0.08	0.71	0.70
time (sec)	N/A	0.189	0.012	0.030	0.000	0.414	1.387	3.795	1.985

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	44	39	0	41	42	31	20
N.S.	1	1.00	0.88	0.78	0.00	0.82	0.84	0.62	0.40
time (sec)	N/A	0.027	0.012	0.028	0.000	0.366	0.042	3.751	1.889

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	355	355	57	44	0	719	26	253	208
N.S.	1	1.00	0.16	0.12	0.00	2.03	0.07	0.71	0.59
time (sec)	N/A	0.144	0.010	0.025	0.000	0.386	1.496	3.720	0.002

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	44	35	38	34	41	38	36
N.S.	1	1.00	1.07	0.85	0.93	0.83	1.00	0.93	0.88
time (sec)	N/A	0.036	0.010	0.027	0.583	0.339	0.068	4.152	1.886

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	47	38	0	229	168	210	58
N.S.	1	1.00	0.17	0.14	0.00	0.82	0.60	0.75	0.21
time (sec)	N/A	0.145	0.011	0.028	0.000	0.357	0.095	4.346	1.860

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	49	82	0	189	76	81	56
N.S.	1	1.00	0.55	0.92	0.00	2.12	0.85	0.91	0.63
time (sec)	N/A	0.059	0.012	0.036	0.000	0.369	0.093	3.562	0.099

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	47	46	0	622	32	258	479
N.S.	1	1.00	0.13	0.12	0.00	1.68	0.09	0.70	1.29
time (sec)	N/A	0.178	0.010	0.036	0.000	0.395	1.415	2.963	0.068

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	283	286	0	1017	0	295	2490
N.S.	1	1.00	1.01	1.02	0.00	3.63	0.00	1.05	8.89
time (sec)	N/A	0.391	0.161	0.235	0.000	18.388	0.000	3.416	6.206

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	218	208	0	788	0	224	2051
N.S.	1	1.00	1.00	0.95	0.00	3.61	0.00	1.03	9.41
time (sec)	N/A	0.254	0.121	0.248	0.000	10.057	0.000	3.638	5.242

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	178	164	0	581	0	185	1367
N.S.	1	1.00	1.01	0.93	0.00	3.30	0.00	1.05	7.77
time (sec)	N/A	0.185	0.134	0.248	0.000	3.314	0.000	3.261	4.339

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	132	130	0	405	0	149	966
N.S.	1	1.00	0.89	0.87	0.00	2.72	0.00	1.00	6.48
time (sec)	N/A	0.139	0.086	0.248	0.000	1.168	0.000	3.317	3.668

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	107	105	0	309	0	127	801
N.S.	1	1.00	0.86	0.85	0.00	2.49	0.00	1.02	6.46
time (sec)	N/A	0.097	0.052	0.214	0.000	0.535	0.000	3.372	3.407

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	105	104	0	313	0	126	521
N.S.	1	1.00	0.85	0.85	0.00	2.54	0.00	1.02	4.24
time (sec)	N/A	0.069	0.054	0.210	0.000	0.550	0.000	3.767	3.817

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	159	152	160	0	508	0	164	2399
N.S.	1	1.01	0.96	1.01	0.00	3.22	0.00	1.04	15.18
time (sec)	N/A	0.173	0.133	0.203	0.000	177.699	0.000	4.422	5.400

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	194	207	0	0	0	210	2388
N.S.	1	1.00	1.01	1.07	0.00	0.00	0.00	1.09	12.37
time (sec)	N/A	0.233	0.123	0.276	0.000	0.000	0.000	4.506	20.389

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	252	277	0	0	0	279	2500
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	1.11	9.92
time (sec)	N/A	0.293	0.161	0.237	0.000	0.000	0.000	4.436	26.162

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	338	360	0	2689	0	565	2500
N.S.	1	1.00	0.99	1.05	0.00	7.84	0.00	1.65	7.29
time (sec)	N/A	0.596	0.248	0.280	0.000	62.844	0.000	3.538	8.039

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	269	290	0	2131	0	476	2495
N.S.	1	1.00	0.98	1.06	0.00	7.78	0.00	1.74	9.11
time (sec)	N/A	0.370	0.204	0.313	0.000	26.559	0.000	3.690	6.003

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	207	228	0	1439	0	412	2037
N.S.	1	1.00	0.84	0.93	0.00	5.85	0.00	1.67	8.28
time (sec)	N/A	0.265	0.155	0.269	0.000	11.038	0.000	2.931	5.112

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	159	188	0	1099	0	331	1585
N.S.	1	1.00	0.82	0.97	0.00	5.66	0.00	1.71	8.17
time (sec)	N/A	0.202	0.161	0.287	0.000	3.856	0.000	5.457	6.091

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	148	187	0	1057	0	323	1768
N.S.	1	1.00	0.81	1.02	0.00	5.78	0.00	1.77	9.66
time (sec)	N/A	0.153	0.173	0.255	0.000	3.503	0.000	4.188	8.070

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	151	197	0	1071	0	331	1782
N.S.	1	1.00	0.80	1.04	0.00	5.67	0.00	1.75	9.43
time (sec)	N/A	0.205	0.149	0.299	0.000	1.939	0.000	4.134	8.109

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	249	246	281	0	0	0	391	2500
N.S.	1	1.00	0.99	1.13	0.00	0.00	0.00	1.58	10.08
time (sec)	N/A	0.271	0.177	0.336	0.000	0.000	0.000	4.066	25.284

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	287	346	0	0	0	487	2500
N.S.	1	1.00	0.99	1.19	0.00	0.00	0.00	1.67	8.59
time (sec)	N/A	0.375	0.237	0.283	0.000	0.000	0.000	3.839	31.159

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	372	372	370	455	0	0	0	587	2500
N.S.	1	1.00	0.99	1.22	0.00	0.00	0.00	1.58	6.72
time (sec)	N/A	0.548	0.296	0.331	0.000	0.000	0.000	3.929	45.611

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	981	981	10904	11938	0	894	0	0	-1
N.S.	1	1.00	11.12	12.17	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	3.919	33.370	0.280	0.000	0.157	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	778	778	7531	9182	0	710	0	0	-1
N.S.	1	1.00	9.68	11.80	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	1.564	32.993	0.186	0.000	0.107	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	636	636	851	6302	0	581	0	0	-1
N.S.	1	1.00	1.34	9.91	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.672	32.361	0.185	0.000	0.144	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	1046	4361	0	475	0	0	-1
N.S.	1	1.00	1.90	7.93	0.00	0.86	0.00	0.00	-0.00
time (sec)	N/A	0.474	28.291	0.232	0.000	0.107	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	955	955	1258	3023	0	0	0	0	-1
N.S.	1	1.00	1.32	3.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.131	27.551	0.230	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	929	929	1372	3553	0	0	0	0	-1
N.S.	1	1.00	1.48	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.870	29.594	0.233	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1287	1287	1392	4957	0	0	0	0	-1
N.S.	1	1.00	1.08	3.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.572	31.663	0.238	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.011	0.180	0.181	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	249	0	0	0	0	0	-1
N.S.	1	1.00	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.161	0.388	0.066	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	189	0	0	0	0	0	-1
N.S.	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.102	0.213	0.064	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	136	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.110	0.055	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.117	0.086	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.207	0.156	0.093	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.286	0.464	0.080	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	201	15	14	1234	1326	216	1203
N.S.	1	1.00	12.56	0.94	0.88	77.12	82.88	13.50	75.19
time (sec)	N/A	0.037	0.121	0.306	0.283	0.355	0.141	3.202	3.341

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	233	17	1240	1240	1384	246	1210
N.S.	1	1.00	12.94	0.94	68.89	68.89	76.89	13.67	67.22
time (sec)	N/A	0.205	0.120	0.088	0.301	0.350	0.173	3.601	3.234

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	233	17	1240	1240	1394	246	1210
N.S.	1	1.00	12.94	0.94	68.89	68.89	77.44	13.67	67.22
time (sec)	N/A	0.191	0.122	0.108	0.289	0.365	0.184	3.742	3.183

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	260	2042	2041	1297	0	1693	1395
N.S.	1	1.00	11.30	88.78	88.74	56.39	0.00	73.61	60.65
time (sec)	N/A	0.034	0.386	0.070	0.333	0.399	0.000	4.377	5.778

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	201	17	16	1238	1326	218	1208
N.S.	1	1.00	11.17	0.94	0.89	68.78	73.67	12.11	67.11
time (sec)	N/A	0.042	0.119	0.235	0.277	0.360	0.150	2.092	1.376

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	233	19	1242	1242	1384	246	1214
N.S.	1	1.00	11.65	0.95	62.10	62.10	69.20	12.30	60.70
time (sec)	N/A	0.212	0.119	0.074	0.286	0.399	0.175	4.803	3.251

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	233	19	1242	1242	1394	246	1214
N.S.	1	1.00	11.65	0.95	62.10	62.10	69.70	12.30	60.70
time (sec)	N/A	0.198	0.120	0.092	0.292	0.334	0.147	5.499	1.282

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	260	2046	2045	1299	0	1693	1401
N.S.	1	1.00	10.40	81.84	81.80	51.96	0.00	67.72	56.04
time (sec)	N/A	0.040	0.401	0.053	0.354	0.400	0.000	3.775	5.776

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	172	14	13	154	175	13	154
N.S.	1	1.00	11.47	0.93	0.87	10.27	11.67	0.87	10.27
time (sec)	N/A	0.010	0.004	0.199	0.286	0.331	0.050	2.895	2.088

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	182	24	156	156	182	15	156
N.S.	1	1.00	11.38	1.50	9.75	9.75	11.38	0.94	9.75
time (sec)	N/A	0.036	0.005	0.226	0.291	0.334	0.052	4.073	2.080

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	186	24	156	156	185	15	156
N.S.	1	1.00	11.62	1.50	9.75	9.75	11.56	0.94	9.75
time (sec)	N/A	0.034	0.005	0.256	0.276	0.325	0.038	3.499	2.079

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	230	229	189	0	189	229
N.S.	1	1.00	1.00	10.95	10.90	9.00	0.00	9.00	10.90
time (sec)	N/A	0.020	0.034	0.184	0.287	0.373	0.000	4.667	2.628

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	10	12	11	11	10	12	11
N.S.	1	1.00	0.91	1.09	1.00	1.00	0.91	1.09	1.00
time (sec)	N/A	0.003	0.003	0.339	0.277	0.352	0.060	4.389	1.962

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.012	0.004	0.014	0.293	0.336	0.141	5.366	1.956

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	15	15	14	16	15
N.S.	1	1.00	1.00	0.94	0.88	0.88	0.82	0.94	0.88
time (sec)	N/A	0.014	0.005	0.015	0.291	0.396	0.240	3.435	0.052

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	23	19	0	19	121
N.S.	1	1.00	1.00	1.26	1.21	1.00	0.00	1.00	6.37
time (sec)	N/A	0.017	0.042	0.032	0.309	0.335	0.000	3.378	2.317

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	14	350	359	14	358
N.S.	1	1.00	0.94	0.94	0.88	21.88	22.44	0.88	22.38
time (sec)	N/A	0.003	0.009	0.264	0.269	0.391	3.260	2.926	3.616

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
N.S.	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.012	0.010	0.201	0.403	0.392	4.881	5.944	12.162

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	352	352	360	16	360
N.S.	1	1.00	1.00	0.94	19.56	19.56	20.00	0.89	20.00
time (sec)	N/A	0.014	0.010	0.085	0.400	0.389	15.872	6.161	18.211

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	416	394	0	21	496
N.S.	1	1.00	1.00	0.96	18.09	17.13	0.00	0.91	21.57
time (sec)	N/A	0.017	0.065	0.071	0.738	0.395	0.000	3.149	23.011

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	12	14	13	13	10	14	13
N.S.	1	1.00	0.92	1.08	1.00	1.00	0.77	1.08	1.00
time (sec)	N/A	0.003	0.004	0.224	0.275	0.337	0.064	2.770	0.049

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.012	0.006	0.017	0.279	0.341	0.167	3.797	0.049

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	17	17	14	18	17
N.S.	1	1.00	1.00	0.95	0.89	0.89	0.74	0.95	0.89
time (sec)	N/A	0.016	0.006	0.017	0.272	0.325	0.210	4.081	0.059

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	26	25	21	0	21	199
N.S.	1	1.00	1.00	1.24	1.19	1.00	0.00	1.00	9.48
time (sec)	N/A	0.018	0.044	0.030	0.325	0.352	0.000	3.708	2.676

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	16	17	16	354	359	16	358
N.S.	1	1.00	0.89	0.94	0.89	19.67	19.94	0.89	19.89
time (sec)	N/A	0.003	0.010	0.260	0.283	0.356	3.201	4.946	5.221

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
N.S.	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.013	0.013	0.207	0.406	0.367	5.170	6.007	11.044

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	356	356	360	18	360
N.S.	1	1.00	1.00	0.95	17.80	17.80	18.00	0.90	18.00
time (sec)	N/A	0.016	0.012	0.085	0.423	0.408	15.965	6.779	16.597

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	419	397	0	23	496
N.S.	1	1.00	1.00	0.96	16.76	15.88	0.00	0.92	19.84
time (sec)	N/A	0.021	0.070	0.063	0.787	0.363	0.000	2.649	22.399

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	8
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.80
time (sec)	N/A	0.003	0.004	0.220	0.281	0.341	0.045	3.949	0.051

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	17	13	12	15	13
N.S.	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.016	0.006	0.152	0.273	0.339	0.100	4.410	0.064

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	15	15	14	17	13	12	15	13
N.S.	1	0.94	0.94	0.88	1.06	0.81	0.75	0.94	0.81
time (sec)	N/A	0.019	0.005	0.161	0.268	0.341	0.085	4.921	1.991

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	19	18	47	17	54	17	28
N.S.	1	1.00	1.27	1.20	3.13	1.13	3.60	1.13	1.87
time (sec)	N/A	0.023	0.025	0.242	0.291	0.346	6.885	5.583	2.225

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	14	177	13	81	87	13	12
N.S.	1	1.00	0.93	11.80	0.87	5.40	5.80	0.87	0.80
time (sec)	N/A	0.003	0.015	0.163	0.273	0.360	0.465	2.994	4.296

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.014	0.021	0.161	0.294	0.347	0.671	3.457	2.332

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	197	81	81	87	15	14
N.S.	1	1.00	1.00	12.31	5.06	5.06	5.44	0.94	0.88
time (sec)	N/A	0.016	0.025	0.224	0.308	0.344	0.930	3.796	5.065

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	203	612	105	0	20	107
N.S.	1	1.00	1.00	9.67	29.14	5.00	0.00	0.95	5.10
time (sec)	N/A	0.020	0.071	0.193	0.348	0.387	0.000	3.643	2.357

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	19	21	20	28	104	20	39
N.S.	1	1.00	0.95	1.05	1.00	1.40	5.20	1.00	1.95
time (sec)	N/A	0.004	0.045	0.220	0.276	0.346	33.788	4.741	2.037

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	201	23	49
N.S.	1	1.00	1.00	0.96	1.32	1.32	8.04	0.92	1.96
time (sec)	N/A	0.014	0.038	0.023	0.322	0.380	129.160	3.586	2.092

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	33	33	0	23	49
N.S.	1	1.00	1.00	0.96	1.32	1.32	0.00	0.92	1.96
time (sec)	N/A	0.015	0.055	0.027	0.309	0.350	0.000	2.339	2.115

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	40	39	38	0	27	56
N.S.	1	1.00	0.96	1.48	1.44	1.41	0.00	1.00	2.07
time (sec)	N/A	0.019	0.138	0.041	0.359	0.359	0.000	2.494	2.569

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	23	22	32	104	22	42
N.S.	1	1.00	0.95	1.05	1.00	1.45	4.73	1.00	1.91
time (sec)	N/A	0.003	0.047	0.274	0.274	0.365	34.757	4.517	2.049

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	201	25	52
N.S.	1	1.00	1.00	0.96	1.37	1.37	7.44	0.93	1.93
time (sec)	N/A	0.013	0.041	0.021	0.309	0.341	130.390	4.209	2.047

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	37	37	0	25	52
N.S.	1	1.00	1.00	0.96	1.37	1.37	0.00	0.93	1.93
time (sec)	N/A	0.016	0.057	0.031	0.313	0.337	0.000	3.850	2.079

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	45	43	42	0	29	59
N.S.	1	1.00	0.97	1.55	1.48	1.45	0.00	1.00	2.03
time (sec)	N/A	0.019	0.148	0.046	0.341	0.370	0.000	2.401	2.536

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	17	20	19	26	46	19	23
N.S.	1	1.00	0.89	1.05	1.00	1.37	2.42	1.00	1.21
time (sec)	N/A	0.003	0.034	0.170	0.277	0.347	0.250	4.965	2.034

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	35	31	75	22	31
N.S.	1	1.00	1.00	1.29	1.46	1.29	3.12	0.92	1.29
time (sec)	N/A	0.010	0.065	0.158	0.311	0.344	10.044	3.915	2.074

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	35	31	0	22	31
N.S.	1	1.00	1.00	1.29	1.46	1.29	0.00	0.92	1.29
time (sec)	N/A	0.013	0.067	0.150	0.304	0.365	0.000	3.623	2.069

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	155	40	36	0	26	34
N.S.	1	1.00	0.92	5.96	1.54	1.38	0.00	1.00	1.31
time (sec)	N/A	0.052	0.109	0.224	0.347	0.360	0.000	3.694	2.126

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	318	0	0	0	0	0	-1
N.S.	1	1.00	1.62	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.186	0.639	0.012	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	374	374	5363	0	0	0	0	0	-1
N.S.	1	1.00	14.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.881	6.657	0.011	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	816	816	20515	0	0	0	0	0	-1
N.S.	1	1.00	25.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.520	7.535	0.013	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	36	34	33	119	0	31
N.S.	1	1.00	1.00	0.77	0.72	0.70	2.53	0.00	0.66
time (sec)	N/A	0.038	0.048	0.013	0.276	0.320	2.151	0.000	2.463

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.187	0.057	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	0.181	0.041	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.130	0.158	0.040	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.119	0.040	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	218	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.527	0.574	0.039	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.160	0.041	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.158	0.153	0.041	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	391	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.403	0.803	0.036	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	273	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.486	0.024	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	181	0	0	0	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.219	0.025	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.243	0.052	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.303	0.053	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [144] had the largest ratio of [59]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	22	0.045
2	A	2	1	1.00	22	0.045
3	A	2	1	1.00	22	0.045
4	A	2	1	1.00	22	0.045
5	A	2	1	1.00	20	0.050
6	A	8	8	1.00	22	0.364
7	A	8	8	1.00	22	0.364
8	A	8	8	1.00	22	0.364
9	A	7	6	1.00	25	0.240
10	A	6	6	1.00	25	0.240
11	A	5	5	1.00	25	0.200
12	A	7	6	1.00	25	0.240
13	A	7	6	1.00	25	0.240
14	A	14	8	1.00	25	0.320
15	A	14	8	1.00	25	0.320
16	A	13	7	1.00	23	0.304
17	A	13	7	1.00	22	0.318
18	A	14	8	1.00	25	0.320
19	A	14	8	1.00	25	0.320
20	A	7	6	1.00	23	0.261
21	A	4	4	1.00	23	0.174
22	A	5	5	1.00	23	0.217
23	A	7	6	1.00	23	0.261
24	A	5	4	1.00	23	0.174
25	A	15	9	1.00	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	15	9	1.00	23	0.391
27	A	14	8	1.00	23	0.348
28	A	13	7	1.00	21	0.333
29	A	13	7	1.00	20	0.350
30	A	14	8	1.00	23	0.348
31	A	15	9	1.00	23	0.391
32	A	5	5	1.00	21	0.238
33	A	7	6	1.00	21	0.286
34	A	8	7	1.00	18	0.389
35	A	6	4	1.00	24	0.167
36	A	5	4	1.00	24	0.167
37	A	4	4	1.00	24	0.167
38	A	3	3	1.00	24	0.125
39	A	3	3	1.00	24	0.125
40	A	3	3	1.00	24	0.125
41	A	4	4	1.00	24	0.167
42	A	5	4	1.00	24	0.167
43	A	8	5	1.00	25	0.200
44	A	5	5	1.00	25	0.200
45	A	7	4	1.00	25	0.160
46	A	4	3	1.00	23	0.130
47	A	7	4	1.00	22	0.182
48	A	7	6	1.00	25	0.240
49	A	8	5	1.00	25	0.200
50	A	5	4	1.00	25	0.160
51	A	8	5	1.00	25	0.200
52	A	20	7	1.00	23	0.304
53	A	5	5	1.00	23	0.217
54	A	21	7	1.00	23	0.304
55	A	4	3	1.00	21	0.143
56	A	19	6	1.00	20	0.300
57	A	7	6	1.00	23	0.261
58	A	20	7	1.00	23	0.304
59	A	11	8	1.00	23	0.348
60	A	21	9	1.00	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	6	1.00	25	0.240
62	A	7	6	1.00	25	0.240
63	A	7	6	1.00	23	0.261
64	A	7	6	1.00	22	0.273
65	A	7	6	1.00	25	0.240
66	A	7	7	1.00	25	0.280
67	A	7	6	1.01	25	0.240
68	A	7	6	1.00	25	0.240
69	A	7	6	1.00	25	0.240
70	A	7	6	1.00	25	0.240
71	A	7	6	1.00	25	0.240
72	A	7	6	1.00	23	0.261
73	A	7	6	1.00	22	0.273
74	A	7	6	1.00	25	0.240
75	A	8	7	1.00	25	0.280
76	A	7	6	1.00	25	0.240
77	A	7	6	1.00	25	0.240
78	A	7	6	1.00	25	0.240
79	A	11	7	1.00	29	0.241
80	A	10	7	1.00	29	0.241
81	A	8	7	1.00	29	0.241
82	A	8	7	1.00	27	0.259
83	A	16	11	1.00	26	0.423
84	A	16	11	1.00	29	0.379
85	A	24	12	1.00	29	0.414
86	A	0	0	0.00	0	0.000
87	A	13	4	1.00	26	0.154
88	A	10	4	1.00	26	0.154
89	A	7	4	1.00	24	0.167
90	A	6	3	1.00	26	0.115
91	A	8	3	1.00	26	0.115
92	A	10	3	1.00	26	0.115
93	A	1	1	1.00	19	0.053
94	A	2	2	1.00	24	0.083
95	A	2	2	1.00	26	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.00	30	0.067
97	A	1	1	1.00	21	0.048
98	A	2	2	1.00	26	0.077
99	A	2	2	1.00	28	0.071
100	A	2	2	1.00	32	0.062
101	A	1	1	1.00	18	0.056
102	A	3	3	1.00	23	0.130
103	A	3	3	1.00	25	0.120
104	A	3	3	1.00	29	0.103
105	A	1	1	1.00	19	0.053
106	A	2	2	1.00	24	0.083
107	A	2	2	1.00	26	0.077
108	A	2	2	1.00	30	0.067
109	A	1	1	1.00	19	0.053
110	A	2	2	1.00	24	0.083
111	A	2	2	1.00	26	0.077
112	A	2	2	1.00	30	0.067
113	A	1	1	1.00	21	0.048
114	A	2	2	1.00	26	0.077
115	A	2	2	1.00	28	0.071
116	A	2	2	1.00	32	0.062
117	A	1	1	1.00	21	0.048
118	A	2	2	1.00	26	0.077
119	A	2	2	1.00	28	0.071
120	A	2	2	1.00	32	0.062
121	A	1	1	1.00	18	0.056
122	A	4	3	0.94	23	0.130
123	A	4	3	0.94	25	0.120
124	A	4	3	1.00	29	0.103
125	A	1	1	1.00	18	0.056
126	A	3	3	1.00	23	0.130
127	A	3	3	1.00	25	0.120
128	A	3	3	1.00	29	0.103
129	A	1	1	1.00	19	0.053
130	A	2	2	1.00	24	0.083

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	2	2	1.00	26	0.077
132	A	2	2	1.00	30	0.067
133	A	1	1	1.00	21	0.048
134	A	2	2	1.00	26	0.077
135	A	2	2	1.00	28	0.071
136	A	2	2	1.00	32	0.062
137	A	1	1	1.00	18	0.056
138	A	1	1	1.00	23	0.043
139	A	1	1	1.00	25	0.040
140	A	2	2	1.00	29	0.069
141	A	4	2	1.00	29	0.069
142	A	5	3	1.00	29	0.103
143	A	6	3	1.00	29	0.103
144	A	3	3	1.00	59	0.051
145	A	5	3	1.00	31	0.097
146	A	5	3	1.00	29	0.103
147	A	5	3	1.00	27	0.111
148	A	5	3	1.00	26	0.115
149	A	8	5	1.00	29	0.172
150	A	5	3	1.00	29	0.103
151	A	5	3	1.00	29	0.103
152	A	10	4	1.00	31	0.129
153	A	7	4	1.00	29	0.138
154	A	2	2	1.00	22	0.091
155	A	0	0	0.00	0	0.000
156	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int (d+ex^3)^5 (a+bx^3+cx^6)\, dx$

Optimal. Leaf size=163

$$ad^5x+\frac{1}{4}d^4(bd+5ae)x^4+\frac{1}{7}d^3\left(cd^2+5e(bd+2ae)\right)x^7+\frac{1}{2}d^2e\left(cd^2+2e(bd+ae)\right)x^{10}+\frac{5}{13}de^2\left(2cd^2+e(2bd+ae\right.$$

```
[Out] a*d^5*x+1/4*d^4*(5*a*e+b*d)*x^4+1/7*d^3*(c*d^2+5*e*(2*a*e+b*d))*x^7+1/2*d^2
*e*(c*d^2+2*e*(a*e+b*d))*x^10+5/13*d*e^2*(2*c*d^2+e*(a*e+2*b*d))*x^13+1/16*
e^3*(10*c*d^2+e*(a*e+5*b*d))*x^16+1/19*e^4*(b*e+5*c*d)*x^19+1/22*c*e^5*x^22
```

Rubi [A]
time = 0.13, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{1421\}$

$$\frac{1}{16}e^3x^{16}\left(e(ae+5bd)+10cd^2\right)+\frac{5}{13}de^2x^{13}\left(e(ae+2bd)+2cd^2\right)+\frac{1}{2}d^2ex^{10}\left(2e(ae+bd)+cd^2\right)+\frac{1}{7}d^3x^7\left(5e(2ae+bd)+cd^2\right)+\frac{1}{4}d^4x^4(5ae+bd)+ad^5x+\frac{1}{19}e^4x^{19}(be+5cd)+\frac{1}{22}ce^5x^{22}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*e*(b*d + 2*a*e))*x^7)
/7 + (d^2*e*(c*d^2 + 2*e*(b*d + a*e))*x^10)/2 + (5*d*e^2*(2*c*d^2 + e*(2*b*
d + a*e))*x^13)/13 + (e^3*(10*c*d^2 + e*(5*b*d + a*e))*x^16)/16 + (e^4*(5*c
*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (d+ex^3)^5 \left(a+bx^3+cx^6\right) dx = \int \left(ad^5+d^4(bd+5ae)x^3+d^3\left(cd^2+5e(bd+2ae)\right)x^6+5d^2e\left(cd^2+2e(b\right.\right.$$
$$= ad^5x+\frac{1}{4}d^4(bd+5ae)x^4+\frac{1}{7}d^3\left(cd^2+5e(bd+2ae)\right)x^7+\frac{1}{2}d^2e\left(cd^2+2e(\right.$$

Mathematica [A]
time = 0.04, size = 164, normalized size = 1.01

$$ad^5x+\frac{1}{4}d^4(bd+5ae)x^4+\frac{1}{7}d^3\left(cd^2+5bde+10ae^2\right)x^7+\frac{1}{2}d^2e\left(cd^2+2bde+2ae^2\right)x^{10}+\frac{5}{13}de^2\left(2cd^2+2bde+ae^2\right)x^{13}+\frac{1}{16}e^3\left(10cd^2+5bde+ae^2\right)x^{16}+\frac{1}{19}e^4(5cd+be)x^{19}+\frac{1}{22}ce^5x^{22}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^5*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^5*x + (d^4*(b*d + 5*a*e)*x^4)/4 + (d^3*(c*d^2 + 5*b*d*e + 10*a*e^2)*x^7
)/7 + (d^2*e*(c*d^2 + 2*b*d*e + 2*a*e^2)*x^10)/2 + (5*d*e^2*(2*c*d^2 + 2*b*
d*e + a*e^2)*x^13)/13 + (e^3*(10*c*d^2 + 5*b*d*e + a*e^2)*x^16)/16 + (e^4*(
5*c*d + b*e)*x^19)/19 + (c*e^5*x^22)/22
```

Maple [A]
time = 0.22, size = 169, normalized size = 1.04

method	result
norman	$a\,d^5x+\left(\frac{5}{4}d^4ea+\frac{1}{4}d^5b\right)x^4+\left(\frac{10}{7}a\,d^3e^2+\frac{5}{7}d^4eb+\frac{1}{7}d^5c\right)x^7+\left(a\,d^2e^3+d^3e^2b+\frac{1}{2}c\,d^4e\right)x^{10}+\left(\frac{5}{13}\right.$
default	$\frac{c\,e^5x^{22}}{22}+\frac{\left(e^5b+5d\,e^4c\right)x^{19}}{19}+\frac{\left(e^5a+5bd\,e^4+10d^2e^3c\right)x^{16}}{16}+\frac{\left(5d\,e^4a+10d^2e^3b+10d^3e^2c\right)x^{13}}{13}+\frac{\left(10a\,d^2e^3+10d^3e^2b+5c\,d^4e\right)x^{1}}{10}$
gosper	$a\,d^5x+\frac{5}{4}x^4d^4ea+\frac{1}{4}x^4d^5b+\frac{10}{7}x^7a\,d^3e^2+\frac{5}{7}x^7d^4eb+\frac{1}{7}x^7d^5c+x^{10}a\,d^2e^3+x^{10}d^3e^2b+\frac{1}{2}x^{10}c\,d^4e$
risch	$a\,d^5x+\frac{5}{4}x^4d^4ea+\frac{1}{4}x^4d^5b+\frac{10}{7}x^7a\,d^3e^2+\frac{5}{7}x^7d^4eb+\frac{1}{7}x^7d^5c+x^{10}a\,d^2e^3+x^{10}d^3e^2b+\frac{1}{2}x^{10}c\,d^4e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^5*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/22*c*e^5*x^22+1/19*(b*e^5+5*c*d*e^4)*x^19+1/16*(a*e^5+5*b*d*e^4+10*c*d^2*
e^3)*x^16+1/13*(5*a*d*e^4+10*b*d^2*e^3+10*c*d^3*e^2)*x^13+1/10*(10*a*d^2*e^
3+10*b*d^3*e^2+5*c*d^4*e)*x^10+1/7*(10*a*d^3*e^2+5*b*d^4*e+c*d^5)*x^7+1/4*(
5*a*d^4*e+b*d^5)*x^4+a*d^5*x
```

Maxima [A]
time = 0.27, size = 157, normalized size = 0.96

$$\frac{1}{22}\,cx^{22}e^5+\frac{1}{19}\left(5\,cde^4+be^5\right)x^{19}+\frac{1}{16}\left(10\,cd^2e^3+5\,bde^4+ae^5\right)x^{16}+\frac{5}{13}\left(2\,cd^3e^2+2\,bd^2e^3+ade^4\right)x^{13}+\frac{1}{2}\left(cd^4e+2\,bd^3e^2+2\,ad^2e^3\right)x^{10}+\frac{1}{7}\left(cd^5+5\,bd^4e+10\,ad^3e^2\right)x^7+ad^5x+\frac{1}{4}\left(bd^5+5\,ad^4e\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/22*c*x^22*e^5 + 1/19*(5*c*d*e^4 + b*e^5)*x^19 + 1/16*(10*c*d^2*e^3 + 5*b*
d*e^4 + a*e^5)*x^16 + 5/13*(2*c*d^3*e^2 + 2*b*d^2*e^3 + a*d*e^4)*x^13 + 1/2
*(c*d^4*e + 2*b*d^3*e^2 + 2*a*d^2*e^3)*x^10 + 1/7*(c*d^5 + 5*b*d^4*e + 10*a
*d^3*e^2)*x^7 + a*d^5*x + 1/4*(b*d^5 + 5*a*d^4*e)*x^4
```

Fricas [A]
time = 0.32, size = 170, normalized size = 1.04

$$\frac{1}{7}\,cd^5x^7+\frac{1}{4}\,bd^5x^4+ad^5x+\frac{1}{3344}\left(152\,cx^{22}+176\,bx^{19}+209\,ax^{16}\right)e^5+\frac{5}{3952}\left(208\,cdx^{19}+247\,bdx^{16}+304\,adx^{13}\right)e^4+\frac{1}{104}\left(65\,cd^2x^{16}+80\,bd^2x^{13}+104\,ad^2x^{10}\right)e^3+\frac{1}{91}\left(70\,cd^3x^{13}+91\,bd^3x^{10}+130\,ad^3x^7\right)e^2+\frac{1}{28}\left(14\,cd^4x^{10}+20\,bd^4x^7+35\,ad^4x^4\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/7*c*d^5*x^7 + 1/4*b*d^5*x^4 + a*d^5*x + 1/3344*(152*c*x^22 + 176*b*x^19 +
 209*a*x^16)*e^5 + 5/3952*(208*c*d*x^19 + 247*b*d*x^16 + 304*a*d*x^13)*e^4
+ 1/104*(65*c*d^2*x^16 + 80*b*d^2*x^13 + 104*a*d^2*x^10)*e^3 + 1/91*(70*c*d
^3*x^13 + 91*b*d^3*x^10 + 130*a*d^3*x^7)*e^2 + 1/28*(14*c*d^4*x^10 + 20*b*d
^4*x^7 + 35*a*d^4*x^4)*e
```

Sympy [A]
time = 0.02, size = 187, normalized size = 1.15

$$ad^5x + \frac{ce^5x^{22}}{22} + x^{19}\left(\frac{be^5}{19} + \frac{5cde^4}{19}\right) + x^{16}\left(\frac{ae^5}{16} + \frac{5bde^4}{16} + \frac{5cd^2e^3}{8}\right) + x^{13}\cdot\left(\frac{5ade^4}{13} + \frac{10bd^2e^3}{13} + \frac{10cd^3e^2}{13}\right) + x^{10}\left(ad^2e^3 + bd^3e^2 + \frac{cd^4e}{2}\right) + x^7\cdot\left(\frac{10ad^3e^2}{7} + \frac{5bd^4e}{7} + \frac{cd^5}{7}\right) + x^4\cdot\left(\frac{5ad^4e}{4} + \frac{bd^5}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**5*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**5*x + c*e**5*x**22/22 + x**19*(b*e**5/19 + 5*c*d*e**4/19) + x**16*(a*e
**5/16 + 5*b*d*e**4/16 + 5*c*d**2*e**3/8) + x**13*(5*a*d*e**4/13 + 10*b*d**
2*e**3/13 + 10*c*d**3*e**2/13) + x**10*(a*d**2*e**3 + b*d**3*e**2 + c*d**4*
e/2) + x**7*(10*a*d**3*e**2/7 + 5*b*d**4*e/7 + c*d**5/7) + x**4*(5*a*d**4*e
/4 + b*d**5/4)
```

Giac [A]
time = 3.35, size = 173, normalized size = 1.06

$$\frac{1}{22}\,cx^{22}e^5 + \frac{5}{19}\,cdx^{19}e^4 + \frac{1}{19}\,bx^{19}e^5 + \frac{5}{8}\,cd^2x^{16}e^3 + \frac{5}{16}\,bdx^{16}e^4 + \frac{1}{16}\,ax^{16}e^5 + \frac{10}{13}\,cd^3x^{13}e^2 + \frac{10}{13}\,bd^2x^{13}e^3 + \frac{5}{13}\,adx^{13}e^4 + \frac{1}{2}\,cd^4x^{10}e + bd^3x^{10}e^2 + ad^2x^{10}e^3 + \frac{1}{7}\,cd^5x^7 + \frac{5}{7}\,bd^4x^7e + \frac{10}{7}\,ad^3x^7e^2 + \frac{1}{4}\,bd^5x^4 + \frac{5}{4}\,ad^4x^4e + ad^5x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^5*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/22*c*x^22*e^5 + 5/19*c*d*x^19*e^4 + 1/19*b*x^19*e^5 + 5/8*c*d^2*x^16*e^3
+ 5/16*b*d*x^16*e^4 + 1/16*a*x^16*e^5 + 10/13*c*d^3*x^13*e^2 + 10/13*b*d^2*
x^13*e^3 + 5/13*a*d*x^13*e^4 + 1/2*c*d^4*x^10*e + b*d^3*x^10*e^2 + a*d^2*x^
10*e^3 + 1/7*c*d^5*x^7 + 5/7*b*d^4*x^7*e + 10/7*a*d^3*x^7*e^2 + 1/4*b*d^5*x
^4 + 5/4*a*d^4*x^4*e + a*d^5*x
```

Mupad [B]
time = 1.60, size = 158, normalized size = 0.97

$$x^4\,\left(\frac{b\,d^5}{4}+\frac{5\,a\,e\,d^4}{4}\right)+x^{19}\,\left(\frac{b\,e^5}{19}+\frac{5\,c\,d\,e^4}{19}\right)+x^7\,\left(\frac{c\,d^5}{7}+\frac{5\,b\,d^4\,e}{7}+\frac{10\,a\,d^3\,e^2}{7}\right)+x^{16}\,\left(\frac{5\,c\,d^2\,e^3}{8}+\frac{5\,b\,d\,e^4}{16}+\frac{a\,e^5}{16}\right)+\frac{c\,e^5\,x^{22}}{22}+a\,d^5\,x+\frac{d^2\,e\,x^{10}\,\left(c\,d^2+2\,b\,d\,e+2\,a\,e^2\right)}{2}+\frac{5\,d\,e^2\,x^{13}\,\left(2\,c\,d^2+2\,b\,d\,e+a\,e^2\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^5*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^5)/4 + (5*a*d^4*e)/4) + x^19*((b*e^5)/19 + (5*c*d*e^4)/19) + x^7*
((c*d^5)/7 + (10*a*d^3*e^2)/7 + (5*b*d^4*e)/7) + x^16*((a*e^5)/16 + (5*c*d^
2*e^3)/8 + (5*b*d*e^4)/16) + (c*e^5*x^22)/22 + a*d^5*x + (d^2*e*x^10*(2*a*e
^2 + c*d^2 + 2*b*d*e))/2 + (5*d*e^2*x^13*(a*e^2 + 2*c*d^2 + 2*b*d*e))/13
```

3.2 $\int (d+ex^3)^4 (a+bx^3+cx^6)\,dx$

Optimal. Leaf size=135

$$ad^4x+\frac{1}{4}d^3(bd+4ae)x^4+\frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7+\frac{1}{5}de\left(2cd^2+e(3bd+2ae)\right)x^{10}+\frac{1}{13}e^2\left(6cd^2+e(4bd+a$$

```
[Out] a*d^4*x+1/4*d^3*(4*a*e+b*d)*x^4+1/7*d^2*(6*a*e^2+4*b*d*e+c*d^2)*x^7+1/5*d*e
*(2*c*d^2+e*(2*a*e+3*b*d))*x^10+1/13*e^2*(6*c*d^2+e*(a*e+4*b*d))*x^13+1/16*
e^3*(b*e+4*c*d)*x^16+1/19*c*e^4*x^19
```

Rubi [A]
time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1421}

$$\frac{1}{13}e^2x^{13}\left(e(ae+4bd)+6cd^2\right)+\frac{1}{7}d^2x^7\left(6ae^2+4bde+cd^2\right)+\frac{1}{5}dex^{10}\left(e(2ae+3bd)+2cd^2\right)+\frac{1}{4}d^3x^4(4ae+bd)+ad^4x+\frac{1}{16}e^3x^{16}(be+4cd)+\frac{1}{19}ce^4x^{19}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + e*(3*b*d + 2*a*e))*x^10)/5 + (e^2*(6*c*d^2 + e*(4*b*d
+ a*e))*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^4\left(a+bx^3+cx^6\right)dx = \int \left(ad^4+d^3(bd+4ae)x^3+d^2\left(cd^2+4bde+6ae^2\right)x^6+2de\left(2cd^2+e(3\right.\right.$$

$$= ad^4x+\frac{1}{4}d^3(bd+4ae)x^4+\frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7+\frac{1}{5}de\left(2cd^2+e(\right.$$

Mathematica [A]
time = 0.03, size = 135, normalized size = 1.00

$$ad^4x+\frac{1}{4}d^3(bd+4ae)x^4+\frac{1}{7}d^2\left(cd^2+4bde+6ae^2\right)x^7+\frac{1}{5}de\left(2cd^2+3bde+2ae^2\right)x^{10}+\frac{1}{13}e^2\left(6cd^2+4bde+ae^2\right)x^{13}+\frac{1}{16}e^3(4cd+be)x^{16}+\frac{1}{19}ce^4x^{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^4*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^4*x + (d^3*(b*d + 4*a*e)*x^4)/4 + (d^2*(c*d^2 + 4*b*d*e + 6*a*e^2)*x^7)
/7 + (d*e*(2*c*d^2 + 3*b*d*e + 2*a*e^2)*x^10)/5 + (e^2*(6*c*d^2 + 4*b*d*e +
 a*e^2)*x^13)/13 + (e^3*(4*c*d + b*e)*x^16)/16 + (c*e^4*x^19)/19
```

Maple [A]
time = 0.25, size = 136, normalized size = 1.01

method	result
norman	$a\,d^4x+\left(d^3ea+\frac{1}{4}d^4b\right)x^4+\left(\frac{6}{7}d^2e^2a+\frac{4}{7}d^3eb+\frac{1}{7}d^4c\right)x^7+\left(\frac{2}{5}d\,e^3a+\frac{3}{5}d^2e^2b+\frac{2}{5}d^3ec\right)x^{10}+\left(\frac{1}{13}e^4c\right.$
default	$\frac{c\,e^4x^{19}}{19}+\frac{(e^4b+4d\,e^3c)x^{16}}{16}+\frac{(e^4a+4d\,e^3b+6d^2e^2c)x^{13}}{13}+\frac{(4d\,e^3a+6d^2e^2b+4d^3ec)x^{10}}{10}+\frac{(6d^2e^2a+4d^3eb+d^4c)x^7}{7}+\frac{(4d^3ea+}{4}$
gosper	$a\,d^4x+x^4d^3ea+\frac{1}{4}x^4d^4b+\frac{6}{7}x^7d^2e^2a+\frac{4}{7}x^7d^3eb+\frac{1}{7}x^7d^4c+\frac{2}{5}x^{10}d\,e^3a+\frac{3}{5}x^{10}d^2e^2b+\frac{2}{5}x^{10}d^3ec+$
risch	$a\,d^4x+x^4d^3ea+\frac{1}{4}x^4d^4b+\frac{6}{7}x^7d^2e^2a+\frac{4}{7}x^7d^3eb+\frac{1}{7}x^7d^4c+\frac{2}{5}x^{10}d\,e^3a+\frac{3}{5}x^{10}d^2e^2b+\frac{2}{5}x^{10}d^3ec+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^4*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/19*c*e^4*x^19+1/16*(b*e^4+4*c*d*e^3)*x^16+1/13*(a*e^4+4*b*d*e^3+6*c*d^2*e
^2)*x^13+1/10*(4*a*d*e^3+6*b*d^2*e^2+4*c*d^3*e)*x^10+1/7*(6*a*d^2*e^2+4*b*d
^3*e+c*d^4)*x^7+1/4*(4*a*d^3*e+b*d^4)*x^4+a*d^4*x
```

Maxima [A]
time = 0.30, size = 129, normalized size = 0.96

$$\frac{1}{19}\,cx^{19}e^4+\frac{1}{16}\,(4\,cde^3+be^4)x^{16}+\frac{1}{13}\,(6\,cd^2e^2+4\,bde^3+ae^4)x^{13}+\frac{1}{5}\,(2\,cd^3e+3\,bd^2e^2+2\,ade^3)x^{10}+\frac{1}{7}\,(cd^4+4\,bd^3e+6\,ad^2e^2)x^7+ad^4x+\frac{1}{4}\,(bd^4+4\,ad^3e)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/19*c*x^19*e^4 + 1/16*(4*c*d*e^3 + b*e^4)*x^16 + 1/13*(6*c*d^2*e^2 + 4*b*d
*e^3 + a*e^4)*x^13 + 1/5*(2*c*d^3*e + 3*b*d^2*e^2 + 2*a*d*e^3)*x^10 + 1/7*(
c*d^4 + 4*b*d^3*e + 6*a*d^2*e^2)*x^7 + a*d^4*x + 1/4*(b*d^4 + 4*a*d^3*e)*x^
4
```

Fricas [A]
time = 0.33, size = 138, normalized size = 1.02

$$\frac{1}{7}\,cd^4x^7+\frac{1}{4}\,bd^4x^4+ad^4x+\frac{1}{3952}\,(208\,cx^{19}+247\,bx^{16}+304\,ax^{13})e^4+\frac{1}{260}\,(65\,cdx^{16}+80\,bdx^{13}+104\,adx^{10})e^3+\frac{3}{455}\,(70\,cd^2x^{13}+91\,bd^2x^{10}+130\,ad^2x^7)e^2+\frac{1}{35}\,(14\,cd^3x^{10}+20\,bd^3x^7+35\,ad^3x^4)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/7*c*d^4*x^7 + 1/4*b*d^4*x^4 + a*d^4*x + 1/3952*(208*c*x^19 + 247*b*x^16 +
 304*a*x^13)*e^4 + 1/260*(65*c*d*x^16 + 80*b*d*x^13 + 104*a*d*x^10)*e^3 + 3
/455*(70*c*d^2*x^13 + 91*b*d^2*x^10 + 130*a*d^2*x^7)*e^2 + 1/35*(14*c*d^3*x
^10 + 20*b*d^3*x^7 + 35*a*d^3*x^4)*e
```

Sympy [A]
time = 0.02, size = 151, normalized size = 1.12

$$ad^4x + \frac{ce^4x^{19}}{19} + x^{16}\left(\frac{be^4}{16} + \frac{cde^3}{4}\right) + x^{13}\left(\frac{ae^4}{13} + \frac{4bde^3}{13} + \frac{6cd^2e^2}{13}\right) + x^{10}\cdot\left(\frac{2ade^3}{5} + \frac{3bd^2e^2}{5} + \frac{2cd^3e}{5}\right) + x^7\cdot\left(\frac{6ad^2e^2}{7} + \frac{4bd^3e}{7} + \frac{cd^4}{7}\right) + x^4\left(ad^3e + \frac{bd^4}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**4*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**4*x + c*e**4*x**19/19 + x**16*(b*e**4/16 + c*d*e**3/4) + x**13*(a*e**4
/13 + 4*b*d*e**3/13 + 6*c*d**2*e**2/13) + x**10*(2*a*d*e**3/5 + 3*b*d**2*e*
*2/5 + 2*c*d**3*e/5) + x**7*(6*a*d**2*e**2/7 + 4*b*d**3*e/7 + c*d**4/7) + x
**4*(a*d**3*e + b*d**4/4)
```

Giac [A]
time = 3.88, size = 141, normalized size = 1.04

$$\frac{1}{19}cx^{19}e^4 + \frac{1}{4}cdx^{16}e^3 + \frac{1}{16}bx^{16}e^4 + \frac{6}{13}cd^2x^{13}e^2 + \frac{4}{13}bdx^{13}e^3 + \frac{1}{13}ax^{13}e^4 + \frac{2}{5}cd^3x^{10}e + \frac{3}{5}bd^2x^{10}e^2 + \frac{2}{5}adx^{10}e^3 + \frac{1}{7}cd^4x^7 + \frac{4}{7}bd^3x^7e + \frac{6}{7}ad^2x^7e^2 + \frac{1}{4}bd^4x^4 + ad^3x^4e + ad^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^4*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/19*c*x^19*e^4 + 1/4*c*d*x^16*e^3 + 1/16*b*x^16*e^4 + 6/13*c*d^2*x^13*e^2
+ 4/13*b*d*x^13*e^3 + 1/13*a*x^13*e^4 + 2/5*c*d^3*x^10*e + 3/5*b*d^2*x^10*e
^2 + 2/5*a*d*x^10*e^3 + 1/7*c*d^4*x^7 + 4/7*b*d^3*x^7*e + 6/7*a*d^2*x^7*e^2
 + 1/4*b*d^4*x^4 + a*d^3*x^4*e + a*d^4*x
```

Mupad [B]
time = 0.06, size = 130, normalized size = 0.96

$$x^4\left(\frac{b\,d^4}{4} + a\,e\,d^3\right) + x^{16}\left(\frac{b\,e^4}{16} + \frac{c\,d\,e^3}{4}\right) + x^7\left(\frac{c\,d^4}{7} + \frac{4\,b\,d^3\,e}{7} + \frac{6\,a\,d^2\,e^2}{7}\right) + x^{13}\left(\frac{6\,c\,d^2\,e^2}{13} + \frac{4\,b\,d\,e^3}{13} + \frac{a\,e^4}{13}\right) + \frac{c\,e^4\,x^{19}}{19} + a\,d^4\,x + \frac{d\,e\,x^{10}\,\left(2\,c\,d^2 + 3\,b\,d\,e + 2\,a\,e^2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^4*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^4)/4 + a*d^3*e) + x^16*((b*e^4)/16 + (c*d*e^3)/4) + x^7*((c*d^4)/
7 + (6*a*d^2*e^2)/7 + (4*b*d^3*e)/7) + x^13*((a*e^4)/13 + (6*c*d^2*e^2)/13
+ (4*b*d*e^3)/13) + (c*e^4*x^19)/19 + a*d^4*x + (d*e*x^10*(2*a*e^2 + 2*c*d^
2 + 3*b*d*e))/5
```

3.3 $\int \left(d+ex^3\right)^3 \left(a+bx^3+cx^6\right) \, dx$

Optimal. Leaf size=103

$$ad^3x+\frac{1}{4}d^2(bd+3ae)x^4+\frac{1}{7}d\left(cd^2+3e(bd+ae)\right)x^7+\frac{1}{10}e\left(3cd^2+e(3bd+ae)\right)x^{10}+\frac{1}{13}e^2(3cd+be)x^{13}+\frac{1}{16}ce^3x^1$$

```
[Out] a*d^3*x+1/4*d^2*(3*a*e+b*d)*x^4+1/7*d*(c*d^2+3*e*(a*e+b*d))*x^7+1/10*e*(3*c
*d^2+e*(a*e+3*b*d))*x^10+1/13*e^2*(b*e+3*c*d)*x^13+1/16*c*e^3*x^16
```

Rubi [A]
time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1421}

$$\frac{1}{10}ex^{10}\left(e(ae+3bd)+3cd^2\right)+\frac{1}{7}dx^7\left(3e(ae+bd)+cd^2\right)+\frac{1}{4}d^2x^4(3ae+bd)+ad^3x+\frac{1}{13}e^2x^{13}(be+3cd)+\frac{1}{16}ce^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*e*(b*d + a*e))*x^7)/7 +
 (e*(3*c*d^2 + e*(3*b*d + a*e))*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 + (c
*e^3*x^16)/16
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^3\right)^3\left(a+bx^3+cx^6\right)\,dx &= \int \left(ad^3+d^2(bd+3ae)x^3+d\left(cd^2+3e(bd+ae)\right)x^6+e\left(3cd^2+e(3bd+\right.\right.\\ &= ad^3x+\frac{1}{4}d^2(bd+3ae)x^4+\frac{1}{7}d\left(cd^2+3e(bd+ae)\right)x^7+\frac{1}{10}e\left(3cd^2+e(3bc\right.\end{aligned}$$

Mathematica [A]
time = 0.02, size = 104, normalized size = 1.01

$$ad^3x+\frac{1}{4}d^2(bd+3ae)x^4+\frac{1}{7}d\left(cd^2+3bde+3ae^2\right)x^7+\frac{1}{10}e\left(3cd^2+3bde+ae^2\right)x^{10}+\frac{1}{13}e^2(3cd+be)x^{13}+\frac{1}{16}ce^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^3*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^3*x + (d^2*(b*d + 3*a*e)*x^4)/4 + (d*(c*d^2 + 3*b*d*e + 3*a*e^2)*x^7)/7
 + (e*(3*c*d^2 + 3*b*d*e + a*e^2)*x^10)/10 + (e^2*(3*c*d + b*e)*x^13)/13 +
(c*e^3*x^16)/16
```

Maple [A]
time = 0.28, size = 103, normalized size = 1.00

method	result
default	$\frac{c\,e^{3} x^{16}}{16}+\frac{\left(e^{3} b+3 d\,e^{2} c\right) x^{13}}{13}+\frac{\left(a\,e^{3}+3 d\,e^{2} b+3 c\,d^{2} e\right) x^{10}}{10}+\frac{\left(3 d\,e^{2} a+3 d^{2} e b+c\,d^{3}\right) x^{7}}{7}+\frac{\left(3 a\,d^{2} e+d^{3} b\right) x^{4}}{4}+a\,d^{3} x$
norman	$a\,d^{3} x+\left(\frac{3}{4} a\,d^{2} e+\frac{1}{4} d^{3} b\right) x^{4}+\left(\frac{3}{7} d\,e^{2} a+\frac{3}{7} d^{2} e b+\frac{1}{7} c\,d^{3}\right) x^{7}+\left(\frac{1}{10} a\,e^{3}+\frac{3}{10} d\,e^{2} b+\frac{3}{10} c\,d^{2} e\right) x^{10}+($
gosper	$a\,d^{3} x+\frac{3}{4} x^{4} a\,d^{2} e+\frac{1}{4} x^{4} d^{3} b+\frac{3}{7} x^{7} d\,e^{2} a+\frac{3}{7} x^{7} d^{2} e b+\frac{1}{7} x^{7} c\,d^{3}+\frac{1}{10} x^{10} a\,e^{3}+\frac{3}{10} x^{10} d\,e^{2} b+\frac{3}{10} x^{10} c\,d^{2}$
risch	$a\,d^{3} x+\frac{3}{4} x^{4} a\,d^{2} e+\frac{1}{4} x^{4} d^{3} b+\frac{3}{7} x^{7} d\,e^{2} a+\frac{3}{7} x^{7} d^{2} e b+\frac{1}{7} x^{7} c\,d^{3}+\frac{1}{10} x^{10} a\,e^{3}+\frac{3}{10} x^{10} d\,e^{2} b+\frac{3}{10} x^{10} c\,d^{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^3*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*c*e^3*x^16+1/13*(b*e^3+3*c*d*e^2)*x^13+1/10*(a*e^3+3*b*d*e^2+3*c*d^2*e
)*x^10+1/7*(3*a*d*e^2+3*b*d^2*e+c*d^3)*x^7+1/4*(3*a*d^2*e+b*d^3)*x^4+a*d^3*
x
```

Maxima [A]
time = 0.27, size = 99, normalized size = 0.96

$$\frac{1}{16}\,cx^{16}e^{3}+\frac{1}{13}\left(3\,cde^{2}+be^{3}\right)x^{13}+\frac{1}{10}\left(3\,cd^{2}e+3\,bde^{2}+ae^{3}\right)x^{10}+\frac{1}{7}\left(cd^{3}+3\,bd^{2}e+3\,ade^{2}\right)x^{7}+ad^{3}x+\frac{1}{4}\left(bd^{3}+3\,ad^{2}e\right)x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/16*c*x^16*e^3 + 1/13*(3*c*d*e^2 + b*e^3)*x^13 + 1/10*(3*c*d^2*e + 3*b*d*e
^2 + a*e^3)*x^10 + 1/7*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*x^7 + a*d^3*x + 1/4*
(b*d^3 + 3*a*d^2*e)*x^4
```

Fricas [A]
time = 0.39, size = 106, normalized size = 1.03

$$\frac{1}{7}\,cd^{3}x^{7}+\frac{1}{4}\,bd^{3}x^{4}+ad^{3}x+\frac{1}{1040}\left(65\,cx^{16}+80\,bx^{13}+104\,ax^{10}\right)e^{3}+\frac{3}{910}\left(70\,cdx^{13}+91\,bdx^{10}+130\,adx^{7}\right)e^{2}+\frac{3}{140}\left(14\,cd^{2}x^{10}+20\,bd^{2}x^{7}+35\,ad^{2}x^{4}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/7*c*d^3*x^7 + 1/4*b*d^3*x^4 + a*d^3*x + 1/1040*(65*c*x^16 + 80*b*x^13 + 1
04*a*x^10)*e^3 + 3/910*(70*c*d*x^13 + 91*b*d*x^10 + 130*a*d*x^7)*e^2 + 3/14
0*(14*c*d^2*x^10 + 20*b*d^2*x^7 + 35*a*d^2*x^4)*e
```

Sympy [A]
time = 0.02, size = 117, normalized size = 1.14

$$ad^3x + \frac{ce^3x^{16}}{16} + x^{13}\left(\frac{be^3}{13} + \frac{3cde^2}{13}\right) + x^{10}\left(\frac{ae^3}{10} + \frac{3bde^2}{10} + \frac{3cd^2e}{10}\right) + x^7 \cdot \left(\frac{3ade^2}{7} + \frac{3bd^2e}{7} + \frac{cd^3}{7}\right) + x^4 \cdot \left(\frac{3ad^2e}{4} + \frac{bd^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**3*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**3*x + c*e**3*x**16/16 + x**13*(b*e**3/13 + 3*c*d*e**2/13) + x**10*(a*e
**3/10 + 3*b*d*e**2/10 + 3*c*d**2*e/10) + x**7*(3*a*d*e**2/7 + 3*b*d**2*e/7
 + c*d**3/7) + x**4*(3*a*d**2*e/4 + b*d**3/4)
```

Giac [A]
time = 4.03, size = 109, normalized size = 1.06

$$\frac{1}{16}cx^{16}e^3 + \frac{3}{13}cdx^{13}e^2 + \frac{1}{13}bx^{13}e^3 + \frac{3}{10}cd^2x^{10}e + \frac{3}{10}bdx^{10}e^2 + \frac{1}{10}ax^{10}e^3 + \frac{1}{7}cd^3x^7 + \frac{3}{7}bd^2x^7e + \frac{3}{7}adx^7e^2 + \frac{1}{4}bd^3x^4 + \frac{3}{4}ad^2x^4e + ad^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^3*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/16*c*x^16*e^3 + 3/13*c*d*x^13*e^2 + 1/13*b*x^13*e^3 + 3/10*c*d^2*x^10*e +
 3/10*b*d*x^10*e^2 + 1/10*a*x^10*e^3 + 1/7*c*d^3*x^7 + 3/7*b*d^2*x^7*e + 3/
7*a*d*x^7*e^2 + 1/4*b*d^3*x^4 + 3/4*a*d^2*x^4*e + a*d^3*x
```

Mupad [B]
time = 0.04, size = 102, normalized size = 0.99

$$x^4\left(\frac{b\,d^3}{4} + \frac{3\,a\,e\,d^2}{4}\right) + x^{13}\left(\frac{b\,e^3}{13} + \frac{3\,c\,d\,e^2}{13}\right) + x^7\left(\frac{c\,d^3}{7} + \frac{3\,b\,d^2\,e}{7} + \frac{3\,a\,d\,e^2}{7}\right) + x^{10}\left(\frac{3\,c\,d^2\,e}{10} + \frac{3\,b\,d\,e^2}{10} + \frac{a\,e^3}{10}\right) + \frac{c\,e^3\,x^{16}}{16} + a\,d^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^3*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^4*((b*d^3)/4 + (3*a*d^2*e)/4) + x^13*((b*e^3)/13 + (3*c*d*e^2)/13) + x^7*
((c*d^3)/7 + (3*a*d*e^2)/7 + (3*b*d^2*e)/7) + x^10*((a*e^3)/10 + (3*b*d*e^2
)/10 + (3*c*d^2*e)/10) + (c*e^3*x^16)/16 + a*d^3*x
```

3.4 $\int (d+ex^3)^2 (a+bx^3+cx^6)\, dx$

Optimal. Leaf size=73

$$ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{13}ce^2x^{13}$$

```
[Out] a*d^2*x+1/4*d*(2*a*e+b*d)*x^4+1/7*(c*d^2+e*(a*e+2*b*d))*x^7+1/10*e*(b*e+2*c
*d)*x^10+1/13*c*e^2*x^13
```

Rubi [A]
time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1421}

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{4}dx^4(2ae+bd)+ad^2x+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex^3)^2(a+bx^3+cx^6)\,dx &= \int \left(ad^2+d(bd+2ae)x^3+\left(cd^2+e(2bd+ae)\right)x^6+e(2cd+be)x^9+ce\right.\\ &= ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{10}e(2cd+be)x^{10}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 73, normalized size = 1.00

$$ad^2x+\frac{1}{4}d(bd+2ae)x^4+\frac{1}{7}\left(cd^2+2bde+ae^2\right)x^7+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{13}ce^2x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^2*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^4)/4 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7 + (e*
(2*c*d + b*e)*x^10)/10 + (c*e^2*x^13)/13
```

Maple [A]
time = 0.19, size = 70, normalized size = 0.96

method	result	size
default	$\frac{c\,e^2x^{13}}{13}+\frac{(e^2b+2cde)x^{10}}{10}+\frac{(a\,e^2+2deb+c\,d^2)x^7}{7}+\frac{(2ade+d^2b)x^4}{4}+a\,d^2x$	70
norman	$\frac{c\,e^2x^{13}}{13}+\left(\frac{1}{10}e^2b+\frac{1}{5}cde\right)x^{10}+\left(\frac{1}{7}a\,e^2+\frac{2}{7}deb+\frac{1}{7}c\,d^2\right)x^7+\left(\frac{1}{2}ade+\frac{1}{4}d^2b\right)x^4+a\,d^2x$	71
gosper	$\frac{1}{13}c\,e^2x^{13}+\frac{1}{10}x^{10}e^2b+\frac{1}{5}x^{10}cde+\frac{1}{7}x^7a\,e^2+\frac{2}{7}x^7deb+\frac{1}{7}x^7c\,d^2+\frac{1}{2}x^4ade+\frac{1}{4}x^4d^2b+a\,d^2x$	77
risch	$\frac{1}{13}c\,e^2x^{13}+\frac{1}{10}x^{10}e^2b+\frac{1}{5}x^{10}cde+\frac{1}{7}x^7a\,e^2+\frac{2}{7}x^7deb+\frac{1}{7}x^7c\,d^2+\frac{1}{2}x^4ade+\frac{1}{4}x^4d^2b+a\,d^2x$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^2*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*c*e^2*x^13+1/10*(b*e^2+2*c*d*e)*x^10+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/4
*(2*a*d*e+b*d^2)*x^4+a*d^2*x
```

Maxima [A]
time = 0.27, size = 69, normalized size = 0.95

$$\frac{1}{13}\,cx^{13}e^2+\frac{1}{10}\,\left(2\,cde+be^2\right)x^{10}+\frac{1}{7}\,\left(cd^2+2\,bde+ae^2\right)x^7+\frac{1}{4}\,\left(bd^2+2\,ade\right)x^4+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/13*c*x^13*e^2 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/7*(c*d^2 + 2*b*d*e + a*e^
2)*x^7 + 1/4*(b*d^2 + 2*a*d*e)*x^4 + a*d^2*x
```

Fricas [A]
time = 0.35, size = 74, normalized size = 1.01

$$\frac{1}{7}\,cd^2x^7+\frac{1}{4}\,bd^2x^4+ad^2x+\frac{1}{910}\,\left(70\,cx^{13}+91\,bx^{10}+130\,ax^7\right)e^2+\frac{1}{70}\,\left(14\,cdx^{10}+20\,bdx^7+35\,adx^4\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/7*c*d^2*x^7 + 1/4*b*d^2*x^4 + a*d^2*x + 1/910*(70*c*x^13 + 91*b*x^10 + 13
0*a*x^7)*e^2 + 1/70*(14*c*d*x^10 + 20*b*d*x^7 + 35*a*d*x^4)*e
```

Sympy [A]
time = 0.01, size = 75, normalized size = 1.03

$$ad^2x+\frac{ce^2x^{13}}{13}+x^{10}\left(\frac{be^2}{10}+\frac{cde}{5}\right)+x^7\left(\frac{ae^2}{7}+\frac{2bde}{7}+\frac{cd^2}{7}\right)+x^4\left(\frac{ade}{2}+\frac{bd^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**2*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**13/13 + x**10*(b*e**2/10 + c*d*e/5) + x**7*(a*e**2/7 +
 2*b*d*e/7 + c*d**2/7) + x**4*(a*d*e/2 + b*d**2/4)
```

Giac [A]
time = 4.46, size = 76, normalized size = 1.04

$$\frac{1}{13}\,cx^{13}e^2+\frac{1}{5}\,cdx^{10}e+\frac{1}{10}\,bx^{10}e^2+\frac{1}{7}\,cd^2x^7+\frac{2}{7}\,bdx^7e+\frac{1}{7}\,ax^7e^2+\frac{1}{4}\,bd^2x^4+\frac{1}{2}\,adx^4e+ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^2*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/13*c*x^13*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/7*c*d^2*x^7 + 2/7*b*
d*x^7*e + 1/7*a*x^7*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + a*d^2*x
```

Mupad [B]
time = 0.04, size = 70, normalized size = 0.96

$$x^7\left(\frac{c\,d^2}{7}+\frac{2\,b\,d\,e}{7}+\frac{a\,e^2}{7}\right)+x^4\left(\frac{b\,d^2}{4}+\frac{a\,e\,d}{2}\right)+x^{10}\left(\frac{b\,e^2}{10}+\frac{c\,d\,e}{5}\right)+\frac{c\,e^2\,x^{13}}{13}+a\,d^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^2*(a + b*x^3 + c*x^6),x)
```

```
[Out] x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^4*((b*d^2)/4 + (a*d*e)/2) + x
^10*((b*e^2)/10 + (c*d*e)/5) + (c*e^2*x^13)/13 + a*d^2*x
```

3.5 $\int (d+ex^3)(a+bx^3+cx^6)\,dx$

Optimal. Leaf size=42

$$adx+\frac{1}{4}(bd+ae)x^4+\frac{1}{7}(cd+be)x^7+\frac{1}{10}cex^{10}$$

```
[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

Rubi [A]
time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1421}

$$\frac{1}{4}x^4(ae+bd)+adx+\frac{1}{7}x^7(be+cd)+\frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Rule 1421

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2
_)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q*(a + b*x^n + c*x^(2*n))
, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex^3)(a+bx^3+cx^6)\,dx &= \int \left(ad+(bd+ae)x^3+(cd+be)x^6+cex^9\right)dx\\ &= adx+\frac{1}{4}(bd+ae)x^4+\frac{1}{7}(cd+be)x^7+\frac{1}{10}cex^{10}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 42, normalized size = 1.00

$$adx+\frac{1}{4}(bd+ae)x^4+\frac{1}{7}(cd+be)x^7+\frac{1}{10}cex^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)*(a + b*x^3 + c*x^6),x]
```

```
[Out] a*d*x + ((b*d + a*e)*x^4)/4 + ((c*d + b*e)*x^7)/7 + (c*e*x^10)/10
```

Maple [A]
time = 0.06, size = 37, normalized size = 0.88

method	result	size
default	$adx+\frac{(ae+bd)x^4}{4}+\frac{(eb+cd)x^7}{7}+\frac{ce\,x^{10}}{10}$	37
norman	$\frac{ce\,x^{10}}{10}+\left(\frac{eb}{7}+\frac{cd}{7}\right)x^7+\left(\frac{ae}{4}+\frac{bd}{4}\right)x^4+adx$	39
gosper	$\frac{1}{10}ce\,x^{10}+\frac{1}{7}x^7eb+\frac{1}{7}x^7cd+\frac{1}{4}x^4ae+\frac{1}{4}x^4bd+adx$	41
risch	$\frac{1}{10}ce\,x^{10}+\frac{1}{7}x^7eb+\frac{1}{7}x^7cd+\frac{1}{4}x^4ae+\frac{1}{4}x^4bd+adx$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
[Out] a*d*x+1/4*(a*e+b*d)*x^4+1/7*(b*e+c*d)*x^7+1/10*c*e*x^10
```

Maxima [A]
time = 0.27, size = 39, normalized size = 0.93

$$\frac{1}{10}\,cx^{10}e+\frac{1}{7}\,(cd+be)x^7+\frac{1}{4}\,(bd+ae)x^4+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="maxima")
[Out] 1/10*c*x^10*e + 1/7*(c*d + b*e)*x^7 + 1/4*(b*d + a*e)*x^4 + a*d*x
```

Fricas [A]
time = 0.33, size = 42, normalized size = 1.00

$$\frac{1}{7}\,cdx^7+\frac{1}{4}\,bdx^4+adx+\frac{1}{140}\left(14\,cx^{10}+20\,bx^7+35\,ax^4\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="fricas")
[Out] 1/7*c*d*x^7 + 1/4*b*d*x^4 + a*d*x + 1/140*(14*c*x^10 + 20*b*x^7 + 35*a*x^4)
*e
```

Sympy [A]
time = 0.01, size = 39, normalized size = 0.93

$$adx+\frac{cex^{10}}{10}+x^7\left(\frac{be}{7}+\frac{cd}{7}\right)+x^4\left(\frac{ae}{4}+\frac{bd}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)*(c*x**6+b*x**3+a),x)
[Out] a*d*x + c*e*x**10/10 + x**7*(b*e/7 + c*d/7) + x**4*(a*e/4 + b*d/4)
```

Giac [A]
time = 4.08, size = 43, normalized size = 1.02

$$\frac{1}{10}\,cx^{10}e+\frac{1}{7}\,cdx^7+\frac{1}{7}\,bx^7e+\frac{1}{4}\,bdx^4+\frac{1}{4}\,ax^4e+adx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)*(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] 1/10*c*x^10*e + 1/7*c*d*x^7 + 1/7*b*x^7*e + 1/4*b*d*x^4 + 1/4*a*x^4*e + a*d
*x
```

Mupad [B]
time = 0.04, size = 38, normalized size = 0.90

$$\frac{c\,e\,x^{10}}{10}+\left(\frac{b\,e}{7}+\frac{c\,d}{7}\right)\,x^7+\left(\frac{a\,e}{4}+\frac{b\,d}{4}\right)\,x^4+a\,d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)*(a + b*x^3 + c*x^6),x)
[Out] x^4*((a*e)/4 + (b*d)/4) + x^7*((b*e)/7 + (c*d)/7) + a*d*x + (c*e*x^10)/10
```

3.6 $\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx$

Optimal. Leaf size=188

$$-\frac{(cd-be)x}{e^2}+\frac{cx^4}{4e}-\frac{(cd^2-bde+ae^2)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{7/3}}+\frac{(cd^2-bde+ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{7/3}}-\frac{(cd^2-}{}$$

```
[Out] -(-b*e+c*d)*x/e^2+1/4*c*x^4/e+1/3*(a*e^2-b*d*e+c*d^2)*ln(d^(1/3)+e^(1/3)*x)
/d^(2/3)/e^(7/3)-1/6*(a*e^2-b*d*e+c*d^2)*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/
3)*x^2)/d^(2/3)/e^(7/3)-1/3*(a*e^2-b*d*e+c*d^2)*arctan(1/3*(d^(1/3)-2*e^(1/
3)*x)/d^(1/3)*3^(1/2))/d^(2/3)/e^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1425, 396, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)(ae^2-bde+cd^2)}{\sqrt{3}\,d^{2/3}e^{7/3}}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)(ae^2-bde+cd^2)}{6d^{2/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)(ae^2-bde+cd^2)}{3d^{2/3}e^{7/3}}-\frac{x(cd-be)}{e^2}+\frac{cx^4}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] -(((c*d - b*e)*x)/e^2) + (c*x^4)/(4*e) - ((c*d^2 - b*d*e + a*e^2)*ArcTan[(d
^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(Sqrt[3]*d^(2/3)*e^(7/3)) + ((c*d
^2 - b*d*e + a*e^2)*Log[d^(1/3) + e^(1/3)*x])/(3*d^(2/3)*e^(7/3)) - ((c*d^2
 - b*d*e + a*e^2)*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/(6*d^(2/3
)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{d+ex^3}\,dx = \frac{cx^4}{4e} + \frac{\int \frac{4ae-(4cd-4be)x^3}{d+ex^3}\,dx}{4e}$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} - \left(-a-\frac{d(cd-be)}{e^2}\right)\int \frac{1}{d+ex^3}\,dx$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{3d^{2/3}} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)\int \frac{2\sqrt[3]{d}-}{d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}}}{3d^{2/3}}$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} + \frac{(cd^2-bde+ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{7/3}} - \frac{(cd^2-bde+ae^2)\int -}{6d^{2/3}}$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} + \frac{(cd^2-bde+ae^2)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{3d^{2/3}e^{7/3}} - \frac{(cd^2-bde+ae^2)\log}{6}$$

$$= -\frac{(cd-be)x}{e^2} + \frac{cx^4}{4e} - \frac{(cd^2-bde+ae^2)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{\sqrt{3}\,d^{2/3}e^{7/3}} + \frac{(cd^2-bde+ae^2)\log}{3d^{2/3}}$$

Mathematica [A]

time = 0.11, size = 176, normalized size = 0.94

$$\frac{12\sqrt[3]{e}\,(-cd+be)x+3ce^{4/3}x^4-\frac{4\sqrt{3}\,(cd^2+e(-bd+ae))\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{2/3}}+\frac{4(cd^2+e(-bd+ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{d^{2/3}}-\frac{2(cd^2+e(-bd+ae))\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{d^{2/3}}}{12e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3),x]
```

```
[Out] (12*e^(1/3)*(-(c*d) + b*e)*x + 3*c*e^(4/3)*x^4 - (4*Sqrt[3]*(c*d^2 + e*(-(b
*d) + a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]])/d^(2/3) + (4*(c*d^
2 + e*(-(b*d) + a*e))*Log[d^(1/3) + e^(1/3)*x])/d^(2/3) - (2*(c*d^2 + e*(-(
b*d) + a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2])/d^(2/3))/(12*e
^(7/3))
```

Maple [A]

time = 0.19, size = 133, normalized size = 0.71

method	result	size
risch	$\frac{c\,x^4}{4e}+\frac{bx}{e}-\frac{cdx}{e^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e\,\_Z^3+d\right)}\frac{\left(a\,e^2-deb+c\,d^2\right)\ln\left(x-\_R\right)}{\_R^2}}{3e^3}$	67

default	$\frac{\frac{1}{4}ce\,x^4+ebx-cdx}{e^2}+\frac{\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^2-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)\left(a\,e^2-deb+c\,d^2\right)}{e^2}$	133

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^2*(1/4*c*e*x^4+e*b*x-c*d*x)+(1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))-1/6/e/
(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1/3/e/(d/e)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)))*(a*e^2-b*d*e+c*d^2)/e^2
```

Maxima [A]
time = 0.50, size = 146, normalized size = 0.78

$$\frac{\sqrt{3}\left(cd^2-bde+ae^2\right)\arctan\left(-\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-2x\right)e^{\frac{1}{3}}}{3d^{\frac{1}{3}}}\right)e^{\left(-\frac{7}{3}\right)}}{3d^{\frac{2}{3}}}-\frac{\left(cd^2-bde+ae^2\right)e^{\left(-\frac{7}{3}\right)}\log\left(-d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^2+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{6d^{\frac{2}{3}}}+\frac{\left(cd^2-bde+ae^2\right)e^{\left(-\frac{7}{3}\right)}\log\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right)}{3d^{\frac{2}{3}}}+\frac{1}{4}\left(cx^4e-4\left(cd-be\right)x\right)e^{\left(-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*arctan(-1/3*sqrt(3)*(d^(1/3)*e^(-1/3) -
 2*x)*e^(1/3)/d^(1/3))*e^(-7/3)/d^(2/3) - 1/6*(c*d^2 - b*d*e + a*e^2)*e^(-7
/3)*log(-d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(2/3) + 1/3*(c*d^2
- b*d*e + a*e^2)*e^(-7/3)*log(d^(1/3)*e^(-1/3) + x)/d^(2/3) + 1/4*(c*x^4*e
- 4*(c*d - b*e)*x)*e^(-2)
```

Fricas [A]
time = 0.37, size = 199, normalized size = 1.06

$$-\frac{\left(12\,cd^3xe-12\sqrt{\frac{1}{3}}\left(cd^3e-bd^2e^2+ade^3\right)\left(d^2\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(d^2\right)^{\frac{2}{3}}xe^{\frac{2}{3}}-\left(d^2\right)^{\frac{1}{3}}de^{\frac{1}{3}}\right)\left(d^2\right)^{\frac{1}{6}}e^{\left(-\frac{1}{3}\right)}}{d^2}\right)e^{\left(-\frac{1}{3}\right)}+2\left(cd^2-bde+ae^2\right)\left(d^2\right)^{\frac{2}{3}}e^{\frac{2}{3}}\log\left(dx^2e-\left(d^2\right)^{\frac{2}{3}}xe^{\frac{2}{3}}+\left(d^2\right)^{\frac{1}{3}}de^{\frac{1}{3}}\right)-4\left(cd^2-bde+ae^2\right)\left(d^2\right)^{\frac{2}{3}}e^{\frac{2}{3}}\log\left(dxe+\left(d^2\right)^{\frac{2}{3}}e^{\frac{2}{3}}\right)-3\left(cd^2x^4+4\,bd^2x\right)e^2\right)e^{\left(-3\right)}}{12\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="fricas")
```

```
[Out] -1/12*(12*c*d^3*x*e - 12*sqrt(1/3)*(c*d^3*e - b*d^2*e^2 + a*d*e^3)*(d^2)^(1
/6)*arctan(sqrt(1/3)*(2*(d^2)^(2/3)*x*e^(2/3) - (d^2)^(1/3)*d*e^(1/3))*(d^2
)^(1/6)*e^(-1/3)/d^2)*e^(-1/3) + 2*(c*d^2 - b*d*e + a*e^2)*(d^2)^(2/3)*e^(2
/3)*log(d*x^2*e - (d^2)^(2/3)*x*e^(2/3) + (d^2)^(1/3)*d*e^(1/3)) - 4*(c*d^2
```

```
 - b*d*e + a*e^2)*(d^2)^(2/3)*e^(2/3)*log(d*x*e + (d^2)^(2/3)*e^(2/3)) - 3*
(c*d^2*x^4 + 4*b*d^2*x)*e^2)*e^(-3)/d^2
```

Sympy [A]
time = 0.45, size = 175, normalized size = 0.93

$$\frac{cx^4}{4e} + x\left(\frac{b}{e} - \frac{cd}{e^2}\right) + \operatorname{RootSum}\left(27t^3d^2e^7 - a^3e^6 + 3a^2bde^5 - 3a^2cd^2e^4 - 3ab^2d^2e^4 + 6abcd^3e^3 - 3ac^2d^4e^2 + b^3d^3e^3 - 3b^2cd^4e^2 + 3bc^2d^5e - c^3d^6, \left(t \mapsto t\log\left(\frac{3tde^2}{ae^2 - bde + cd^2} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d),x)
```

```
[Out] c*x**4/(4*e) + x*(b/e - c*d/e**2) + RootSum(27*_t**3*d**2*e**7 - a**3*e**6
+ 3*a**2*b*d*e**5 - 3*a**2*c*d**2*e**4 - 3*a*b**2*d**2*e**4 + 6*a*b*c*d**3*
e**3 - 3*a*c**2*d**4*e**2 + b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 + 3*b*c**2*
d**5*e - c**3*d**6, Lambda(_t, _t*log(3*_t*d*e**2/(a*e**2 - b*d*e + c*d**2)
 + x)))
```

Giac [A]
time = 4.15, size = 173, normalized size = 0.92

$$-\frac{\sqrt{3}\left(cd^2 - bde + ae^2\right)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{3\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(cd^2 - bde + ae^2\right)e^{(-1)}\log\left(x^2 + \left(-de^{(-1)}\right)^{\frac{1}{3}}x + \left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{6\left(-de^2\right)^{\frac{2}{3}}} - \frac{\left(cd^2e^2 - bde^3 + ae^4\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-4)}\log\left(\left|x - \left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{3d} + \frac{1}{4}\left(cx^4e^3 - 4cdxe^2 + 4bxe^3\right)e^{(-4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*(c*d^2 - b*d*e + a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(-1))^
(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/(-d*e^2)^(2/3) - 1/6*(c*d^2 - b*d*e + a*e^
2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3))/(-d*e^2)^(2/3)
 - 1/3*(c*d^2*e^2 - b*d*e^3 + a*e^4)*(-d*e^(-1))^(1/3)*e^(-4)*log(abs(x - (
-d*e^(-1))^(1/3)))/d + 1/4*(c*x^4*e^3 - 4*c*d*x*e^2 + 4*b*x*e^3)*e^(-4)
```

Mupad [B]
time = 0.27, size = 165, normalized size = 0.88

$$x\left(\frac{b}{e} - \frac{c\,d}{e^2}\right) + \frac{c\,x^4}{4\,e} + \frac{\ln\left(e^{1/3}\,x + d^{1/3}\right)\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}} + \frac{\ln\left(2\,e^{1/3}\,x - d^{1/3} + \sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(-\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}} - \frac{\ln\left(d^{1/3} - 2\,e^{1/3}\,x + \sqrt{3}\,d^{1/3}\,1\mathrm{i}\right)\,\left(\frac{1}{2} + \frac{\sqrt{3}\,1\mathrm{i}}{2}\right)\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{3\,d^{2/3}\,e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3),x)
```

```
[Out] x*(b/e - (c*d)/e^2) + (c*x^4)/(4*e) + (log(e^(1/3)*x + d^(1/3))*(a*e^2 + c*
d^2 - b*d*e))/(3*d^(2/3)*e^(7/3)) + (log(3^(1/2)*d^(1/3)*1i + 2*e^(1/3)*x -
 d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3
)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2
)*(a*e^2 + c*d^2 - b*d*e))/(3*d^(2/3)*e^(7/3))
```

3.7 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^2}\,dx$

Optimal. Leaf size=213

$$\frac{cx}{e^2}+\frac{\left(cd^2-bde+ae^2\right)x}{3de^2\left(d+ex^3\right)}+\frac{\left(4cd^2-e(bd+2ae)\right)\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{3\sqrt{3}\,d^{5/3}e^{7/3}}-\frac{\left(4cd^2-e(bd+2ae)\right)\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{9d^{5/3}e^{7/3}}$$

```
[Out] c*x/e^2+1/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)-1/9*(4*c*d^2-e*(2*a*e+b*d
))*ln(d^(1/3)+e^(1/3)*x)/d^(5/3)/e^(7/3)+1/18*(4*c*d^2-e*(2*a*e+b*d))*ln(d^
(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/d^(5/3)/e^(7/3)+1/9*(4*c*d^2-e*(2*a*e+
b*d))*arctan(1/3*(d^(1/3)-2*e^(1/3)*x)/d^(1/3)*3^(1/2))/d^(5/3)/e^(7/3)*3^(
1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1423, 396, 206, 31, 648, 631, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)\left(4cd^2-e(2ae+bd)\right)}{3\sqrt{3}\,d^{5/3}e^{7/3}}+\frac{x\left(ae^2-bde+cd^2\right)}{3de^2\left(d+ex^3\right)}+\frac{\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)\left(4cd^2-e(2ae+bd)\right)}{18d^{5/3}e^{7/3}}-\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\left(4cd^2-e(2ae+bd)\right)}{9d^{5/3}e^{7/3}}+\frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*(d + e*x^3)) + ((4*c*d^2 -
 e*(b*d + 2*a*e))*ArcTan[(d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(3*Sqr
t[3]*d^(5/3)*e^(7/3)) - ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*
x])/(9*d^(5/3)*e^(7/3)) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(18*d^(5/3)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^2}\,dx = \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)} - \frac{\int \frac{cd^2-e(bd+2ae)-3cdex^3}{d+ex^3}\,dx}{3de^2}$$

$$= \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)} - \frac{(4cd^2-e(bd+2ae))\int \frac{1}{d+ex^3}\,dx}{3de^2}$$

$$= \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)} - \frac{(4cd^2-e(bd+2ae))\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{9d^{5/3}e^2} - \frac{(4cd^2-e(bd+2\ldots}{\ldots}$$

$$= \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)} - \frac{(4cd^2-e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{9d^{5/3}e^{7/3}} + \frac{(4cd^2-e(bd\ldots}{\ldots}$$

$$= \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)} - \frac{(4cd^2-e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{9d^{5/3}e^{7/3}} + \frac{(4cd^2-e(bd\ldots}{\ldots}$$

$$= \frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{3de^2\,(d+ex^3)} + \frac{(4cd^2-e(bd+2ae))\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{3\sqrt{3}\,d^{5/3}e^{7/3}} - \frac{(4cd^2-e(b\ldots}{\ldots}$$

Mathematica [A]
time = 0.14, size = 199, normalized size = 0.93

$$\frac{18c\sqrt[3]{e}\,x + \frac{6\sqrt[3]{e}\,(cd^2+e(-bd+ae))x}{d(d+ex^3)} + \frac{2\sqrt{3}\,(4cd^2-e(bd+2ae))\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right)}{d^{5/3}} - \frac{2(4cd^2-e(bd+2ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{d^{5/3}} + \frac{(4cd^2-e(bd+2ae))\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{d^{5/3}}}{18e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^2,x]
```

```
[Out] (18*c*e^(1/3)*x + (6*e^(1/3)*(c*d^2 + e*(-(b*d) + a*e))*x)/(d*(d + e*x^3))
+ (2*Sqrt[3]*(4*c*d^2 - e*(b*d + 2*a*e))*ArcTan[(1 - (2*e^(1/3)*x)/d^(1/3))
/Sqrt[3]])/d^(5/3) - (2*(4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(1/3) + e^(1/3)*x
])/d^(5/3) + ((4*c*d^2 - e*(b*d + 2*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x +
 e^(2/3)*x^2])/d^(5/3))/(18*e^(7/3))
```

Maple [A]
time = 0.24, size = 156, normalized size = 0.73

method	result	size

risch	$\frac{cx}{e^{2}}+\frac{\left(a\,e^{2}-deb+c\,d^{2}\right)x}{3d\,e^{2}\left(e\,x^{3}+d\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^{3}+d\right)}\frac{\left(2a\,e^{2}+deb-4c\,d^{2}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{9e^{3}d}$	88
default	$\frac{cx}{e^{2}}+\frac{\frac{\left(a\,e^{2}-deb+c\,d^{2}\right)x}{3d\left(e\,x^{3}+d\right)}+\frac{\left(2a\,e^{2}+deb-4c\,d^{2}\right)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{e}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\right)}{3d}}{e^{2}}$	156

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/3*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^3+d)+1/3*(2*a*e^2+b*d*e-4*c
*d^2)/d*(1/3/e/(d/e)^(2/3)*ln(x+(d/e)^(1/3))-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)
^(1/3)*x+(d/e)^(2/3))+1/3/e/(d/e)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)
^(1/3)*x-1))))
```

Maxima [A]
time = 0.50, size = 168, normalized size = 0.79

$$cxe^{(-2)}-\frac{\sqrt{3}\,(4\,cd^{2}-bde-2\,ae^{2})\arctan\left(-\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-2\,x\right)e^{\frac{1}{3}}}{3\,d^{\frac{1}{3}}}\right)e^{\left(-\frac{7}{3}\right)}}{9\,d^{\frac{5}{3}}}+\frac{(4\,cd^{2}-bde-2\,ae^{2})e^{\left(-\frac{7}{3}\right)}\log\left(-d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^{2}+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{18\,d^{\frac{5}{3}}}-\frac{(4\,cd^{2}-bde-2\,ae^{2})e^{\left(-\frac{7}{3}\right)}\log\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right)}{9\,d^{\frac{5}{3}}}+\frac{(cd^{2}-bde+ae^{2})x}{3\,(dx^{3}e^{3}+d^{2}e^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="maxima")
```

```
[Out] c*x*e^(-2) - 1/9*sqrt(3)*(4*c*d^2 - b*d*e - 2*a*e^2)*arctan(-1/3*sqrt(3)*(d
^(1/3)*e^(-1/3) - 2*x)*e^(1/3)/d^(1/3))*e^(-7/3)/d^(5/3) + 1/18*(4*c*d^2 -
b*d*e - 2*a*e^2)*e^(-7/3)*log(-d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))
/d^(5/3) - 1/9*(4*c*d^2 - b*d*e - 2*a*e^2)*e^(-7/3)*log(d^(1/3)*e^(-1/3) +
x)/d^(5/3) + 1/3*(c*d^2 - b*d*e + a*e^2)*x/(d*x^3*e^3 + d^2*e^2)
```

Fricas [A]
time = 0.36, size = 323, normalized size = 1.52

$$\frac{24\,cd^{4}xe+6\,ad^{2}xe^{3}+6\,\sqrt{\frac{1}{3}}\,(2\,adx^{3}e^{4}-4\,cd^{4}e+(bd^{2}x^{3}+2\,ad^{2})e^{3}-(4\,cd^{3}x^{3}-bd^{3})e^{2})(d^{2})^{\frac{1}{6}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\,(d^{2})^{\frac{2}{3}}xe^{\frac{2}{3}}-(d^{2})^{\frac{1}{3}}de^{\frac{1}{3}}\right)(d^{2})^{\frac{1}{6}}e^{\left(-\frac{1}{3}\right)}}{d^{2}}\right)e^{\left(-\frac{1}{3}\right)}-(2\,ax^{3}e^{3}-4\,cd^{3}+(bdx^{3}+2\,ad)e^{2}-(4\,cd^{2}x^{3}-bd^{2})e)(d^{2})^{\frac{2}{3}}e^{\frac{2}{3}}\log\left(dx^{2}e-(d^{2})^{\frac{2}{3}}xe^{\frac{2}{3}}+(d^{2})^{\frac{1}{3}}de^{\frac{1}{3}}\right)+2\,(2\,ax^{3}e^{3}-4\,cd^{3}+(bdx^{3}+2\,ad)e^{2}-(4\,cd^{2}x^{3}-bd^{2})e)(d^{2})^{\frac{2}{3}}e^{\frac{2}{3}}\log\left(dxe+(d^{2})^{\frac{2}{3}}e^{\frac{2}{3}}\right)+6\,(3\,cd^{3}x^{4}-bd^{3}x)e^{2}}{18\,(d^{3}x^{3}e^{4}+d^{4}e^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(24*c*d^4*x*e + 6*a*d^2*x*e^3 + 6*sqrt(1/3)*(2*a*d*x^3*e^4 - 4*c*d^4*e
 + (b*d^2*x^3 + 2*a*d^2)*e^3 - (4*c*d^3*x^3 - b*d^3)*e^2)*(d^2)^(1/6)*arcta
n(sqrt(1/3)*(2*(d^2)^(2/3)*x*e^(2/3) - (d^2)^(1/3)*d*e^(1/3))*(d^2)^(1/6)*e
^(-1/3)/d^2)*e^(-1/3) - (2*a*x^3*e^3 - 4*c*d^3 + (b*d*x^3 + 2*a*d)*e^2 - (4
*c*d^2*x^3 - b*d^2)*e)*(d^2)^(2/3)*e^(2/3)*log(d*x^2*e - (d^2)^(2/3)*x*e^(2
/3) + (d^2)^(1/3)*d*e^(1/3)) + 2*(2*a*x^3*e^3 - 4*c*d^3 + (b*d*x^3 + 2*a*d)
*e^2 - (4*c*d^2*x^3 - b*d^2)*e)*(d^2)^(2/3)*e^(2/3)*log(d*x*e + (d^2)^(2/3)
*e^(2/3)) + 6*(3*c*d^3*x^4 - b*d^3*x)*e^2)/(d^3*x^3*e^4 + d^4*e^3)
```

Sympy [A]
time = 0.90, size = 206, normalized size = 0.97

$$\frac{cx}{e^2}+\frac{x(ae^2-bde+cd^2)}{3d^2e^2+3de^3x^3}+\operatorname{RootSum}\left(729t^3d^5e^7-8a^3e^6-12a^2bde^5+48a^2cd^2e^4-6ab^2d^2e^4+48abcd^3e^3-96ac^2d^4e^2-b^3d^3e^3+12b^2cd^4e^2-48bc^2d^5e+64c^3d^6,\left(t\mapsto t\log\left(\frac{9td^2e^2}{2ae^2+bde-4cd^2}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(3*d**2*e**2 + 3*d*e**3*x**3) + Root
Sum(729*_t**3*d**5*e**7 - 8*a**3*e**6 - 12*a**2*b*d*e**5 + 48*a**2*c*d**2*e
**4 - 6*a*b**2*d**2*e**4 + 48*a*b*c*d**3*e**3 - 96*a*c**2*d**4*e**2 - b**3*
d**3*e**3 + 12*b**2*c*d**4*e**2 - 48*b*c**2*d**5*e + 64*c**3*d**6, Lambda(_
t, _t*log(9*_t*d**2*e**2/(2*a*e**2 + b*d*e - 4*c*d**2) + x)))
```

Giac [A]
time = 3.95, size = 199, normalized size = 0.93

$$cxe^{(-2)}+\frac{\sqrt{3}\,(4\,cd^2-bde-2\,ae^2)\arctan\left(\frac{\sqrt{3}\,\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\,\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{9\,\left(-de^2\right)^{\frac{2}{3}}\,d}+\frac{(4\,cd^2-bde-2\,ae^2)e^{(-1)}\log\left(x^2+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{18\,\left(-de^2\right)^{\frac{2}{3}}\,d}+\frac{(4\,cd^2-bde-2\,ae^2)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-2)}\log\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{9\,d^2}+\frac{(cd^2x-bdxe+axe^2)e^{(-2)}}{3\,(x^3e+d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) + 1/9*sqrt(3)*(4*c*d^2 - b*d*e - 2*a*e^2)*arctan(1/3*sqrt(3)*(2*
x + (-d*e^(-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/((-d*e^2)^(2/3)*d) + 1/18*
(4*c*d^2 - b*d*e - 2*a*e^2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-
1))^(2/3))/((-d*e^2)^(2/3)*d) + 1/9*(4*c*d^2 - b*d*e - 2*a*e^2)*(-d*e^(-1))
^(1/3)*e^(-2)*log(abs(x - (-d*e^(-1))^(1/3)))/d^2 + 1/3*(c*d^2*x - b*d*x*e
+ a*x*e^2)*e^(-2)/((x^3*e + d)*d)
```

Mupad [B]
time = 1.80, size = 187, normalized size = 0.88

$$\frac{c\,x}{e^2}+\frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\,\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}}+\frac{x\,\left(c\,d^2-b\,d\,e+a\,e^2\right)}{3\,d\,\left(e^3\,x^3+d\,e^2\right)}+\frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}}-\frac{\ln\left(d^{1/3}-2\,e^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(-4\,c\,d^2+b\,d\,e+2\,a\,e^2\right)}{9\,d^{5/3}\,e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^2,x)
```

```
[Out] (c*x)/e^2 + (log(e^(1/3)*x + d^(1/3))*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^(5/
3)*e^(7/3)) + (x*(a*e^2 + c*d^2 - b*d*e))/(3*d*(d*e^2 + e^3*x^3)) + (log(3^
(1/2)*d^(1/3)*1i + 2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(2*a*e^2 -
 4*c*d^2 + b*d*e))/(9*d^(5/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3
)*x + d^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(2*a*e^2 - 4*c*d^2 + b*d*e))/(9*d^(5/
3)*e^(7/3))
```

3.8 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^3} \, dx$

Optimal. Leaf size=242

$$\frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^3)^2}-\frac{(7cd^2-e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)}-\frac{(2cd^2+e(bd+5ae))\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)}{9\sqrt{3}\,d^{8/3}e^{7/3}}+\frac{(2cd^2+e(bd+5a}{27}$$

```
[Out] 1/6*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^2-1/18*(7*c*d^2-e*(5*a*e+b*d))*x/
d^2/e^2/(e*x^3+d)+1/27*(2*c*d^2+e*(5*a*e+b*d))*ln(d^(1/3)+e^(1/3)*x)/d^(8/3
)/e^(7/3)-1/54*(2*c*d^2+e*(5*a*e+b*d))*ln(d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)
*x^2)/d^(8/3)/e^(7/3)-1/27*(2*c*d^2+e*(5*a*e+b*d))*arctan(1/3*(d^(1/3)-2*e^
(1/3)*x)/d^(1/3)*3^(1/2))/d^(8/3)/e^(7/3)*3^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1423, 393, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{e}\,x}{\sqrt{3}\,\sqrt[3]{d}}\right)(e(5ae+bd)+2cd^2)}{9\sqrt{3}\,d^{8/3}e^{7/3}}-\frac{x(7cd^2-e(5ae+bd))}{18d^2e^2\,(d+ex^3)}+\frac{x(ae^2-bde+cd^2)}{6de^2\,(d+ex^3)^2}-\frac{\log\left(d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2\right)(e(5ae+bd)+2cd^2)}{54d^{8/3}e^{7/3}}+\frac{\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)(e(5ae+bd)+2cd^2)}{27d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(6*d*e^2*(d + e*x^3)^2) - ((7*c*d^2 - e*(b*d +
5*a*e))*x)/(18*d^2*e^2*(d + e*x^3)) - ((2*c*d^2 + e*(b*d + 5*a*e))*ArcTan[(
d^(1/3) - 2*e^(1/3)*x)/(Sqrt[3]*d^(1/3))])/(9*Sqrt[3]*d^(8/3)*e^(7/3)) + ((
2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(1/3) + e^(1/3)*x])/(27*d^(8/3)*e^(7/3)) -
 ((2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2
])/(54*d^(8/3)*e^(7/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^3+cx^6}{(d+ex^3)^3}\,dx &= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^3)^2}-\frac{\int \frac{cd^2-e(bd+5ae)-6cdex^3}{(d+ex^3)^2}\,dx}{6de^2}\\
&= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^3)^2}-\frac{(7cd^2-e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)}+\frac{(2cd^2+e(bd+5ae))\int \frac{1}{d+ex^3}\,dx}{9d^2e^2}\\
&= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^3)^2}-\frac{(7cd^2-e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)}+\frac{(2cd^2+e(bd+5ae))\int \frac{1}{\sqrt[3]{d}+\sqrt[3]{e}\,x}\,dx}{27d^{8/3}e^2}\\
&= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^3)^2}-\frac{(7cd^2-e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)}+\frac{(2cd^2+e(bd+5ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}\right.}{27d^{8/3}e^{7/3}}\\
&= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^3)^2}-\frac{(7cd^2-e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)}+\frac{(2cd^2+e(bd+5ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}\right.}{27d^{8/3}e^{7/3}}\\
&= \frac{(cd^2-bde+ae^2)\,x}{6de^2\,(d+ex^3)^2}-\frac{(7cd^2-e(bd+5ae))\,x}{18d^2e^2\,(d+ex^3)}-\frac{(2cd^2+e(bd+5ae))\tan^{-1}\left(\frac{\sqrt[3]{d}-2\sqrt[3]{}}{\sqrt{3}\sqrt[3]{}}\right.}{9\sqrt{3}\,d^{8/3}e^{7/3}}\end{aligned}$$

Mathematica [A]

time = 0.19, size = 209, normalized size = 0.86

$$\frac{-\frac{3d^{2/3}\sqrt[3]{e}\,x\left(cd^2(4d+7ex^3)-e\left(bd(-2d+ex^3)+ae(8d+5ex^3)\right)\right)}{(d+ex^3)^2}-2\sqrt{3}\,(2cd^2+e(bd+5ae))\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{e}\,x}{\sqrt[3]{d}}}{\sqrt{3}}\right)+2(2cd^2+e(bd+5ae))\log\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)-(2cd^2+e(bd+5ae))\log\left(d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2\right)}{54d^{8/3}e^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^3,x]
```

```
[Out] ((-3*d^(2/3)*e^(1/3)*x*(c*d^2*(4*d + 7*e*x^3) - e*(b*d*(-2*d + e*x^3) + a*e
*(8*d + 5*e*x^3))))/(d + e*x^3)^2 - 2*Sqrt[3]*(2*c*d^2 + e*(b*d + 5*a*e))*A
rcTan[(1 - (2*e^(1/3)*x)/d^(1/3))/Sqrt[3]] + 2*(2*c*d^2 + e*(b*d + 5*a*e))*
Log[d^(1/3) + e^(1/3)*x] - (2*c*d^2 + e*(b*d + 5*a*e))*Log[d^(2/3) - d^(1/3
)*e^(1/3)*x + e^(2/3)*x^2])/(54*d^(8/3)*e^(7/3))
```

Maple [A]

time = 0.23, size = 183, normalized size = 0.76

method	result

risch	$\frac{\frac{\left(5a\,e^{2}+deb-7c\,d^{2}\right)x^{4}}{18d^{2}e}+\frac{\left(4a\,e^{2}-deb-2c\,d^{2}\right)x}{9d\,e^{2}}}{\left(e\,x^{3}+d\right)^{2}}+\frac{\sum_{\_R=\mathrm{RootOf}\left(e\_Z^{3}+d\right)}\frac{\left(5a\,e^{2}+deb+2c\,d^{2}\right)\ln\left(x-\_R\right)}{\_R^{2}}}{27d^{2}e^{3}}$
default	$\frac{\frac{\left(5a\,e^{2}+deb-7c\,d^{2}\right)x^{4}}{18d^{2}e}+\frac{\left(4a\,e^{2}-deb-2c\,d^{2}\right)x}{9d\,e^{2}}}{\left(e\,x^{3}+d\right)^{2}}+\frac{\left(5a\,e^{2}+deb+2c\,d^{2}\right)\left(\frac{\ln\left(x+\left(\frac{d}{e}\right)^{\frac{1}{3}}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}-\frac{\ln\left(x^{2}-\left(\frac{d}{e}\right)^{\frac{1}{3}}x+\left(\frac{d}{e}\right)^{\frac{2}{3}}\right)}{6e\left(\frac{d}{e}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\cdots\right.}{\cdots}\right)}{3e\left(\frac{d}{e}\right)^{\frac{2}{3}}}\cdots\right)}{9d^{2}e^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/18*(5*a*e^2+b*d*e-7*c*d^2)/d^2/e*x^4+1/9*(4*a*e^2-b*d*e-2*c*d^2)/d/e^2*x
)/(e*x^3+d)^2+1/9*(5*a*e^2+b*d*e+2*c*d^2)/d^2/e^2*(1/3/e/(d/e)^(2/3)*ln(x+(
d/e)^(1/3))-1/6/e/(d/e)^(2/3)*ln(x^2-(d/e)^(1/3)*x+(d/e)^(2/3))+1/3/e/(d/e)
^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(d/e)^(1/3)*x-1)))
```

Maxima [A]
time = 0.51, size = 203, normalized size = 0.84

$$\frac{\sqrt{3}\left(2\,cd^{2}+bde+5\,ae^{2}\right)\arctan\left(-\frac{\sqrt{3}\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}-2\,x\right)e^{\frac{1}{3}}}{3\,d^{\frac{1}{3}}}\right)e^{\left(-\frac{7}{3}\right)}}{27\,d^{\frac{8}{3}}}-\frac{\left(2\,cd^{2}+bde+5\,ae^{2}\right)e^{\left(-\frac{7}{3}\right)}\log\left(-d^{\frac{1}{3}}xe^{\left(-\frac{1}{3}\right)}+x^{2}+d^{\frac{2}{3}}e^{\left(-\frac{2}{3}\right)}\right)}{54\,d^{\frac{8}{3}}}+\frac{\left(2\,cd^{2}+bde+5\,ae^{2}\right)e^{\left(-\frac{7}{3}\right)}\log\left(d^{\frac{1}{3}}e^{\left(-\frac{1}{3}\right)}+x\right)}{27\,d^{\frac{8}{3}}}-\frac{\left(7\,cd^{2}e-bde^{2}-5\,ae^{3}\right)x^{4}+2\left(2\,cd^{3}+bd^{2}e-4\,ade^{2}\right)x}{18\left(d^{2}x^{6}e^{4}+2\,d^{3}x^{3}e^{3}+d^{4}e^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="maxima")
```

```
[Out] 1/27*sqrt(3)*(2*c*d^2 + b*d*e + 5*a*e^2)*arctan(-1/3*sqrt(3)*(d^(1/3)*e^(-1
/3) - 2*x)*e^(1/3)/d^(1/3))*e^(-7/3)/d^(8/3) - 1/54*(2*c*d^2 + b*d*e + 5*a*
e^2)*e^(-7/3)*log(-d^(1/3)*x*e^(-1/3) + x^2 + d^(2/3)*e^(-2/3))/d^(8/3) + 1
/27*(2*c*d^2 + b*d*e + 5*a*e^2)*e^(-7/3)*log(d^(1/3)*e^(-1/3) + x)/d^(8/3)
- 1/18*((7*c*d^2*e - b*d*e^2 - 5*a*e^3)*x^4 + 2*(2*c*d^3 + b*d^2*e - 4*a*d*
e^2)*x)/(d^2*x^6*e^4 + 2*d^3*x^3*e^3 + d^4*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(202) = 404.
time = 0.35, size = 442, normalized size = 1.83

$$\frac{15\,ad^{2}x^{4}e^{4}-12\,cd^{5}xe+6\sqrt{\frac{1}{3}}\left(5\,adx^{6}e^{5}+2\,cd^{5}e+\left(bd^{2}x^{6}+10\,ad^{2}x^{3}\right)e^{4}+\left(2\,cd^{3}x^{6}+2\,bd^{3}x^{3}+5\,ad^{3}\right)e^{3}+\left(4\,cd^{4}x^{3}+bd^{4}\right)e^{2}\right)\left(d^{2}\right)^{\frac{1}{6}}\arctan\left(\frac{\sqrt{\frac{1}{3}}\left(2\left(d^{2}\right)^{\frac{2}{3}}xe^{\frac{2}{3}}-\left(d^{2}\right)^{\frac{1}{3}}de^{\frac{1}{3}}\right)\left(d^{2}\right)^{\frac{1}{6}}e^{\left(-\frac{1}{3}\right)}}{d^{2}}\right)e^{\left(-\frac{1}{3}\right)}-\left(5\,ax^{6}e^{4}+2\,cd^{4}+\left(bdx^{6}+10\,adx^{3}\right)e^{3}+\left(2\,cd^{2}x^{6}+2\,bd^{2}x^{3}+5\,ad^{2}\right)e^{2}+\left(4\,cd^{3}x^{3}+bd^{3}\right)e\right)\left(d^{2}\right)^{\frac{2}{3}}e^{\frac{2}{3}}\log\left(dx^{2}e-\left(d^{2}\right)^{\frac{2}{3}}xe^{\frac{2}{3}}+\left(d^{2}\right)^{\frac{1}{3}}de^{\frac{1}{3}}\right)+2\left(5\,ax^{6}e^{4}+2\,cd^{4}+\left(bdx^{6}+10\,adx^{3}\right)e^{3}+\left(2\,cd^{2}x^{6}+2\,bd^{2}x^{3}+5\,ad^{2}\right)e^{2}+\left(4\,cd^{3}x^{3}+bd^{3}\right)e\right)\left(d^{2}\right)^{\frac{2}{3}}e^{\frac{2}{3}}\log\left(dxe+\left(d^{2}\right)^{\frac{2}{3}}e^{\frac{2}{3}}\right)+3\left(bd^{3}x^{4}+8\,ad^{3}x\right)e^{3}-3\left(7\,cd^{4}x^{4}+2\,bd^{4}x\right)e^{2}}{54\left(d^{4}x^{6}e^{5}+2\,d^{5}x^{3}e^{4}+d^{6}e^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="fricas")
```

```
[Out] 1/54*(15*a*d^2*x^4*e^4 - 12*c*d^5*x*e + 6*sqrt(1/3)*(5*a*d*x^6*e^5 + 2*c*d^
5*e + (b*d^2*x^6 + 10*a*d^2*x^3)*e^4 + (2*c*d^3*x^6 + 2*b*d^3*x^3 + 5*a*d^3
)*e^3 + (4*c*d^4*x^3 + b*d^4)*e^2)*(d^2)^(1/6)*arctan(sqrt(1/3)*(2*(d^2)^(2
/3)*x*e^(2/3) - (d^2)^(1/3)*d*e^(1/3))*(d^2)^(1/6)*e^(-1/3)/d^2)*e^(-1/3) -
 (5*a*x^6*e^4 + 2*c*d^4 + (b*d*x^6 + 10*a*d*x^3)*e^3 + (2*c*d^2*x^6 + 2*b*d
^2*x^3 + 5*a*d^2)*e^2 + (4*c*d^3*x^3 + b*d^3)*e)*(d^2)^(2/3)*e^(2/3)*log(d*
x^2*e - (d^2)^(2/3)*x*e^(2/3) + (d^2)^(1/3)*d*e^(1/3)) + 2*(5*a*x^6*e^4 + 2
*c*d^4 + (b*d*x^6 + 10*a*d*x^3)*e^3 + (2*c*d^2*x^6 + 2*b*d^2*x^3 + 5*a*d^2)
*e^2 + (4*c*d^3*x^3 + b*d^3)*e)*(d^2)^(2/3)*e^(2/3)*log(d*x*e + (d^2)^(2/3)
*e^(2/3)) + 3*(b*d^3*x^4 + 8*a*d^3*x)*e^3 - 3*(7*c*d^4*x^4 + 2*b*d^4*x)*e^2
)/(d^4*x^6*e^5 + 2*d^5*x^3*e^4 + d^6*e^3)
```

Sympy [A]
time = 11.25, size = 246, normalized size = 1.02

$$\frac{x^{4}\cdot\left(5ae^{3}+bde^{2}-7cd^{2}e\right)+x\left(8ade^{2}-2bd^{2}e-4cd^{3}\right)}{18d^{4}e^{2}+36d^{3}e^{3}x^{3}+18d^{2}e^{4}x^{6}}+\operatorname{RootSum}\left(19683t^{3}d^{8}e^{7}-125a^{3}e^{6}-75a^{2}bde^{5}-150a^{2}cd^{2}e^{4}-15ab^{2}d^{2}e^{4}-60abcd^{3}e^{3}-60ac^{2}d^{4}e^{2}-b^{3}d^{3}e^{3}-6b^{2}cd^{4}e^{2}-12bc^{2}d^{5}e-8c^{3}d^{6},\left(t\mapsto t\log\left(\frac{27td^{3}e^{2}}{5ae^{2}+bde+2cd^{2}}+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**3,x)
```

```
[Out] (x**4*(5*a*e**3 + b*d*e**2 - 7*c*d**2*e) + x*(8*a*d*e**2 - 2*b*d**2*e - 4*c
*d**3))/(18*d**4*e**2 + 36*d**3*e**3*x**3 + 18*d**2*e**4*x**6) + RootSum(19
683*_t**3*d**8*e**7 - 125*a**3*e**6 - 75*a**2*b*d*e**5 - 150*a**2*c*d**2*e*
*4 - 15*a*b**2*d**2*e**4 - 60*a*b*c*d**3*e**3 - 60*a*c**2*d**4*e**2 - b**3*
d**3*e**3 - 6*b**2*c*d**4*e**2 - 12*b*c**2*d**5*e - 8*c**3*d**6, Lambda(_t,
 _t*log(27*_t*d**3*e**2/(5*a*e**2 + b*d*e + 2*c*d**2) + x)))
```

Giac [A]
time = 3.75, size = 224, normalized size = 0.93

$$-\frac{\sqrt{3}\left(2\,cd^{2}+bde+5\,ae^{2}\right)\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-de^{(-1)}\right)^{\frac{1}{3}}\right)}{3\left(-de^{(-1)}\right)^{\frac{1}{3}}}\right)e^{(-1)}}{27\left(-de^{2}\right)^{\frac{2}{3}}d^{2}}-\frac{\left(2\,cd^{2}+bde+5\,ae^{2}\right)e^{(-1)}\log\left(x^{2}+\left(-de^{(-1)}\right)^{\frac{1}{3}}x+\left(-de^{(-1)}\right)^{\frac{2}{3}}\right)}{54\left(-de^{2}\right)^{\frac{2}{3}}d^{2}}-\frac{\left(2\,cd^{2}+bde+5\,ae^{2}\right)\left(-de^{(-1)}\right)^{\frac{1}{3}}e^{(-2)}\log\left(\left|x-\left(-de^{(-1)}\right)^{\frac{1}{3}}\right|\right)}{27\,d^{3}}-\frac{\left(7\,cd^{2}x^{4}e-bdx^{4}e^{2}-5\,ax^{4}e^{3}+4\,cd^{3}x+2\,bd^{2}xe-8\,adxe^{2}\right)e^{(-2)}}{18\left(x^{3}e+d\right)^{2}d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^3,x, algorithm="giac")
```

```
[Out] -1/27*sqrt(3)*(2*c*d^2 + b*d*e + 5*a*e^2)*arctan(1/3*sqrt(3)*(2*x + (-d*e^(
-1))^(1/3))/(-d*e^(-1))^(1/3))*e^(-1)/((-d*e^2)^(2/3)*d^2) - 1/54*(2*c*d^2
+ b*d*e + 5*a*e^2)*e^(-1)*log(x^2 + (-d*e^(-1))^(1/3)*x + (-d*e^(-1))^(2/3)
)/((-d*e^2)^(2/3)*d^2) - 1/27*(2*c*d^2 + b*d*e + 5*a*e^2)*(-d*e^(-1))^(1/3)
*e^(-2)*log(abs(x - (-d*e^(-1))^(1/3)))/d^3 - 1/18*(7*c*d^2*x^4*e - b*d*x^4
*e^2 - 5*a*x^4*e^3 + 4*c*d^3*x + 2*b*d^2*x*e - 8*a*d*x*e^2)*e^(-2)/((x^3*e
+ d)^2*d^2)
```

Mupad [B]
time = 0.29, size = 221, normalized size = 0.91

$$\frac{\ln\left(e^{1/3}\,x+d^{1/3}\right)\,\left(2\,c\,d^{2}+b\,d\,e+5\,a\,e^{2}\right)}{27\,d^{8/3}\,e^{7/3}}-\frac{\frac{x\,\left(2\,c\,d^{2}+b\,d\,e-4\,a\,e^{2}\right)}{9\,d\,e^{2}}-\frac{x^{4}\,\left(-7\,c\,d^{2}+b\,d\,e+5\,a\,e^{2}\right)}{18\,d^{2}\,e}}{d^{2}+2\,d\,e\,x^{3}+e^{2}\,x^{6}}+\frac{\ln\left(2\,e^{1/3}\,x-d^{1/3}+\sqrt{3}\,d^{1/3}\,1{}\mathrm{i}\right)\,\left(-\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,c\,d^{2}+b\,d\,e+5\,a\,e^{2}\right)}{27\,d^{8/3}\,e^{7/3}}-\frac{\ln\left(d^{1/3}-2\,e^{1/3}\,x+\sqrt{3}\,d^{1/3}\,1{}\mathrm{i}\right)\,\left(\frac{1}{2}+\frac{\sqrt{3}\,1{}\mathrm{i}}{2}\right)\,\left(2\,c\,d^{2}+b\,d\,e+5\,a\,e^{2}\right)}{27\,d^{8/3}\,e^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^3,x)
```

```
[Out] (log(e^(1/3)*x + d^(1/3))*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3))
 - ((x*(2*c*d^2 - 4*a*e^2 + b*d*e))/(9*d*e^2) - (x^4*(5*a*e^2 - 7*c*d^2 + b
*d*e))/(18*d^2*e))/(d^2 + e^2*x^6 + 2*d*e*x^3) + (log(3^(1/2)*d^(1/3)*1i +
2*e^(1/3)*x - d^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/
(27*d^(8/3)*e^(7/3)) - (log(3^(1/2)*d^(1/3)*1i - 2*e^(1/3)*x + d^(1/3))*((3
^(1/2)*1i)/2 + 1/2)*(5*a*e^2 + 2*c*d^2 + b*d*e))/(27*d^(8/3)*e^(7/3))
```

3.9 $\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=132

$$\frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{(b^2cd-2ac^2d-b^3e+3abce)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}-\frac{(bcd-b^2e+ace)\log\left(a+bx^3+cx\right.}{6c^3}$$

```
[Out] 1/3*(-b*e+c*d)*x^3/c^2+1/6*e*x^6/c-1/6*(a*c*e-b^2*e+b*c*d)*ln(c*x^6+b*x^3+a
)/c^3-1/3*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b
^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1488, 814, 648, 632, 212, 642}

$$-\frac{(ace+b^2(-e)+bcd)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{(3abce-2ac^2d+b^3(-e)+b^2cd)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^6}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((c*d - b*e)*x^3)/(3*c^2) + (e*x^6)/(6*c) - ((b^2*c*d - 2*a*c^2*d - b^3*e +
 3*a*b*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*c^3*Sqrt[b^2 - 4*a
*c]) - ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^8(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{1}{3}\text{Subst}\left(\int \frac{x^2(d+ex)}{a+bx+cx^2}\,dx,x,x^3\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{cd-be}{c^2}+\frac{ex}{c}-\frac{a(cd-be)+(bcd-b^2e+ace)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx,x,x^3\right)$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{\text{Subst}\left(\int \frac{a(cd-be)+(bcd-b^2e+ace)x}{a+bx+cx^2}\,dx,x,x^3\right)}{3c^2}$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{(bcd-b^2e+ace)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6c^3}+\frac{(b^2cd-2ac^2d}{}$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{(bcd-b^2e+ace)\log\left(a+bx^3+cx^6\right)}{6c^3}-\frac{(b^2cd-2ac^2d-b^3e+}{}$$
$$= \frac{(cd-be)x^3}{3c^2}+\frac{ex^6}{6c}-\frac{(b^2cd-2ac^2d-b^3e+3abce)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^3\sqrt{b^2-4ac}}-\frac{(bcd-}{}$$

Mathematica [A]
time = 0.05, size = 126, normalized size = 0.95

$$\frac{2c(cd-be)x^3+c^2ex^6+\frac{2(b^2cd-2ac^2d-b^3e+3abce)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bcd+b^2e-ace)\log\left(a+bx^3+cx^6\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*(c*d - b*e)*x^3 + c^2*e*x^6 + (2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*c*
d) + b^2*e - a*c*e)*Log[a + b*x^3 + c*x^6])/(6*c^3)
```

Maple [A]
time = 0.13, size = 136, normalized size = 1.03

method	result	size
default	$-\frac{-\frac{1}{2} c e \,x^{6}+b e \,x^{3}-c d \,x^{3}}{3 c^{2}}+\frac{\frac{\left(-a c e +b^{2} e -b c d \right) \ln \left(c \,x^{6}+b \,x^{3}+a \right)}{2 c}+\frac{2 \left(a b e -a c d -\frac{\left(-a c e +b^{2} e -b c d \right) b}{2 c}\right) \arctan \left(\frac{2 c \,x^{3}+b}{\sqrt{4 a c -b^{2}}}\right)}{\sqrt{4 a c -b^{2}}}}{3 c^{2}}$	136
risch	Expression too large to display	2131

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/c^2*(-1/2*c*e*x^6+b*e*x^3-c*d*x^3)+1/3/c^2*(1/2*(-a*c*e+b^2*e-b*c*d)/c
*ln(c*x^6+b*x^3+a)+2*(a*b*e-a*c*d-1/2*(-a*c*e+b^2*e-b*c*d)*b/c)/(4*a*c-b^2)
^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.59, size = 440, normalized size = 3.33

$$\left[\frac{2 \,{\left(b^{2} c^{2} - 4 \, a c^{3}\right)} d x^{3} + \sqrt{b^{2} - 4 \, a c} {\left({\left(b^{2} c - 2 \, a c^{2}\right)} d - {\left(b^{3} - 3 \, a b c\right)} e\right)} \log\left(\frac{2 \, c^{2} x^{6} + 2 \, b c x^{3} + b^{2} - 2 \, a c - {\left(2 \, c x^{3} + b\right)} \sqrt{b^{2} - 4 \, a c}}{c x^{6} + b x^{3} + a}\right) + {\left({\left(b^{2} c^{2} - 4 \, a c^{3}\right)} x^{6} - 2 \, {\left(b^{3} c - 4 \, a b c^{2}\right)} x^{3}\right)} e - {\left({\left(b^{3} c - 4 \, a b c^{2}\right)} d - {\left(b^{4} - 5 \, a b^{2} c + 4 \, a^{2} c^{2}\right)} e\right)} \log\left(c x^{6} + b x^{3} + a\right)}{6 \, {\left(b^{2} c^{3} - 4 \, a c^{4}\right)}}, \frac{2 \,{\left(b^{2} c^{2} - 4 \, a c^{3}\right)} d x^{3} - 2 \, \sqrt{-b^{2} + 4 \, a c} {\left({\left(b^{2} c - 2 \, a c^{2}\right)} d - {\left(b^{3} - 3 \, a b c\right)} e\right)} \arctan\left(-\frac{{\left(2 \, c x^{3} + b\right)} \sqrt{-b^{2} + 4 \, a c}}{b^{2} - 4 \, a c}\right) + {\left({\left(b^{2} c^{2} - 4 \, a c^{3}\right)} x^{6} - 2 \, {\left(b^{3} c - 4 \, a b c^{2}\right)} x^{3}\right)} e - {\left({\left(b^{3} c - 4 \, a b c^{2}\right)} d - {\left(b^{4} - 5 \, a b^{2} c + 4 \, a^{2} c^{2}\right)} e\right)} \log\left(c x^{6} + b x^{3} + a\right)}{6 \, {\left(b^{2} c^{3} - 4 \, a c^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c^2 - 4*a*c^3)*d*x^3 + sqrt(b^2 - 4*a*c)*((b^2*c - 2*a*c^2)*d
- (b^3 - 3*a*b*c)*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c - (2*c*x^3 +
b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)) + ((b^2*c^2 - 4*a*c^3)*x^6 - 2*(
b^3*c - 4*a*b*c^2)*x^3)*e - ((b^3*c - 4*a*b*c^2)*d - (b^4 - 5*a*b^2*c + 4*a
^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4), 1/6*(2*(b^2*c^2 - 4
*a*c^3)*d*x^3 - 2*sqrt(-b^2 + 4*a*c)*((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)
*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4
*a*c^3)*x^6 - 2*(b^3*c - 4*a*b*c^2)*x^3)*e - ((b^3*c - 4*a*b*c^2)*d - (b^4
- 5*a*b^2*c + 4*a^2*c^2)*e)*log(c*x^6 + b*x^3 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.08, size = 131, normalized size = 0.99

$$\frac{cx^6e + 2\,cdx^3 - 2\,bx^3e}{6\,c^2} - \frac{(bcd - b^2e + ace)\log\left(cx^6 + bx^3 + a\right)}{6\,c^3} + \frac{(b^2cd - 2\,ac^2d - b^3e + 3\,abce)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(c*x^6*e + 2*c*d*x^3 - 2*b*x^3*e)/c^2 - 1/6*(b*c*d - b^2*e + a*c*e)*log
(c*x^6 + b*x^3 + a)/c^3 + 1/3*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arc
tan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [**B**]
time = 2.40, size = 2500, normalized size = 18.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] x^3*(d/(3*c) - (b*e)/(3*c^2)) + (e*x^6)/(6*c) - (log(a + b*x^3 + c*x^6)*(3*
b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c
^4 - 9*b^2*c^3)) - (atan((4*c^6*(4*a*c - b^2)^(3/2)*(x^3*((b*((b^5*c^3*d^3
- b^8*e^3 - 2*a*b^3*c^4*d^3 + a^2*b*c^5*d^3 + a^3*c^5*d^2*e - 3*b^6*c^2*d^2
*e - 8*a^2*b^4*c^2*e^3 + 4*a^3*b^2*c^3*e^3 + 5*a*b^6*c*e^3 + 3*b^7*c*d*e^2
```

+ 9*a*b^4*c^3*d^2*e - 12*a*b^5*c^2*d*e^2 - 4*a^3*b*c^4*d*e^2 - 7*a^2*b^2*c^
4*d^2*e + 14*a^2*b^3*c^3*d*e^2)/c^6 - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d^2 + 1
2*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*e^2 -
24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3*c^7*d
- 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e
+ 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b
^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e)
)/(2*(36*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*
c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((45*b^3*c^7*d - 45*
b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e + 12*
a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3
))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) -
(9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e -
3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^4
- 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c -
b^2)^(1/2)) + (3*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^2*(3*b^4*e +
12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(4*c^3*(4*a*c - b
^2)*(36*a*c^4 - 9*b^2*c^3))))/(4*a^2*c) - ((2*a*c - b^2)*((((((45*b^3*c^7*d
- 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3*b^4*e
+ 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b
^2*c^3))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1
/2)) - (9*b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c
^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(4*a*c - b^2)^(1/2)*(36
*a*c^4 - 9*b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d -
15*a*b^2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((6*a^2*c^7*d^2 + 12*b^4*c^5*d
^2 + 12*b^6*c^3*e^2 - 18*a*b^2*c^6*d^2 - 42*a*b^4*c^4*e^2 + 36*a^2*b^2*c^5*
e^2 - 24*b^5*c^4*d*e + 60*a*b^3*c^5*d*e - 30*a^2*b*c^6*d*e)/c^6 - (((45*b^3
*c^7*d - 45*b^4*c^6*e - 36*a*b*c^8*d + 81*a*b^2*c^7*e)/c^6 - (27*b^2*c^3*(3
*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4
- 9*b^2*c^3))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^
2*c*e))/(2*(36*a*c^4 - 9*b^2*c^3)))*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*
e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (b^2*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b
*c*e)^3)/(4*c^6*(4*a*c - b^2)^(3/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))) - (b*
((a*b^7*e^3 - a*b^4*c^3*d^3 - 4*a^2*b^5*c*e^3 - 2*a^4*b*c^3*e^3 + a^4*c^4*d
*e^2 + a^2*b^2*c^4*d^3 + 5*a^3*b^3*c^2*e^3 - 3*a*b^6*c*d*e^2 + 3*a*b^5*c^2*
d^2*e + 2*a^3*b*c^4*d^2*e - 6*a^2*b^3*c^3*d^2*e + 9*a^2*b^4*c^2*d*e^2 - 7*a
^3*b^2*c^3*d*e^2)/c^6 + (((15*a*b^3*c^5*d^2 - 12*a^2*b*c^6*d^2 + 15*a*b^5*c
^3*e^2 + 27*a^3*b*c^5*e^2 - 42*a^2*b^3*c^4*e^2 - 12*a^3*c^6*d*e - 30*a*b^4*
c^4*d*e + 54*a^2*b^2*c^5*d*e)/c^6 + (((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a
*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3
*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(3*b^4*e +
12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*(36*a*c^4 - 9*
b^2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*
e))/(2*(36*a*c^4 - 9*b^2*c^3)) - (((((36*a^2*c^8*d - 72*a*b^2*c^7*d + 72*a*
b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c^2*e - 3*

```
b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(b^3*e + 2*
a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (9*a*b*(b^3*e
 + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 1
2*a*b*c^2*d - 15*a*b^2*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^2*c^3)))*
(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) - (3
*a*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)^2*(3*b^4*e + 12*a^2*c^2*e -
3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(2*c^3*(4*a*c - b^2)*(36*a*c^4 -
9*b^2*c^3))))/(4*a^2*c) + ((2*a*c - b^2)*((((((36*a^2*c^8*d - 72*a*b^2*c^7*
d + 72*a*b^3*c^6*e - 108*a^2*b*c^7*e)/c^6 + (54*a*b*c^3*(3*b^4*e + 12*a^2*c
^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/(36*a*c^4 - 9*b^2*c^3))*(b
^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e))/(6*c^3*(4*a*c - b^2)^(1/2)) + (9*a
*b*(b^3*e + 2*a*c^2*d - b^2*c*d - 3*a*b*c*e)*(3*b^4*e + 12*a^2*c^2*e - 3*b^
3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^4 - 9*b^
2*c^3)))*(3*b^4*e + 12*a^2*c^2*e - 3*b^3*c*d + 12*a*b*c^2*d - 15*a*b^2*c*e)
)/(2*(36*a*c^4 - 9*b^2*c^3)) + (((15*a*b^3*c^5*d^2 - 12*a^2*b*c^6*d^2 + 15*
a*b^5*c^3*e^2 + 27*a^3*b*c^5*e^2 - 42*a^2*b^3*c^4*e^2 - 12*a^3*c^6*d*e - 30
*a*b^4*c^4*d*e + 54*a^2*b^2*c^5*d*e)/c^6 + (((3...
```

3.10 $\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=97

$$\frac{ex^3}{3c}+\frac{(bcd-b^2e+2ace)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}$$

```
[Out] 1/3*e*x^3/c+1/6*(-b*e+c*d)*ln(c*x^6+b*x^3+a)/c^2+1/3*(2*a*c*e-b^2*e+b*c*d)*
arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1488, 787, 648, 632, 212, 642}

$$\frac{(2ace+b^2(-e)+bcd)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{ex^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^3)/(3*c) + ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 -
 4*a*c]])/(3*c^2*Sqrt[b^2 - 4*a*c]) + ((c*d - b*e)*Log[a + b*x^3 + c*x^6])/
(6*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(d+ex^3)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{x(d+ex)}{a+bx+cx^2}\,dx,x,x^3\right)\\ &= \frac{ex^3}{3c}+\frac{\text{Subst}\left(\int \frac{-ae+(cd-be)x}{a+bx+cx^2}\,dx,x,x^3\right)}{3c}\\ &= \frac{ex^3}{3c}+\frac{(cd-be)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6c^2}-\frac{(bcd-b^2e+2ace)\,\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,d\right.}{6c^2}\\ &= \frac{ex^3}{3c}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}+\frac{(bcd-b^2e+2ace)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b\right.}{3c^2}\\ &= \frac{ex^3}{3c}+\frac{(bcd-b^2e+2ace)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c^2\sqrt{b^2-4ac}}+\frac{(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 93, normalized size = 0.96

$$\frac{2cex^3+\frac{2(-bcd+b^2e-2ace)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(cd-be)\log\left(a+bx^3+cx^6\right)}{6c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*c*e*x^3 + (2*(-(b*c*d) + b^2*e - 2*a*c*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2
 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*d - b*e)*Log[a + b*x^3 + c*x^6])/(6*c^2
)
```

Maple [A]
time = 0.09, size = 98, normalized size = 1.01

method	result
default	$\frac{e\,x^{3}}{3c}+\frac{\frac{\left(-eb+cd\right)\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2c}+\frac{2\left(-ae-\frac{\left(-eb+cd\right)b}{2c}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3c}$
risch	$\frac{e\,x^{3}}{3c}-\frac{2\ln\left(\left(\left(-8a^{2}c^{2}e+6a\,b^{2}ce-4ab\,c^{2}d-b^{4}e+b^{3}cd+\sqrt{-\left(4ac-b^{2}\right)\left(2ace-b^{2}e+bcd\right)^{2}}\,b\right)x^{3}+2\sqrt{-\left(4ac-b^{2}\right.}\right.\right.}{3c\left(4ac-b^{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*e*x^3/c+1/3/c*(1/2*(-b*e+c*d)/c*ln(c*x^6+b*x^3+a)+2*(-a*e-1/2*(-b*e+c*d
)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.42, size = 311, normalized size = 3.21

$$\left[\frac{2\,(b^{2}c-4\,ac^{2})x^{3}e+(bcd-(b^{2}-2\,ac)e)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{6}+2\,bcx^{3}+b^{2}-2\,ac+(2\,cx^{3}+b)\sqrt{b^{2}-4\,ac}}{cx^{6}+bx^{3}+a}\right)+((b^{2}c-4\,ac^{2})d-(b^{3}-4\,abc)e)\log\left(cx^{6}+bx^{3}+a\right)}{6\,(b^{2}c^{2}-4\,ac^{3})},\ \frac{2\,(b^{2}c-4\,ac^{2})x^{3}e+2\,(bcd-(b^{2}-2\,ac)e)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{3}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)+((b^{2}c-4\,ac^{2})d-(b^{3}-4\,abc)e)\log\left(cx^{6}+bx^{3}+a\right)}{6\,(b^{2}c^{2}-4\,ac^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(b^2*c - 4*a*c^2)*x^3*e + (b*c*d - (b^2 - 2*a*c)*e)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*
```

```
c))/(c*x^6 + b*x^3 + a)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3), 1/6*(2*(b^2*c - 4*a*c^2)*x^3*e + 2*(
b*c*d - (b^2 - 2*a*c)*e)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*d - (b^3 - 4*a*b*c)*e)*log(c*
x^6 + b*x^3 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(94) = 188$.
time = 96.43, size = 434, normalized size = 4.47

$$\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(-\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)}{2ace-b^2e+bcd}\right)+\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)\log\left(x^3+\frac{-abe-12ac^2\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)+2acd+3b^2c\left(\frac{\sqrt{-4ac+b^2}\cdot(2ace-b^2e+bcd)}{6c^2\cdot(4ac-b^2)}-\frac{be-cd}{6c^2}\right)}{2ace-b^2e+bcd}\right)+\frac{ex^3}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) -
(b*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(-sqrt(-4*a*c + b**2)*
(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2))
+ 2*a*c*d + 3*b**2*c*(-sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c*
*2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + (
sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b
*e - c*d)/(6*c**2))*log(x**3 + (-a*b*e - 12*a*c**2*(sqrt(-4*a*c + b**2)*(2*
a*c*e - b**2*e + b*c*d)/(6*c**2*(4*a*c - b**2)) - (b*e - c*d)/(6*c**2)) + 2
*a*c*d + 3*b**2*c*(sqrt(-4*a*c + b**2)*(2*a*c*e - b**2*e + b*c*d)/(6*c**2*(
4*a*c - b**2)) - (b*e - c*d)/(6*c**2)))/(2*a*c*e - b**2*e + b*c*d)) + e*x**
3/(3*c)
```

Giac [**A**]
time = 3.24, size = 95, normalized size = 0.98

$$\frac{x^3e}{3\,c}+\frac{(cd-be)\log\left(cx^6+bx^3+a\right)}{6\,c^2}-\frac{\left(bcd-b^2e+2\,ace\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*x^3*e/c + 1/6*(c*d - b*e)*log(c*x^6 + b*x^3 + a)/c^2 - 1/3*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

Mupad [**B**]
time = 2.95, size = 2624, normalized size = 27.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^3))/(a + b*x^3 + c*x^6),x)

[Out] (e*x^3)/(3*c) + (log(a + b*x^3 + c*x^6)*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d -
 12*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) + (atan((4*c^3*(4*a*c - b^2)^(3/2)
*(x^3*((b*((b^2*c^3*d^3 - b^5*e^3 - a^2*b*c^2*e^3 + a^2*c^3*d*e^2 - 3*b^3*c
^2*d^2*e + 2*a*b^3*c*e^3 + 3*b^4*c*d*e^2 + 2*a*b*c^3*d^2*e - 4*a*b^2*c^2*d*
e^2)/c^3 - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b^2*c^
3*e^2 - 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b^3*c^4
*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a
*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((45*b^2*c^5*d - 45*b^3*c^4*e +
36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*
c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^
2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a*c^2*d - 3*b^
2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c
*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*b^2*(2*a*c*e - b^2*e
+ b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(4*c*(4*a*c - b
^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*((((((45*b^2*c^5*d
 - 45*b^3*c^4*e + 36*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3
*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/
(6*c^2*(4*a*c - b^2)^(1/2)) - (9*b^2*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e +
 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(4*a*c - b^2)^(1/2)*(36*a*c^3 - 9
*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 -
 9*b^2*c^2)) + (b^2*(2*a*c*e - b^2*e + b*c*d)^3)/(4*c^3*(4*a*c - b^2)^(3/2)
) - (((6*a^2*c^4*e^2 + 12*b^2*c^4*d^2 + 12*b^4*c^2*e^2 - 18*a*b^2*c^3*e^2 -
 24*b^3*c^3*d*e + 18*a*b*c^4*d*e)/c^3 - (((45*b^2*c^5*d - 45*b^3*c^4*e + 36
*a*b*c^5*e)/c^3 - (27*b^2*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*
e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e)
)/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^
2)^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2))) + (b*((a^2*b^2*c*e^3 - a*b^4*e^3
 + a^2*c^3*d^2*e + a*b*c^3*d^3 + 3*a*b^3*c*d*e^2 - 3*a*b^2*c^2*d^2*e - 2*a^
2*b*c^2*d*e^2)/c^3 - (((15*a*b^3*c^2*e^2 - 12*a^2*b*c^3*e^2 + 15*a*b*c^4*d^
2 + 12*a^2*c^4*d*e - 30*a*b^2*c^3*d*e)/c^3 - (((36*a^2*c^5*e + 72*a*b*c^5*d
 - 72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a
*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12
*a*b*c*e))/(2*(36*a*c^3 - 9*b^2*c^2)) - (((((36*a^2*c^5*e + 72*a*b*c^5*d -
72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*
b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c -
b^2)^(1/2)) - (9*a*b*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e + 12*a*c^2*d - 3*
b^2*c*d - 12*a*b*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c
*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b^2)^(1/2)) + (3*a*b*(2*a*c*e - b^2*e
+ b*c*d)^2*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*c*(4*a*c - b
^2)*(36*a*c^3 - 9*b^2*c^2))))/(4*a^2*c) + ((2*a*c - b^2)*((((((36*a^2*c^5*e
```

```
 + 72*a*b*c^5*d - 72*a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d -
 3*b^2*c*d - 12*a*b*c*e))/(36*a*c^3 - 9*b^2*c^2))*(2*a*c*e - b^2*e + b*c*d)
)/(6*c^2*(4*a*c - b^2)^(1/2)) - (9*a*b*c*(2*a*c*e - b^2*e + b*c*d)*(3*b^3*e
 + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/((4*a*c - b^2)^(1/2)*(36*a*c^3 - 9
*b^2*c^2)))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e))/(2*(36*a*c^3 -
 9*b^2*c^2)) - (((15*a*b^3*c^2*e^2 - 12*a^2*b*c^3*e^2 + 15*a*b*c^4*d^2 + 12
*a^2*c^4*d*e - 30*a*b^2*c^3*d*e)/c^3 - (((36*a^2*c^5*e + 72*a*b*c^5*d - 72*
a*b^2*c^4*e)/c^3 - (54*a*b*c^3*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c
*e))/(36*a*c^3 - 9*b^2*c^2))*(3*b^3*e + 12*a*c^2*d - 3*b^2*c*d - 12*a*b*c*e
))/(2*(36*a*c^3 - 9*b^2*c^2)))*(2*a*c*e - b^2*e + b*c*d))/(6*c^2*(4*a*c - b
^2)^(1/2)) + (a*b*(2*a*c*e - b^2*e + b*c*d)^3)/(2*c^3*(4*a*c - b^2)^(3/2)))
)/(4*a^2*c*(4*a*c - b^2)^(1/2))))/(8*a^3*c^3*e^3 - b^6*e^3 + b^3*c^3*d^3 -
3*b^4*c^2*d^2*e - 12*a^2*b^2*c^2*e^3 + 6*a*b^4*c*e^3 + 3*b^5*c*d*e^2 + 6*a*
b^2*c^3*d^2*e - 12*a*b^3*c^2*d*e^2 + 12*a^2*b*c^3*d*e^2))*(2*a*c*e - b^2*e
+ b*c*d))/(3*c^2*(4*a*c - b^2)^(1/2))
```

3.11 $\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=72

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{e\log\left(a+bx^3+cx^6\right)}{6c}$$

```
[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c-1/3*(-b*e+2*c*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1482, 648, 632, 212, 642}

$$\frac{e\log\left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -1/3*((2*c*d - b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 -
 4*a*c]) + (e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(d+ex^3)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx,x,x^3\right)\\
&= \frac{e\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6c}+\frac{(2cd-be)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6c}\\
&= \frac{e\log\left(a+bx^3+cx^6\right)}{6c}-\frac{(2cd-be)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3c}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3c\sqrt{b^2-4ac}}+\frac{e\log\left(a+bx^3+cx^6\right)}{6c}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.99

$$\frac{-\frac{2(-2cd+be)\tan^{-1}\left(\frac{b+2cx^3}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+e\log\left(a+bx^3+cx^6\right)}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^3)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^3 + c*x^6])/(6*c)
```

Maple [A]
time = 0.05, size = 66, normalized size = 0.92

method	result

default	$\frac{e\ln(c\,x^6+b\,x^3+a)}{6c}+\frac{2\left(d-\frac{be}{2c}\right)\arctan\left(\frac{2c\,x^3+b}{\sqrt{4ac-b^2}}\right)}{3\sqrt{4ac-b^2}}$
risch	$\frac{2\ln\left(\left(\left(-4abce+8a\,c^2d+b^3e-2b^2cd+\sqrt{-\left(eb-2cd\right)^2\left(4ac-b^2\right)}\,b\right)x^3+2\sqrt{-\left(eb-2cd\right)^2\left(4ac-b^2\right)}\,a\right)ae}{3(4ac-b^2)}$ ln − −

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*e*ln(c*x^6+b*x^3+a)/c+2/3*(d-1/2/c*b*e)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^3+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 220, normalized size = 3.06

$$\left[\frac{(b^2-4\,ac)e\log\left(cx^6+bx^3+a\right)-\sqrt{b^2-4\,ac}\,(2\,cd-be)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac+(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{6\,(b^2c-4\,ac^2)},\frac{(b^2-4\,ac)e\log\left(cx^6+bx^3+a\right)-2\sqrt{-b^2+4\,ac}\,(2\,cd-be)\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{6\,(b^2c-4\,ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((b^2 - 4*a*c)*e*log(c*x^6 + b*x^3 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a
*c))/(c*x^6 + b*x^3 + a)))/(b^2*c - 4*a*c^2), 1/6*((b^2 - 4*a*c)*e*log(c*x^
6 + b*x^3 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^3 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(65) = 130$.
time = 9.69, size = 287, normalized size = 3.99

$$\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c\,(4ac-b^2)}\right)\log\left(x^3+\frac{-12ac\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c(4ac-b^2)}\right)-bd}{be-2cd}\right)+\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c\,(4ac-b^2)}\right)\log\left(x^3+\frac{-12ac\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c(4ac-b^2)}\right)+2ae+3b^2\left(\frac{e}{6c}+\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{6c(4ac-b^2)}\right)-bd}{be-2cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] (e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3
 + (-12*a*c*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2
))) + 2*a*e + 3*b**2*(e/(6*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(6*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(6*c*(4*a*c - b**2)))*log(x**3 + (-12*a*c*(e/(6*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) + 2*a*e + 3*b**2*(e/(6*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(6*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

Giac [A]
time = 3.16, size = 70, normalized size = 0.97

$$\frac{e\log\left(cx^{6}+bx^{3}+a\right)}{6\,c}+\frac{\left(2\,cd-be\right)\arctan\left(\frac{2\,cx^{3}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{3\,\sqrt{-b^{2}+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*e*log(c*x^6 + b*x^3 + a)/c + 1/3*(2*c*d - b*e)*arctan((2*c*x^3 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

Mupad [B]
time = 2.63, size = 1632, normalized size = 22.67

$$[illegible]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] - (log(a + b*x^3 + c*x^6)*(3*b^2*e - 12*a*c*e))/(2*(36*a*c^2 - 9*b^2*c)) -
(atan((b*(4*a*c - b^2)^(3/2)*(a*c*d*e^2 - a*b*e^3 - ((3*b^2*e - 12*a*c*e)*(
((3*b^2*e - 12*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 1
2*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 -
12*a*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) + ((((b*e - 2*c*d)*(72*a*b*c^2*e -
36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*
(4*a*c - b^2)^(1/2)) + (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*
a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(6*c*(4*a*c - b^2)^(1
/2)) + (3*a*b*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(2*(36*a*c^2 - 9*b^2*
c)*(4*a*c - b^2))))/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*
d*e^2)) - (4*x^3*((b*(b^2*e^3 + c^2*d^2*e + ((3*b^2*e - 12*a*c*e)*(6*c^3*d^
```

```
2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e
 - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^
2 - 18*b*c^2*d*e))/(2*(36*a*c^2 - 9*b^2*c)) - 2*b*c*d*e^2 - ((((b*e - 2*c*d
)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2
- 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^2*c^2*(3*b^2*e - 12*a*c*e)*(b
*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(
6*c*(4*a*c - b^2)^(1/2)) - (3*b^2*c*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d)^2)/(
4*(36*a*c^2 - 9*b^2*c)*(4*a*c - b^2))))/(4*a^2*c) - ((2*a*c - b^2)*(((3*b^2
*e - 12*a*c*e)*(((b*e - 2*c*d)*(45*b^2*c^2*e - 36*b*c^3*d + (27*b^2*c^3*(3*
b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) + (9*b^
2*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/(2*(36*a*c^2 - 9*b^2*c)*(4*a*c -
b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) - (b^2*(b*e - 2*c*d)^3)/(4*(4*a*c -
b^2)^(3/2)) + ((b*e - 2*c*d)*(6*c^3*d^2 + ((3*b^2*e - 12*a*c*e)*(45*b^2*c^2
*e - 36*b*c^3*d + (27*b^2*c^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/
(2*(36*a*c^2 - 9*b^2*c)) + 12*b^2*c*e^2 - 18*b*c^2*d*e))/(6*c*(4*a*c - b^2)
^(1/2))))/(4*a^2*c*(4*a*c - b^2)^(1/2)))*(4*a*c - b^2)^(3/2))/(b^3*e^3 - 8*
c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2) + ((2*a*c - b^2)*(4*a*c - b^2)*((
(3*b^2*e - 12*a*c*e)*(((b*e - 2*c*d)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c
^3*(3*b^2*e - 12*a*c*e))/(36*a*c^2 - 9*b^2*c)))/(6*c*(4*a*c - b^2)^(1/2)) +
 (9*a*b*c^2*(3*b^2*e - 12*a*c*e)*(b*e - 2*c*d))/((36*a*c^2 - 9*b^2*c)*(4*a*
c - b^2)^(1/2))))/(2*(36*a*c^2 - 9*b^2*c)) + ((b*e - 2*c*d)*(((3*b^2*e - 12
*a*c*e)*(72*a*b*c^2*e - 36*a*c^3*d + (54*a*b*c^3*(3*b^2*e - 12*a*c*e))/(36*
a*c^2 - 9*b^2*c)))/(2*(36*a*c^2 - 9*b^2*c)) + 15*a*b*c*e^2 - 12*a*c^2*d*e))
/(6*c*(4*a*c - b^2)^(1/2)) - (a*b*(b*e - 2*c*d)^3)/(2*(4*a*c - b^2)^(3/2)))
)/(a^2*c*(b^3*e^3 - 8*c^3*d^3 + 12*b*c^2*d^2*e - 6*b^2*c*d*e^2)))*(b*e - 2*
c*d))/(3*c*(4*a*c - b^2)^(1/2))
```

3.12 $\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}$$

```
[Out] d*ln(x)/a-1/6*d*ln(c*x^6+b*x^3+a)/a+1/3*(-2*a*e+b*d)*arctanh((2*c*x^3+b)/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1488, 814, 648, 632, 212, 642}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2 - 4*a*c]])/(3*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^3 + c*x^6])/(6*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^3}{x\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^3\right)\\
&= \frac{d\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx,x,x^3\right)}{3a}\\
&= \frac{d\log(x)}{a}-\frac{d\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}+\frac{(-bd+2ae)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^3\right)}{6a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}-\frac{(-bd+2ae)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^3\right)}{3a}\\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^3+cx^6\right)}{6a}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 80, normalized size = 1.03

$$\frac{d\log(x)}{a}-\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a)
```

Maple [A]
time = 0.06, size = 75, normalized size = 0.96

method	result
default	$\frac{-\frac{d\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2}+\frac{2\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a}+\frac{d\ln(x)}{a}$
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c-a\,b^{2}\right)\_Z^{2}+\left(4acd-b^{2}d\right)\_Z+a\,e^{2}-deb+c\,d^{2}\right)}\_R\ln\left(\left(\left(-14ac+4b^{2}\right)\_R^{2}+\left(eb-7cd\right)\_R-3e^{2}\right)x^{3}+b\right.}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*(-1/2*d*ln(c*x^6+b*x^3+a)+2*(a*e-1/2*b*d)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^3+b)/(4*a*c-b^2)^(1/2)))+d*ln(x)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.47, size = 242, normalized size = 3.10

$$\left[-\frac{(b^2-4\,ac)d\log\left(cx^6+bx^3+a\right)-6\,(b^2-4\,ac)d\log\left(x\right)+\sqrt{b^2-4\,ac}\,(bd-2\,ae)\log\left(\frac{2\,c^2x^6+2\,bcx^3+b^2-2\,ac-(2\,cx^3+b)\sqrt{b^2-4\,ac}}{cx^6+bx^3+a}\right)}{6\,(ab^2-4\,a^2c)},-\frac{(b^2-4\,ac)d\log\left(cx^6+bx^3+a\right)-6\,(b^2-4\,ac)d\log\left(x\right)-2\,\sqrt{-b^2+4\,ac}\,(bd-2\,ae)\arctan\left(-\frac{(2\,cx^3+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{6\,(ab^2-4\,a^2c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [-1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^6 + 2*b*c*x^3 + b^2 - 2*a*c -
(2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 + a)))/(a*b^2 - 4*a^2*c), -
1/6*((b^2 - 4*a*c)*d*log(c*x^6 + b*x^3 + a) - 6*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.46, size = 76, normalized size = 0.97

$$-\frac{d\log\left(cx^6+bx^3+a\right)}{6\,a}+\frac{d\log\left(|x|\right)}{a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] -1/6*d*log(c*x^6 + b*x^3 + a)/a + d*log(abs(x))/a - 1/3*(b*d - 2*a*e)*arcta
n((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

Mupad [**B**]
time = 6.76, size = 2500, normalized size = 32.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (d*log(x))/a - (log(a + b*x^3 + c*x^6)*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 -
36*a^2*c)) - (atan((48*a^4*x^3*(4*a*c - b^2)^2*(((((((3*b^2*d - 12*a*c*d)*(
((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*
a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*
a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*
a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 -
 36*a^2*c)) - ((2*a*e - b*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12
*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 -
```

```
 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 3
6*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(3*b^2*d - 12*a*c*d))
/(2*(9*a*b^2 - 36*a^2*c)) + ((2*a*e - b*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*
c*d)*(42*a*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*
a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4
*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e
))/(2*(9*a*b^2 - 36*a^2*c)) + 7*c^4*d*e^2))/(6*a*(4*a*c - b^2)^(1/2)) - (((
(2*a*e - b*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b
^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c
^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378
*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2)))
)/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^
2*c^4)*(2*a*e - b*d)^2)/(72*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(2*a*e
 - b*d))/(6*a*(4*a*c - b^2)^(1/2)) - ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 3
78*a*b^2*c^4)*(2*a*e - b*d)^3)/(432*a^3*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^
(3/2)))*(4*b^4*d + 7*a^2*c^2*d - a*b^3*e - 15*a*b^2*c*d + 2*a^2*b*c*e))/(16
*a^4*c^3*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e)) - ((c^3*e^4 - ((3*b
^2*d - 12*a*c*d)*(5*b*c^3*e^3 - ((3*b^2*d - 12*a*c*d)*(42*a*c^4*e^2 - 9*b^2
*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*
a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*
b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(2*(9*a*b^2 - 36*a^2*c)
) + 7*c^4*d*e^2))/(2*(9*a*b^2 - 36*a^2*c)) + ((((2*a*e - b*d)*(((2*a*e - b*
d)*(((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a
^2*c)) + 63*b^2*c^4*d - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(
1/2)) + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/
(12*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(6*a*(4*a*c - b^2)^(1/2))
 + ((3*b^2*d - 12*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d)^2)/(72
*a^2*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)))*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2
 - 36*a^2*c)) + ((((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*(((3*b^2*d - 12*a*c
*d)*(108*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d
 - 81*b^3*c^3*e + 252*a*b*c^4*e))/(6*a*(4*a*c - b^2)^(1/2)) + ((3*b^2*d - 12
*a*c*d)*(108*b^4*c^3 - 378*a*b^2*c^4)*(2*a*e - b*d))/(12*a*(9*a*b^2 - 36*a^
2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) - ((2*a*e - b*d)*(42*a
*c^4*e^2 - 9*b^2*c^3*e^2 - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(10
8*b^4*c^3 - 378*a*b^2*c^4))/(2*(9*a*b^2 - 36*a^2*c)) + 63*b^2*c^4*d - 81*b^
3*c^3*e + 252*a*b*c^4*e))/(2*(9*a*b^2 - 36*a^2*c)) + 42*b*c^4*d*e))/(6*a*(4
*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(6*a*(4*a*c - b^2)^(1/2)) - ((108*b^4*c^
3 - 378*a*b^2*c^4)*(2*a*e - b*d)^4)/(1296*a^4*(4*a*c - b^2)^2))*(4*b^5*d -
2*a^3*c^2*e - a*b^4*e - 23*a*b^3*c*d + 29*a^2*b*c^2*d + 4*a^2*b^2*c*e))/(16
*a^4*c^3*(4*a*c - b^2)^(1/2)*(a^2*e^2 - 12*b^2*d^2 + 49*a*c*d^2 - a*b*d*e))
))/(8*a^3*c^3*e^3 - b^3*c^3*d^3 + 6*a*b^2*c^3*d^2*e - 12*a^2*b*c^3*d*e^2) -
 (3*(4*a*c - b^2)^(3/2)*(c^3*d*e^3 + ((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*
(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 1
2*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3
*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^
```

```
(1/2))))/(6*a*(4*a*c - b^2)^(1/2)) + (3*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e
 - b*d)^2)/(8*a*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2))))/(2*(9*a*b^2 - 36*a^2*
c)) - ((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(((3*b^2*d - 12*a*c*d)*(
27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a
*b^2 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d
*e))/(2*(9*a*b^2 - 36*a^2*c)) - a*c^3*e^3 + 9*b*c^3*d*e^2))/(2*(9*a*b^2 - 3
6*a^2*c)) + ((((3*b^2*d - 12*a*c*d)*(((2*a*e - b*d)*(27*b^3*c^3*d - 27*a*b^
2*c^3*e + (27*a*b^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(6
*a*(4*a*c - b^2)^(1/2)) + (9*b^3*c^3*(3*b^2*d - 12*a*c*d)*(2*a*e - b*d))/(4
*(9*a*b^2 - 36*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(9*a*b^2 - 36*a^2*c)) + ((2
*a*e - b*d)*(((3*b^2*d - 12*a*c*d)*(27*b^3*c^3*d - 27*a*b^2*c^3*e + (27*a*b
^3*c^3*(3*b^2*d - 12*a*c*d))/(2*(9*a*b^2 - 36*a^2*c))))/(2*(9*a*b^2 - 36*a^
2*c)) + 9*a*b*c^3*e^2 - 27*b^2*c^3*d*e))/(6*a*(4*a*c - b^2)^(1/2)))*(2*a*e
- b*d))/(6*a*(4*a*c - b^2)^(1/2)) - (b^3*c^3*(2...
```

3.13 $\int \frac{d+ex^3}{x^4(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=112

$$-\frac{d}{3ax^3}-\frac{(b^2d-2acd-abe)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}$$

```
[Out] -1/3*d/a/x^3-(-a*e+b*d)*ln(x)/a^2+1/6*(-a*e+b*d)*ln(c*x^6+b*x^3+a)/a^2-1/3*
(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^3+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+
b^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1488, 814, 648, 632, 212, 642}

$$-\frac{(-abe-2acd+b^2d)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*d/(a*x^3) - ((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^3)/Sqrt[b^2
- 4*a*c]])/(3*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b*d - a
*e)*Log[a + b*x^3 + c*x^6])/(6*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^3}{x^4\left(a+bx^3+cx^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{d+ex}{x^2\left(a+bx+cx^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{d}{ax^2}+\frac{-bd+ae}{a^2x}+\frac{b^2d-acd-abe+c(bd-ae)x}{a^2\left(a+bx+cx^2\right)}\right)\,dx,x,x^3\right)\\
&= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2d-acd-abe+c(bd-ae)x}{a+bx+cx^2}\,dx,x,x^3\right)}{3a^2}\\
&= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)}{6a^2}+\frac{\left(b^2d-2acd\right.}{}\\
&= -\frac{d}{3ax^3}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^3+cx^6\right)}{6a^2}-\frac{\left(b^2d-2acd-abe\right)}{}\\
&= -\frac{d}{3ax^3}-\frac{\left(b^2d-2acd-abe\right)\tanh^{-1}\left(\frac{b+2cx^3}{\sqrt{b^2-4ac}}\right)}{3a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-}{}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.04, size = 130, normalized size = 1.16

$$-\frac{d}{3ax^3}+\frac{(-bd+ae)\log(x)}{a^2}+\frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{b^2d\log(x-\#1)-acd\log(x-\#1)-abe\log(x-\#1)+bcd\log(x-\#1)\#1^3-ace\log(x-\#1)\#1^3}{b+2c\#1^3}\&\right]}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/3*d/(a*x^3) + ((-(b*d) + a*e)*Log[x])/a^2 + RootSum[a + b*#1^3 + c*#1^6
& , (b^2*d*Log[x - #1] - a*c*d*Log[x - #1] - a*b*e*Log[x - #1] + b*c*d*Log[
x - #1]*#1^3 - a*c*e*Log[x - #1]*#1^3)/(b + 2*c*#1^3) & ]/(3*a^2)
```

Maple [A]
time = 0.08, size = 126, normalized size = 1.12

method	result
default	$-\frac{\frac{(ace-bcd)\ln\left(c\,x^{6}+b\,x^{3}+a\right)}{2c}+\frac{2\left(abe+acd-b^{2}d-\frac{(ace-bcd)b}{2c}\right)\arctan\left(\frac{2c\,x^{3}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{3a^{2}}-\frac{d}{3a\,x^{3}}+\frac{(ae-bd)\ln(x)}{a^{2}}$
risch	$-\frac{d}{3a\,x^{3}}+\frac{\ln(x)e}{a}-\frac{\ln(x)bd}{a^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{3}c-a^{2}b^{2}\right)\_Z^{2}+\left(4a^{2}ce-a\,b^{2}e-4abcd+b^{3}d\right)\_Z+ac\,e^{2}-bcde+c^{2}d^{2}\right)}\_R\ln\left(\left(\left(-14a\right.\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/a^2*(1/2*(a*c*e-b*c*d)/c*ln(c*x^6+b*x^3+a)+2*(a*b*e+a*c*d-b^2*d-1/2*(a
*c*e-b*c*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^3+b)/(4*a*c-b^2)^(1/2)))-1
/3*d/a/x^3+(a*e-b*d)/a^2*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.72, size = 409, normalized size = 3.65

$$\left[\frac{(abx^{3}e-(b^{2}-2\,ac)dx^{3})\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{6}+2\,bcx^{3}+b^{2}-2\,ac+(2\,cx^{3}+b)\sqrt{b^{2}-4\,ac}}{cx^{6}+bx^{3}+a}\right)-2\,(ab^{2}-4\,a^{2}c)d+((b^{3}-4\,abc)dx^{3}-(ab^{2}-4\,a^{2}c)x^{3}e)\log\left(cx^{6}+bx^{3}+a\right)-6\,((b^{3}-4\,abc)dx^{3}-(ab^{2}-4\,a^{2}c)x^{3}e)\log\left(x\right)}{6\,(a^{2}b^{2}-4\,a^{3}c)x^{3}},\ \frac{2\,(abx^{3}e-(b^{2}-2\,ac)dx^{3})\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{3}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-2\,(ab^{2}-4\,a^{2}c)d+((b^{3}-4\,abc)dx^{3}-(ab^{2}-4\,a^{2}c)x^{3}e)\log\left(cx^{6}+bx^{3}+a\right)-6\,((b^{3}-4\,abc)dx^{3}-(ab^{2}-4\,a^{2}c)x^{3}e)\log\left(x\right)}{6\,(a^{2}b^{2}-4\,a^{3}c)x^{3}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] [1/6*((a*b*x^3*e - (b^2 - 2*a*c)*d*x^3)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^6 +
2*b*c*x^3 + b^2 - 2*a*c + (2*c*x^3 + b)*sqrt(b^2 - 4*a*c))/(c*x^6 + b*x^3 +
 a)) - 2*(a*b^2 - 4*a^2*c)*d + ((b^3 - 4*a*b*c)*d*x^3 - (a*b^2 - 4*a^2*c)*x
^3*e)*log(c*x^6 + b*x^3 + a) - 6*((b^3 - 4*a*b*c)*d*x^3 - (a*b^2 - 4*a^2*c)
*x^3*e)*log(x))/((a^2*b^2 - 4*a^3*c)*x^3), 1/6*(2*(a*b*x^3*e - (b^2 - 2*a*c
)*d*x^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^3 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) - 2*(a*b^2 - 4*a^2*c)*d + ((b^3 - 4*a*b*c)*d*x^3 - (a*b^2 - 4*a^2*
c)*x^3*e)*log(c*x^6 + b*x^3 + a) - 6*((b^3 - 4*a*b*c)*d*x^3 - (a*b^2 - 4*a^
2*c)*x^3*e)*log(x))/((a^2*b^2 - 4*a^3*c)*x^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**4/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.91, size = 128, normalized size = 1.14

$$\frac{(bd-ae)\log\left(cx^6+bx^3+a\right)}{6\,a^2}-\frac{(bd-ae)\log\left(|x|\right)}{a^2}+\frac{\left(b^2d-2\,acd-abe\right)\arctan\left(\frac{2\,cx^3+b}{\sqrt{-b^2+4\,ac}}\right)}{3\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bdx^3-ax^3e-ad}{3\,a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^4/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/6*(b*d - a*e)*log(c*x^6 + b*x^3 + a)/a^2 - (b*d - a*e)*log(abs(x))/a^2 +
1/3*(b^2*d - 2*a*c*d - a*b*e)*arctan((2*c*x^3 + b)/sqrt(-b^2 + 4*a*c))/(sqr
t(-b^2 + 4*a*c)*a^2) + 1/3*(b*d*x^3 - a*x^3*e - a*d)/(a^2*x^3)
```

Mupad [B]
time = 9.57, size = 2500, normalized size = 22.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x^4*(a + b*x^3 + c*x^6)),x)
```

```
[Out] (log(x)*(a*e - b*d))/a^2 - (log(((((((a*e - b*d + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*b^2*c^3*(a*b*e - b^2*d + a*c*d))
/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a*c*d))/a + (9*b^2*c^3*(a*b + 4*b
```

```
^2*x^3 - 14*a*c*x^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4
*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) - (3*c^5*d*x^3*(11*b^2*d - 14*a*b*e
 + 14*a*c*d))/a^2 + (9*b*c^4*d*(3*a*b*e - 3*b^2*d + a*c*d))/a^2)*(a*e - b*d
 + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) +
 (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3 + (c^6*d^2*x^3*(7*a*e - 12*b*d))
/a^3)*(a*e - b*d + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(
1/2)))/(6*a^2) + (c^6*d^3*(a*e - b*d))/a^4 - (c^7*d^4*x^3)/a^4)*(((((((b*d
- a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((27*
b^2*c^3*(a*b*e - b^2*d + a*c*d))/a + (9*b*c^4*x^3*(2*b^2*d + 7*a*b*e - 28*a
*c*d))/a - (9*b^2*c^3*(a*b + 4*b^2*x^3 - 14*a*c*x^3)*(b*d - a*e + a^2*(-(a*
b*e - b^2*d + 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(2*a^2)))/(6*a^2) + (
3*c^5*d*x^3*(11*b^2*d - 14*a*b*e + 14*a*c*d))/a^2 - (9*b*c^4*d*(3*a*b*e - 3
*b^2*d + a*c*d))/a^2)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2*a*c*d)^2/(a^4*(
4*a*c - b^2)))^(1/2)))/(6*a^2) + (c^5*d^2*(9*a*b*e - 9*b^2*d + a*c*d))/a^3
+ (c^6*d^2*x^3*(7*a*e - 12*b*d))/a^3)*(b*d - a*e + a^2*(-(a*b*e - b^2*d + 2
*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(6*a^2) - (c^6*d^3*(a*e - b*d))/a^4
+ (c^7*d^4*x^3)/a^4))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(
36*a^3*c - 9*a^2*b^2)) - d/(3*a*x^3) - (atan((48*a^8*x^3*(((((((((18*a^3*b
^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 37
8*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*
a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2))
 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d
- 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3
*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*(36*a
^3*c - 9*a^2*b^2)) - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*
d*e)/a^4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 +
((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12
*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*
e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*
a^2*(4*a*c - b^2)^(1/2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/
(2*(36*a^3*c - 9*a^2*b^2)) - (((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 2
52*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b
^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b
^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*
b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a
*b*c*d))/(12*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*
d + 2*a*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2
*c^4)*(a*b*e - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*
b*c*d))/(72*a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a
*c*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + (((7*a^2*c^6*d^2*e - 12*a*b*c^6*d^3)/a
^4 - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*d*e)/a^4 - (((18
*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a^4*b^4*c^
3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(2*a^
4*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))
/(2*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d
```

```
))/(2*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d))/(6*a^2*(4*a*c - b
^2)^(1/2)) - ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e - b^2*d + 2*a*c*d)
^3*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(432*a^10*(4*a*c - b^2)
^(3/2)*(36*a^3*c - 9*a^2*b^2)))*(4*b^5*d - 7*a^3*c^2*e - 4*a*b^4*e - 16*a*b
^3*c*d + 9*a^2*b*c^2*d + 15*a^2*b^2*c*e))/(16*a^4*c^3*(49*a^3*c*e^2 - 12*b^
4*d^2 - 12*a^2*b^2*e^2 + a^2*c^2*d^2 + 24*a*b^3*d*e + 48*a*b^2*c*d^2 - 97*a
^2*b*c*d*e)) - (((((((((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5
*d)/a^4 + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^2*e + 12*a^
2*c*e - 12*a*b*c*d))/(2*a^4*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c
*d))/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*
b*e - b^2*d + 2*a*c*d)*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(12
*a^6*(4*a*c - b^2)^(1/2)*(36*a^3*c - 9*a^2*b^2)))*(a*b*e - b^2*d + 2*a*c*d)
)/(6*a^2*(4*a*c - b^2)^(1/2)) + ((108*a^4*b^4*c^3 - 378*a^5*b^2*c^4)*(a*b*e
 - b^2*d + 2*a*c*d)^2*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*e - 12*a*b*c*d))/(72*
a^8*(4*a*c - b^2)*(36*a^3*c - 9*a^2*b^2)))*(3*b^3*d - 3*a*b^2*e + 12*a^2*c*
e - 12*a*b*c*d))/(2*(36*a^3*c - 9*a^2*b^2)) - (((7*a^2*c^6*d^2*e - 12*a*b*c
^6*d^3)/a^4 - (((42*a^3*c^6*d^2 + 33*a^2*b^2*c^5*d^2 - 42*a^3*b*c^5*d*e)/a^
4 - (((18*a^3*b^3*c^4*d + 63*a^4*b^2*c^4*e - 252*a^4*b*c^5*d)/a^4 + ((108*a
^4*b^4*c^3 - 378*a^5*b^2*c^4)*(3*b^3*d - 3*a*b^...
```

3.14 $\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=723

$$\frac{ex^2}{2c}-\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{\ldots}{\sqrt[3]{b\ldots}}}{\ldots}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b+\sqrt{b^2-4a\ldots}}}$$

```
[Out] 1/2*e*x^2/c-1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(c*d-b*e
+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)
^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)
^(1/2))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)
/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*arcta
n(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d-b*
e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3
))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-
4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-
4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(2*a*c*e-b^2
*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-
1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2)
)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(5/3)*3^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A]
time = 1.13, antiderivative size = 723, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1516, 1524, 298, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\,2^{2/3}c^{5/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{ex^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x^2)/(2*c) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/
(2^(2/3)*Sqrt[3]*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*
c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)
/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(5/3)*(b + Sqr
t[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 -
```

```
 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)
*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((c*d - b*e + (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/
3)*x])/(3*2^(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e - (b
*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3
) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])
/(6*2^(2/3)*c^(5/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + ((c*d - b*e + (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(2/3)*c^(5/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex^2}{2c} - \frac{\int \frac{x(2ae-2(cd-be)x^3)}{a+bx^3+cx^6}\,dx}{2c}$$

$$= \frac{ex^2}{2c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)}{2}$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\ 2^{2/3}c^{4/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} + \left(cd-be-\frac{bcd-b^2e}{\sqrt{b^2-}}\right.$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \left(cd-be+\right.$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \left(cd-be+\right.$$

$$= \frac{ex^2}{2c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{5/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(cd-be+\frac{bcd-b^2}{\sqrt{b^2-}}\right.}{2^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 88, normalized size = 0.12

$$\frac{3ex^2-2\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^3+be\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{6c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (3*e*x^2 - 2*RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x - #1]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ])/(6*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.20, size = 70, normalized size = 0.10

method	result	size
default	$\frac{e\,x^{2}}{2c}-\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(eb-cd\right)\_R^{4}+ae\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	70
risch	$\frac{e\,x^{2}}{2c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-eb+cd\right)\_R^{4}-ae\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e*x^2/c-1/3/c*sum(((b*e-c*d)*_R^4+a*e*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*e/c + integrate(((c*d - b*e)*x^4 - a*x*e)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 42.01, size = 2500, normalized size = 3.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((2^(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a
*c*e^2 - 2*b*c*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a
^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d
^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d
^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*
d^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e
^3*(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*
a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*
(-(4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*
a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)
^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4
*a*c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e^3 + 16*a^4*
c^4*e^3 - b^5*c^3*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3
- 16*a^2*b*c^5*d^3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*
e + 3*b^6*c^2*d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3
*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3
*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(
4*a*c - b^2)^3)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2
*c^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1
/2) + 12*a*b^2*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*(4*a*c - b^2)*(b*
```

```
e - c*d)*(b^4*e^2 - a*c^3*d^2 + 3*a^2*c^2*e^2 + b^2*c^2*d^2 - 2*b^3*c*d*e -
 4*a*b^2*c*e^2 + 5*a*b*c^2*d*e))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3
*d^3 + b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^
3 + 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2
*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*
b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(c^5*(4*a*c - b^2)^3))^(2/3))/18 - (a^2*x*(a*e^2 + c*d^2 - b*d*e)^2*(a*c*
e - b^2*e + b*c*d))/c^2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3*d^3 + b^5*e^
3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^3 + 2*a*c^4*d
^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2*e + 41*a^2*b
^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 - b^2*c^3*d^3*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^6*c*e^3 - 3*b^7*c*d*e^2 - 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 27*
a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 - 3*b^4*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) + 72*a^2
*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 - 6*a^2*c^3*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a*b^2*c^2*d*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^3
*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)))^(1/3) + log((2^(1/3)*((2^
(2/3)*(27*a^2*c*x*(4*a*c - b^2)*(b^2*e^2 + 2*c^2*d^2 - 2*a*c*e^2 - 2*b*c*d*
e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*
c^3*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5
*d^3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*
d^2*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2
)^3)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 +
 3*b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3
)^(1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2
*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(c^5*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^8*e^3 + 16*a^4*c^4*e^3 - b^5*c^3
*d^3 - b^5*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^4*d^3 - 16*a^2*b*c^5*d^
3 - 2*a*c^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^5*d^2*e + 3*b^6*c^2*d^2
*e + 41*a^2*b^4*c^2*e^3 - 56*a^3*b^2*c^3*e^3 + b^2*c^3*d^3*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^6*c*e^3 - 3*b^7*c*d*e^2 + 5*a*b^3*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 27*a*b^4*c^3*d^2*e + 30*a*b^5*c^2*d*e^2 + 96*a^3*b*c^4*d*e^2 + 3*
b^4*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a^2*b*c^2*e^3*(-(4*a*c - b^2)^3)^(
1/2) + 72*a^2*b^2*c^4*d^2*e - 96*a^2*b^3*c^3*d*e^2 + 6*a^2*c^3*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 3*b^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a*b^2*c^
2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2)
)/(c^5*(4*a*c - b^2)^3))^(1/3))/6 - (9*a*(4*a*c...
```

3.15 $\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=718

$$\frac{ex}{c}-\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\ldots}}{\sqrt[3]{b+\ldots}}}{\ldots}\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b+\sqrt{b^2-4ac}\right)^{2\ldots}}$$

```
[Out] e*x/c+1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(c*d-b*e+(-2*a
*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2)
)^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2)
)^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a
*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*
(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d-b*e+(-2*
a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(2/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(c*
d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+
b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+
b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(c*d-b*e+(2*a*c*e-b^2*e+b*c
*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*ar
ctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(c*d
-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(4/3)*3^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A]
time = 0.95, antiderivative size = 718, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1516, 1436, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt[3]{2}\sqrt{3}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{ex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(1 - (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/
3)*Sqrt[3]*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d -
b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b +
Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(4/3)*(b + Sqrt[b^2
- 4*a*c])^(2/3)) + ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c
```

```
])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*c^(4/
3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)
/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])
/(3*2^(1/3)*c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e - (b*c*d -
 b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^
(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^
(1/3)*c^(4/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - ((c*d - b*e + (b*c*d - b^2*e
 + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*
c^(4/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^3(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^3}{a+bx^3+cx^6}\,dx}{c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)}{2}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b-\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,c\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \left(cd-be-\frac{bcd-b^2e}{\sqrt{b^2-}}\right.$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + (cd-be+$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + (cd-be+$$

$$= \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\,\sqrt{3}\,c^{4/3}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(cd-be+\frac{bcd-b}{\sqrt{b^2}}\right.}{\sqrt[3]{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 88, normalized size = 0.12

$$\frac{ex}{c} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^3+be\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^3 + c*#1^6 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^3 + b*e*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 67, normalized size = 0.09

method	result	size
default	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-eb+cd\right)\_R^{3}-ae\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67
risch	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\left(-eb+cd\right)\_R^{3}-ae\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3c}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] e*x/c+1/3/c*sum(((-b*e+c*d)*_R^3-a*e)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(
_Z^6*c+_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] x*e/c + integrate(((c*d - b*e)*x^3 - a*e)/(c*x^6 + b*x^3 + a), x)/c
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((x^3*e + d)*x^3/(c*x^6 + b*x^3 + a), x)
```

Mupad [B]
time = 30.15, size = 2500, normalized size = 3.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^3*d
^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3*e
+ 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/3)*
((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)
^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c - b^2)^3)^(1
/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2*a^2*c^2*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*a*b^2*c*e^3
*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48*a^
2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c*d*e^2*(-(4
*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-(4*a*c - b^
2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^2)^
3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*a*c -
b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 + 2
*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 - 4*
a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*
e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3*b^3*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^2*e*(-
(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*
a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2*c^
2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2 + 6
```

*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b^4*e^3*(-(4
*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^3*d^3*(-(4*
a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e
^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^
2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*
c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 3
*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3*b^2*c^2*d^
2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c
^4*(4*a*c - b^2)^3))^(1/3))/6)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 + b
^4*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 - b*c^
3*d^3*(-(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^
2*b^3*c^2*e^3 + 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3
*b^6*c*d*e^2 - 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e
+ 27*a*b^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e - 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3
)^(1/2) - 3*b^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 + 3
*b^2*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3
)^(1/2))/(54*(64*a^3*c^7 - b^6*c^4 + 12*a*b^4*c^5 - 48*a^2*b^2*c^6)))^(1/3)
 + log((3*a*x*(a*b^4*e^4 - 2*a*c^4*d^4 - b^5*d*e^3 + 2*a^3*c^2*e^4 + b^2*c^
3*d^4 - 4*a^2*b^2*c*e^4 - 3*b^3*c^2*d^3*e + 3*b^4*c*d^2*e^2 + 8*a*b*c^3*d^3
*e + 2*a*b^3*c*d*e^3 + 4*a^2*b*c^2*d*e^3 - 9*a*b^2*c^2*d^2*e^2))/c - (2^(2/
3)*((2^(1/3)*(81*a*c^3*d*x*(4*a*c - b^2)^2 - (81*2^(2/3)*a*b*c^3*(4*a*c - b
^2)^2*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*c - b^2)^3)
^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3 - 2*a^2*c^2
e^3(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 + 4*a*b^2*c*
e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2*d*e^2 + 48
*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^3*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*(4*a*c - b^
2)^3))^(1/3))/2)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-(4*a*c
 - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^2*e^3
- 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d*e^2 +
 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b^4*c^2
*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 3*b^
3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^2*b^2*c^3*d*e^2 - 3*b^2*c^2*d^2*e
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b*c^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(c^4*
(4*a*c - b^2)^3))^(2/3))/18 + (9*a*(4*a*c - b^2)*(b^4*e^3 - b*c^3*d^3 + a^2
*c^2*e^3 + 3*b^2*c^2*d^2*e - 3*a*b^2*c*e^3 - 3*a*c^3*d^2*e - 3*b^3*c*d*e^2
+ 6*a*b*c^2*d*e^2))/c)*((b^7*e^3 - 16*a^2*c^5*d^3 - b^4*c^3*d^3 - b^4*e^3*(
-(4*a*c - b^2)^3)^(1/2) + 8*a*b^2*c^4*d^3 - 32*a^3*b*c^3*e^3 + b*c^3*d^3*(-
(4*a*c - b^2)^3)^(1/2) + 48*a^3*c^4*d*e^2 + 3*b^5*c^2*d^2*e + 32*a^2*b^3*c^
2*e^3 - 2*a^2*c^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*e^3 - 3*b^6*c*d
*e^2 + 4*a*b^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 24*a*b^3*c^3*d^2*e + 27*a*b
^4*c^2*d*e^2 + 48*a^2*b*c^4*d^2*e + 6*a*c^3*d^2...

3.16 $\int \frac{x(d+ex^3)}{a+bx^3+cx^6} \, dx$

Optimal. Leaf size=634

$$-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(e+(-b*e+2*c*d)/(-4
*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/
3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^
2)^(1/2))^(2/3))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b-(-4
*a*c+b^2)^(1/2))^(1/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2
)^(1/2))^(1/3))*3^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3
)*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+
b^2)^(1/2))^(1/3))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b+(-
4*a*c+b^2)^(1/2))^(1/3)+1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-
4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(e+(b*e-2*c*d)/(-4*a*
c+b^2)^(1/2))*2^(1/3)/c^(2/3)/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/6*arctan(1/3*(
1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(e+(b*e-2*c*d)
/(-4*a*c+b^2)^(1/2))*2^(1/3)/c^(2/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/3)
```

Rubi [A]

time = 0.42, antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1524, 298, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3]*c^(2/3)*(b - Sqrt
[b^2 - 4*a*c])^(1/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sq
rt[3]*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/
3)*c^(2/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(2/3)*
```

```
c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(2/3)*c^(2/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x(d+ex^3)}{a+bx^3+cx^6}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}-\frac{1}{2}}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\ 2^{2/3}\sqrt[3]{c}\,\sqrt[3]{b+\sqrt{b^2-4ac}}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{\left(b+\sqrt{b^2\right.}}}{3\ 2^{2/3}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\mathrm{lo}}{3\ 2^{2/3}c}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\,\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\mathrm{lo}}{3\ 2^{2/3}c}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\,\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{b-\sqrt{b^2-4ac}}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\Bigg(}{2^{2/3}\sqrt{3}\,c^{2/3}\sqrt[3]{b-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 59, normalized size = 0.09

$$\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1
+ 2*c*#1^4) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 49, normalized size = 0.08

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}e+\_Rd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	49
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(\_R^{4}e+\_Rd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}\right)}{3}$	49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum((_R^4*e+_R*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((x^3*e + d)*x/(c*x^6 + b*x^3 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**3+d)/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((x^3*e + d)*x/(c*x^6 + b*x^3 + a), x)
```

Mupad [B]
time = 24.56, size = 2500, normalized size = 3.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 +
 a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 -
b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) -
 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2
) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(
4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(
2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*b*c^3*(4*a*c -
 b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^
2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2
*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a
^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 2
4*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c
 - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*
c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c
^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3
)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2
)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))
/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3
 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*
d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2)*((a*b^5*e^3 + 16*a^
2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1
/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
```

```
2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*
e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b
*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4
*c^3 - 48*a^3*b^2*c^4)))^(1/3) + log((2^(1/3)*((a*b^5*e^3 + 16*a^2*c^4*d^3
+ b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^
2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2
*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*
c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(
-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*(36*a^3*c^3*e^3 - (
2^(2/3)*(27*c^3*x*(4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*
e) - (27*2^(1/3)*a*b*c^3*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4
*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3
*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/2)*((a*b^5*e^3 + 16*a^2
*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 - a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 + b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2
) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e
^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*
c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(1/3))/6 - 108*a
^2*c^4*d^2*e - 45*a^2*b^2*c^2*e^3 + 9*a*b^4*c*e^3 + 27*a*b^2*c^3*d^2*e - 27
*a*b^3*c^2*d*e^2 + 108*a^2*b*c^3*d*e^2))/18 + c*x*(b*e - c*d)*(a*e^2 + c*d^
2 - b*d*e)^2)*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3
- a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 +
 b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2)
- 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 - 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 24*a^2*b^2*c^2*d*e^2 + 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(54*(64
*a^4*c^5 - a*b^6*c^2 + 12*a^2*b^4*c^3 - 48*a^3*b^2*c^4)))^(1/3) - log(c*x*(
b*e - c*d)*(a*e^2 + c*d^2 - b*d*e)^2 + (2^(1/3)*(3^(1/2)*1i - 1)*((a*b^5*e^
3 + 16*a^2*c^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 2*a^2*c*e^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a
*b^4*c*d*e^2 + 6*a*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^
2 - 3*a*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3)*
(36*a^3*c^3*e^3 - 108*a^2*c^4*d^2*e + (2^(2/3)*(3^(1/2)*1i + 1)*(27*c^3*x*(
4*a*c - b^2)*(2*a^2*e^2 + b^2*d^2 - 2*a*c*d^2 - 2*a*b*d*e) - (27*2^(1/3)*a*
b*c^3*(3^(1/2)*1i - 1)*(4*a*c - b^2)^2*((a*b^5*e^3 + 16*a^2*c^4*d^3 + b^4*c
^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c
*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 2*a^2*c*e^3*
(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d*e^2 - 3*a*b^4*c*d*e^2 + 6*a*c^2*d^2
*e*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d*e^2 - 3*a*b*c*d*e^2*(-(4*a*c
 - b^2)^3)^(1/2))/(a*c^2*(4*a*c - b^2)^3))^(2/3))/4)*((a*b^5*e^3 + 16*a^2*c
^4*d^3 + b^4*c^2*d^3 - 8*a*b^2*c^3*d^3 + a*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^2*b^3*c*e^3 + 16*a^3*b*c^2*e^3 - b*c^2*d...
```

3.17 $\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=634

$$-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b+\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b+\sqrt{b^2-4ac}\right)^{2/3}}$$

```
[Out] 1/6*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(e+(-b*e+2*c*d)/(-4*
a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3
)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/3)+(b-(-4*a*c+b^2
)^(1/2))^(2/3))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b-(-4*
a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b-(-4*a*c+b^2)
^(1/2))^(1/3))*3^(1/2))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)
*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*ln(2^(1/3)*c^(1/3)*x+(b+(-4*a*c+b
^2)^(1/2))^(1/3))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b+(-4
*a*c+b^2)^(1/2))^(2/3)-1/12*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b+(-4
*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))*(e+(b*e-2*c*d)/(-4*a*c
+b^2)^(1/2))*2^(2/3)/c^(1/3)/(b+(-4*a*c+b^2)^(1/2))^(2/3)-1/6*arctan(1/3*(1
-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(e+(b*e-2*c*d)/
(-4*a*c+b^2)^(1/2))*2^(2/3)/c^(1/3)*3^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A]
time = 0.44, antiderivative size = 634, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1436, 206, 31, 648, 631, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(\sqrt{b^2-4ac}+b\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] -(((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/
(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*c^(1/3)*(b - Sqrt
[b^2 - 4*a*c])^(2/3))) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + ((e + (2*c*d - b*e)/Sqrt[b^2
 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/
3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + ((e - (2*c*d - b*e)/Sqrt[b^2 -
4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)*c^(1/3)*x])/(3*2^(1/3)*
```

```
c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a
*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b - Sqrt[b^2 - 4*
a*c])^(2/3)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4
*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^
(2/3)*x^2])/(6*2^(1/3)*c^(1/3)*(b + Sqrt[b^2 - 4*a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{d+ex^3}{a+bx^3+cx^6}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}-\frac{1}{2}}$$

$$= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\left(b+\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{\left(b+\sqrt{b^2-\right.}}{2}}}{3\sqrt[3]{2}\,(}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log}{3\sqrt[3]{2}\sqrt[3]{c}}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\log}{3\sqrt[3]{2}\sqrt[3]{c}}$$

$$= -\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt[3]{2}\sqrt{3}\sqrt[3]{c}\,\left(b+\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.02, size = 61, normalized size = 0.10

$$\frac{1}{3}\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(a + b*x^3 + c*x^6),x]
```

```
[Out] RootSum[a + b*#1^3 + c*#1^6 & , (d*Log[x - #1] + e*Log[x - #1]*#1^3)/(b*#1^
2 + 2*c*#1^5) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 47, normalized size = 0.07

method	result	size
default	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^3e+d\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}\right)}{3}$	47
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^6c+\_Z^3b+a\right)}\frac{\left(\_R^3e+d\right)\ln\left(x-\_R\right)}{2\_R^5c+\_R^2b}\right)}{3}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum((_R^3*e+d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_Z^3*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((x^3*e + d)/(c*x^6 + b*x^3 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/(c*x**6+b*x**3+a),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate((x^3*e + d)/(c*x^6 + b*x^3 + a), x)
```

Mupad [**B**]
time = 18.96, size = 2500, normalized size = 3.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(a + b*x^3 + c*x^6),x)
[Out] log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*
e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3
 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2
) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a
*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(
4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2
)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1/3)*(81*c^3*x*(4*a*c - b^2
)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^
4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(
-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*
e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2
*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*
c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3))/2)*(-(b^5
*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3
- 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2)
 - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e
```

```
- 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2
*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(
2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3*c^2*e^3 - 36*a^2*b*c^3*e^
3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(-(b^5*c*d^3 + a^2*b^4*e^3
+ 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*
c - b^2)^3)^(1/2) - a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 +
b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6
*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 -
48*a^4*b^2*c^3)))^(1/3) + log(3*c^2*x*(2*c^3*d^4 + a*b^2*e^4 - 2*a^2*c*e^4
- b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) - (2^(2/3)*(-(b^5*c*d^3 + a^
2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d
^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^
2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c
*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*
a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*((2^(1
/3)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(4*a*c - b^
2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2
*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^
2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3
*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*
a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^2*c*(4*a*c -
 b^2)^3))^(1/3))/2)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c
^2*d^3 + 16*a^2*b*c^3*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^2*c*(4*a*c - b^2)^3))^(2/3))/18 - 36*a*c^5*d^3 + 9*b^2*c^4*d^3 + 9*a*b^3
*c^2*e^3 - 36*a^2*b*c^3*e^3 + 108*a^2*c^4*d*e^2 - 27*a*b^2*c^3*d*e^2))/6)*(
-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3
*d^3 + 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + a^2*b*e^3*(-(4*a*c - b^2)^3)^
(1/2) - 8*a^3*b^2*c*e^3 - b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 48*a^3*c^3*d
^2*e - 3*a*b^4*c*d^2*e - 6*a^2*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 24*a^2*b^
2*c^2*d^2*e + 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(54*(64*a^5*c^4 - a^2
*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)))^(1/3) + log(3*c^2*x*(2*c^3*d^4
+ a*b^2*e^4 - 2*a^2*c*e^4 - b^3*d*e^3 + 3*b^2*c*d^2*e^2 - 4*b*c^2*d^3*e) +
(2^(2/3)*(3^(1/2)*1i - 1)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a
*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^
2*b*e^3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c
- b^2)^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(
1/2))/(a^2*c*(4*a*c - b^2)^3))^(1/3)*(36*a*c^5*d^3 - 9*b^2*c^4*d^3 - 9*a*b^
3*c^2*e^3 + 36*a^2*b*c^3*e^3 - 108*a^2*c^4*d*e^2 + (2^(1/3)*(3^(1/2)*1i + 1
)*(81*c^3*x*(4*a*c - b^2)^2*(a*e - b*d) - (81*2^(2/3)*a*b*c^3*(3^(1/2)*1i -
 1)*(4*a*c - b^2)^2*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*e^3 - 8*a*b^3*c
```

```
^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - a^2*b*e^
3*(-(4*a*c - b^2)^3)^(1/2) - 8*a^3*b^2*c*e^3 + b^2*c*d^3*(-(4*a*c - b^2)^3)
^(1/2) - 48*a^3*c^3*d^2*e - 3*a*b^4*c*d^2*e + 6*a^2*c*d*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 24*a^2*b^2*c^2*d^2*e - 3*a*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/
(a^2*c*(4*a*c - b^2)^3))^(1/3))/4)*(-(b^5*c*d^3 + a^2*b^4*e^3 + 16*a^4*c^2*
e^3 - 8*a*b^3*c^2*d^3 + 16*a^2*b*c^3*d^3 - 2*a*...
```

3.18 $\int \frac{d+ex^3}{x^2(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=653

$$-\frac{d}{ax}+\frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}}{\sqrt[3]{b+\sqrt{b}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b+\sqrt{b^2-4ac}}}$$

```
[Out] -d/a/x+1/6*c^(1/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))*(d+(-
2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)-1/12*
c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2))^(1/
3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3
)/a/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3
)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/
2))*2^(1/3)/a*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/3)+1/6*c^(1/3)*ln(2^(1/3)*c
^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2
^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)-1/12*c^(1/3)*ln(2^(2/3)*c^(2/3)*x^2-2
^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^(2/3))
*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a/(b+(-4*a*c+b^2)^(1/2))^(1/3)+
1/6*c^(1/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^(1/3))
*3^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/3)/a*3^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/3)
```

Rubi [A]
time = 0.70, antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1518, 1524, 298, 31, 648, 631, 210, 642}

$$\frac{\sqrt[3]{c}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\,\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2^{2/3}\sqrt{3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}-\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}+\frac{\sqrt[3]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}}+\frac{\sqrt[3]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}x\right)}{3\,2^{2/3}a\sqrt[3]{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) + (c^(1/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 - (2*
2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(2/3)*Sqrt[3
]*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 -
 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3))/S
qrt[3]])/(2^(2/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d +
(b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3
)*c^(1/3)*x])/(3*2^(2/3)*a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) + (c^(1/3)*(d - (
```

```
b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(1/3)
*c^(1/3)*x])/(3*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d + (b
*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*
c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*
a*(b - Sqrt[b^2 - 4*a*c])^(1/3)) - (c^(1/3)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4
*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^2 -
4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(2/3)*a*(b + Sqrt[b^2 - 4*a*c]
)^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{d+ex^3}{x^2\left(a+bx^3+cx^6\right)}\,dx = -\frac{d}{ax} - \frac{\int \frac{x\left(bd-ae+cdx^3\right)}{a+bx^3+cx^6}\,dx}{a}$$

$$= -\frac{d}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{d}{ax} + \frac{\left(c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\ 2^{2/3}a\sqrt[3]{b+\sqrt{b^2-4ac}}} - \frac{\left(c^{2/3}\left(d-\right.\right.}{}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d\right.}{}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\ 2^{2/3}a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d\right.}{}$$

$$= -\frac{d}{ax} + \frac{\sqrt[3]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}\,a\sqrt[3]{b-\sqrt{b^2-4ac}}} + \frac{\sqrt[3]{c}\left(d-\frac{}{\sqrt{}}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 85, normalized size = 0.13

$$-\frac{d}{ax} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b\#1+2c\#1^4}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -(d/(a*x)) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log[x -
 #1] + c*d*Log[x - #1]*#1^3)/(b*#1 + 2*c*#1^4) & ]/(3*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 71, normalized size = 0.11

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^6 c+\_Z^3 b+a\right)}\frac{\left(-cd\_R^4+(ae-bd)\_R\right)\ln\left(x-\_R\right)}{2\_R^5 c+\_R^2 b}}{3a}-\frac{d}{ax}$
risch	$-\frac{d}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^7c^3-48b^2c^2a^6+12b^4ca^5-b^6a^4\right)\_Z^6+\left(-16a^5c^2e^3+8a^4b^2ce^3+48a^4bc^2de^2+48a^4c^3d^2e-a^3b^4e^3-24a^3b^3cde^2-72\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*sum((-c*d*_R^4+(a*e-b*d)*_R)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^
6*c+_Z^3*b+a))-d/a/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^4 + (b*d - a*e)*x)/(c*x^6 + b*x^3 + a), x)/a - d/(a*x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**2/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^2/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((x^3*e + d)/((c*x^6 + b*x^3 + a)*x^2), x)
```

Mupad [B]
time = 38.02, size = 2500, normalized size = 3.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x^2*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) -
16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*
a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2
*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*
a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/
2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c
*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*
a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^4*d^2 - 2*a^3
*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c*d^2 + 6*a^2*
b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 - a^3*b^4*e^3
+ b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^
3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a
^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2)
- 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) -
 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c
*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^
2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2
*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*(-(b^7*d^3 -
a^3*b^4*e^3 + b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*
c^3*d^3 - a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*
d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^
```

```
2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e -
24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9
*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(1/3))/6 +
36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a^8*b^2*c^5*d^
3 + 108*a^9*b*c^5*d^2*e - 27*a^8*b^3*c^4*d^2*e + 27*a^9*b^2*c^4*d*e^2))/18
+ a^7*c^4*e*x*(a*e^2 + c*d^2 - b*d*e)^2)*((b^7*d^3 - a^3*b^4*e^3 + b^4*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 - a^3*b*e^3*(-
(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2
*e + 32*a^2*b^3*c^2*d^3 + 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5
*c*d^3 - 3*a*b^6*d^2*e - 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^3*d
^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 4
8*a^4*b*c^2*d*e^2 - 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^2*d*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e + 9*a^2*b*c*d^2*e*(-(4*a*
c - b^2)^3)^(1/2))/(54*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^
2)))^(1/3) + log((2^(1/3)*(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2
)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^
3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^
3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b
^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c
- b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*
e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1
/2))/(a^4*(4*a*c - b^2)^3))^(2/3)*((2^(2/3)*(27*a^7*c^3*x*(4*a*c - b^2)*(b^
4*d^2 - 2*a^3*c*e^2 + a^2*b^2*e^2 + 2*a^2*c^2*d^2 - 2*a*b^3*d*e - 4*a*b^2*c
*d^2 + 6*a^2*b*c*d*e) - (27*2^(1/3)*a^10*b*c^3*(4*a*c - b^2)^2*(-(b^7*d^3 -
 a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^3 - 32*a^3*b
*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3 + 3*a^2*b^5
*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*(-(4*a*c - b
^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b^4*c*d^2*e -
 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*c^2*d^2*e -
9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3))^(2/3))/2)*
(-(b^7*d^3 - a^3*b^4*e^3 - b^4*d^3*(-(4*a*c - b^2)^3)^(1/2) - 16*a^5*c^2*e^
3 - 32*a^3*b*c^3*d^3 + a^3*b*e^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^2*c*e^3
 + 3*a^2*b^5*d*e^2 + 48*a^4*c^3*d^2*e + 32*a^2*b^3*c^2*d^3 - 2*a^2*c^2*d^3*
(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^5*c*d^3 - 3*a*b^6*d^2*e + 4*a*b^2*c*d^3*(
-(4*a*c - b^2)^3)^(1/2) + 3*a*b^3*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 27*a^2*b
^4*c*d^2*e - 24*a^3*b^3*c*d*e^2 + 48*a^4*b*c^2*d*e^2 + 6*a^3*c*d*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 3*a^2*b^2*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 72*a^3*b^2*
c^2*d^2*e - 9*a^2*b*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2))/(a^4*(4*a*c - b^2)^3)
)^(1/3))/6 + 36*a^9*c^6*d^3 - 108*a^10*c^5*d*e^2 + 9*a^7*b^4*c^4*d^3 - 45*a
^8*b^2*c^5*d^3 + 108*a^9*b*c^5*d^2*e - 27*a^8*b...
```

3.19 $\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx$

Optimal. Leaf size=655

$$-\frac{d}{2ax^2}+\frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{\,}}{\sqrt[3]{b+}}}{}\right)}{\sqrt[3]{2}\sqrt{3}\,a\left(b+\sqrt{b^2-4ac}\right)}$$

```
[Out] -1/2*d/a/x^2-1/6*c^(2/3)*ln(2^(1/3)*c^(1/3)*x+(b-(-4*a*c+b^2)^(1/2))^(1/3))
*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)
+1/12*c^(2/3)*ln(2^(2/3)*c^(2/3)*x^2-2^(1/3)*c^(1/3)*x*(b-(-4*a*c+b^2)^(1/2
))^(1/3)+(b-(-4*a*c+b^2)^(1/2))^(2/3))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))*
2^(2/3)/a/(b-(-4*a*c+b^2)^(1/2))^(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*
c^(1/3)*x/(b-(-4*a*c+b^2)^(1/2))^(1/3))*3^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^
2)^(1/2))*2^(2/3)/a*3^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(2/3)-1/6*c^(2/3)*ln(2^(
1/3)*c^(1/3)*x+(b+(-4*a*c+b^2)^(1/2))^(1/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1
/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^(2/3)+1/12*c^(2/3)*ln(2^(2/3)*c^(2/3)
*x^2-2^(1/3)*c^(1/3)*x*(b+(-4*a*c+b^2)^(1/2))^(1/3)+(b+(-4*a*c+b^2)^(1/2))^
(2/3))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a/(b+(-4*a*c+b^2)^(1/2))^
(2/3)+1/6*c^(2/3)*arctan(1/3*(1-2*2^(1/3)*c^(1/3)*x/(b+(-4*a*c+b^2)^(1/2))^
(1/3))*3^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(2/3)/a*3^(1/2)/(b+(-4
*a*c+b^2)^(1/2))^(2/3)
```

Rubi [A]
time = 0.73, antiderivative size = 655, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1518, 1436, 206, 31, 648, 631, 210, 642}

$$\frac{c^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{\sqrt[3]{2}\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\text{ArcTan}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{\sqrt{b^2-4ac}+b}}}{\sqrt{3}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{\sqrt[3]{2}\sqrt{3}\,a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}+\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{b-\sqrt{b^2-4ac}}+\left(b-\sqrt{b^2-4ac}\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}+\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(-\sqrt[3]{2}\sqrt[3]{c}\,x\sqrt[3]{\sqrt{b^2-4ac}+b}+\left(\sqrt{b^2-4ac}+b\right)^{2/3}+2^{2/3}c^{2/3}x^2\right)}{6\sqrt[3]{2}\,a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{c^{2/3}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}}-\frac{c^{2/3}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{\sqrt{b^2-4ac}+b}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(\sqrt{b^2-4ac}+b\right)^{2/3}}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*d/(a*x^2) + (c^(2/3)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(1 -
 (2*2^(1/3)*c^(1/3)*x)/(b - Sqrt[b^2 - 4*a*c])^(1/3))/Sqrt[3]])/(2^(1/3)*Sq
rt[3]*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(1 - (2*2^(1/3)*c^(1/3)*x)/(b + Sqrt[b^2 - 4*a*c])^(1/3
))/Sqrt[3]])/(2^(1/3)*Sqrt[3]*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(
d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(1/3) + 2^
(1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) - (c^(2/3)*(d
```

```
 - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(1/3) + 2^(
1/3)*c^(1/3)*x])/(3*2^(1/3)*a*(b + Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Log[(b - Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1
/3)*c^(1/3)*(b - Sqrt[b^2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1
/3)*a*(b - Sqrt[b^2 - 4*a*c])^(2/3)) + (c^(2/3)*(d - (b*d - 2*a*e)/Sqrt[b^2
 - 4*a*c])*Log[(b + Sqrt[b^2 - 4*a*c])^(2/3) - 2^(1/3)*c^(1/3)*(b + Sqrt[b^
2 - 4*a*c])^(1/3)*x + 2^(2/3)*c^(2/3)*x^2])/(6*2^(1/3)*a*(b + Sqrt[b^2 - 4*
a*c])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{d+ex^3}{x^3(a+bx^3+cx^6)}\,dx = -\frac{d}{2ax^2} - \frac{\int \frac{2(bd-ae)+2cdx^3}{a+bx^3+cx^6}\,dx}{2a}$$

$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^3}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{d}{2ax^2} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{\sqrt[3]{b+\sqrt{b^2-4ac}}}{\sqrt[3]{2}}+\sqrt[3]{c}\,x}\,dx}{3\sqrt[3]{2}\,a\left(b+\sqrt{b^2-4ac}\right)^{2/3}} - \frac{\left(c\left(d-\frac{b}{\sqrt{b^2}}\right.\right.}{}$$

$$= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(\right.}{}$$

$$= -\frac{d}{2ax^2} - \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\log\left(\sqrt[3]{b-\sqrt{b^2-4ac}}+\sqrt[3]{2}\sqrt[3]{c}\,x\right)}{3\sqrt[3]{2}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} - \frac{c^{2/3}\left(\right.}{}$$

$$= -\frac{d}{2ax^2} + \frac{c^{2/3}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{2}\sqrt[3]{c}\,x}{\sqrt[3]{b-\sqrt{b^2-4ac}}}}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}\,a\left(b-\sqrt{b^2-4ac}\right)^{2/3}} + \frac{c^{2/3}\left(d-\frac{}{\sqrt{}}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 89, normalized size = 0.14

$$-\frac{d}{2ax^2} - \frac{\text{RootSum}\left[a+b\#1^3+c\#1^6\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^3}{b\#1^2+2c\#1^5}\&\right]}{3a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x]
```

```
[Out] -1/2*d/(a*x^2) - RootSum[a + b*#1^3 + c*#1^6 & , (b*d*Log[x - #1] - a*e*Log
[x - #1] + c*d*Log[x - #1]*#1^3)/(b*#1^2 + 2*c*#1^5) & ]/(3*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 68, normalized size = 0.10

method	result
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}c+\_Z^{3}b+a\right)}\frac{\left(-\_R^{3}cd+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^{5}c+\_R^{2}b}}{3a}-\frac{d}{2a\,x^{2}}$
risch	$-\frac{d}{2a\,x^{2}}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64c^{3}a^{8}-48b^{2}c^{2}a^{7}+12b^{4}c\,a^{6}-b^{6}a^{5}\right)\_Z^{6}+\left(16a^{5}b\,c^{2}e^{3}+48a^{5}c^{3}d\,e^{2}-8a^{4}b^{3}c\,e^{3}-72a^{4}b^{2}c^{2}d\,e^{2}-96a^{4}b\,c^{3}d^{2}e-16a^{4}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*sum((-_R^3*c*d+a*e-b*d)/(2*_R^5*c+_R^2*b)*ln(x-_R),_R=RootOf(_Z^6*c+_
Z^3*b+a))-1/2*d/a/x^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^3 + b*d - a*e)/(c*x^6 + b*x^3 + a), x)/a - 1/2*d/(a*x^2)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)/x**3/(c*x**6+b*x**3+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)/x^3/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((x^3*e + d)/((c*x^6 + b*x^3 + a)*x^3), x)
```

Mupad [B]
time = 37.90, size = 2500, normalized size = 3.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)/(x^3*(a + b*x^3 + c*x^6)),x)
```

```
[Out] log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 + b^5*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*a^4*c*e^3*(-(4*a*c
 - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3
- 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^
3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^4*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4
*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 - 6*a
^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))
^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (8
1*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d
^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
+ 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2
 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^
3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a
^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/
(a^5*(4*a*c - b^2)^3))^(1/3))/2)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 +
```

b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 + 2*
a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 4
1*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/
2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^
4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) +
3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b
^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c*d^2*
e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5
*(4*a*c - b^2)^3))^(2/3))/18 + 36*a^10*c^5*e^3 + 72*a^8*b*c^6*d^3 - 108*a^9
*c^6*d^2*e + 9*a^6*b^5*c^4*d^3 - 54*a^7*b^3*c^5*d^3 - 9*a^9*b^2*c^4*e^3 - 1
08*a^9*b*c^5*d*e^2 - 27*a^7*b^4*c^4*d^2*e + 135*a^8*b^2*c^5*d^2*e + 27*a^8*
b^3*c^4*d*e^2))/6 - 3*a^6*c^5*x*(2*a^3*e^4 - 2*a*c^2*d^4 + b^2*c*d^4 - b^3*
d^3*e + 3*a*b^2*d^2*e^2 - 4*a^2*b*d*e^3))*(-(b^8*d^3 - a^3*b^5*e^3 + 16*a^4
*c^4*d^3 + b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^
2*e^3 + 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3
*d*e^2 + 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 - a^3*b^2*e^3*(-(4*a*c - b
^2)^3)^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e - 5*a*b^3*c*d^3*(-(4*a*c - b^
2)^3)^(1/2) - 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e -
27*a^3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e + 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^
3)^(1/2) + 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e
+ 72*a^4*b^2*c^2*d*e^2 - 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b^2*c*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(
1/2))/(54*(a^5*b^6 - 64*a^8*c^3 - 12*a^6*b^4*c + 48*a^7*b^2*c^2)))^(1/3) +
log(- (2^(2/3)*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d^3 - b^5*d^3*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3 - 2*a^4*c*e^3*(-(4*a*c
- b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2 + 41*a^2*b^4*c^2*d^3
- 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^6*c*d^
3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^4*d^2*e*
(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^3*b^4*c*d*e^2 + 96*a^4
*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^3*d*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a^4*b^2*c^2*d*e^2 + 6*a
^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*d^2*e*(-(4*a*c - b^2)^
3)^(1/2) + 9*a^3*b*c*d*e^2*(-(4*a*c - b^2)^3)^(1/2))/(a^5*(4*a*c - b^2)^3))
^(1/3)*((2^(1/3)*(81*a^8*c^3*x*(4*a*c - b^2)^2*(a*b*e - b^2*d + a*c*d) + (8
1*2^(2/3)*a^10*b*c^3*(4*a*c - b^2)^2*((b^8*d^3 - a^3*b^5*e^3 + 16*a^4*c^4*d
^3 - b^5*d^3*(-(4*a*c - b^2)^3)^(1/2) + 8*a^4*b^3*c*e^3 - 16*a^5*b*c^2*e^3
- 2*a^4*c*e^3*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b^6*d*e^2 - 48*a^5*c^3*d*e^2
+ 41*a^2*b^4*c^2*d^3 - 56*a^3*b^2*c^3*d^3 + a^3*b^2*e^3*(-(4*a*c - b^2)^3)
^(1/2) - 11*a*b^6*c*d^3 - 3*a*b^7*d^2*e + 5*a*b^3*c*d^3*(-(4*a*c - b^2)^3)^
(1/2) + 3*a*b^4*d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 30*a^2*b^5*c*d^2*e - 27*a^
3*b^4*c*d*e^2 + 96*a^4*b*c^3*d^2*e - 5*a^2*b*c^2*d^3*(-(4*a*c - b^2)^3)^(1/
2) - 3*a^2*b^3*d*e^2*(-(4*a*c - b^2)^3)^(1/2) - 96*a^3*b^3*c^2*d^2*e + 72*a
^4*b^2*c^2*d*e^2 + 6*a^3*c^2*d^2*e*(-(4*a*c - b^2)^3)^(1/2) - 12*a^2*b^2*c*
d^2*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^3*b*c*d*e^...

3.20 $\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=46

$$-\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] -1/6*x^6+1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.261$, Rules used = {1488, 814, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{x^6}{6}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/6*x^6 - ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8(1-x^3)}{1-x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(1-x)x^2}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(-x+\frac{x}{1-x+x^2}\right)\,dx,x,x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)\\
&= -\frac{x^6}{6}+\frac{1}{6}\log\left(1-x^3+x^6\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{x^6}{6}-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [A]
time = 0.01, size = 46, normalized size = 1.00

$$-\frac{x^6}{6}+\frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] -1/6*x^6 + ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A]
time = 0.02, size = 38, normalized size = 0.83

method	result	size
default	$-\frac{x^{6}}{6}+\frac{\ln \left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\, \arctan \left(\frac{\left(2 x^{3}-1\right) \sqrt{3}}{3}\right)}{9}$	38
risch	$-\frac{x^{6}}{6}+\frac{\ln \left(4 x^{6}-4 x^{3}+4\right)}{6}+\frac{\sqrt{3}\, \arctan \left(\frac{\left(2 x^{3}-1\right) \sqrt{3}}{3}\right)}{9}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/6*x^6+1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 37, normalized size = 0.80

$$-\frac{1}{6}\, x^{6}+\frac{1}{9}\, \sqrt{3}\, \arctan \left(\frac{1}{3}\, \sqrt{3}\, \left(2\, x^{3}-1\right)\right)+\frac{1}{6}\, \log \left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

Fricas [A]
time = 0.40, size = 37, normalized size = 0.80

$$-\frac{1}{6}\, x^{6}+\frac{1}{9}\, \sqrt{3}\, \arctan \left(\frac{1}{3}\, \sqrt{3}\, \left(2\, x^{3}-1\right)\right)+\frac{1}{6}\, \log \left(x^{6}-x^{3}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

Sympy [A]
time = 0.05, size = 42, normalized size = 0.91

$$-\frac{x^{6}}{6}+\frac{\log \left(x^{6}-x^{3}+1\right)}{6}+\frac{\sqrt{3}\, \operatorname{atan}\left(\frac{2 \sqrt{3}\, x^{3}}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**6/6 + log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/
3)/9
```

Giac [A]
time = 3.31, size = 37, normalized size = 0.80

$$-\frac{1}{6}\,x^6+\frac{1}{9}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^3-1\right)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/6*x^6 + 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3
+ 1)
```

Mupad [B]
time = 0.06, size = 39, normalized size = 0.85

$$\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}-\frac{x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^8*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^
6/6
```

3.21 $\int \frac{x^5(1-x^3)}{1-x^3+x^6} dx$

Optimal. Leaf size=31

$$-\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/3*x^3-2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1488, 787, 632, 210}

$$-\frac{2\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*x^3 - (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
```

```
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(1-x^3)}{1-x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{(1-x)x}{1-x+x^2}\,dx, x, x^3\right)\\ &= -\frac{x^3}{3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)\\ &= -\frac{x^3}{3}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\ &= -\frac{x^3}{3}-\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 31, normalized size = 1.00

$$-\frac{x^3}{3}+\frac{2\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*x^3 + (2*ArcTan[(-1 + 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A]
time = 0.04, size = 25, normalized size = 0.81

method	result	size
default	$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	25
risch	$-\frac{x^3}{3}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*x^3+2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 24, normalized size = 0.77

$$-\frac{1}{3}x^3 + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Fricas [A]
time = 0.41, size = 24, normalized size = 0.77

$$-\frac{1}{3}x^3 + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Sympy [A]
time = 0.04, size = 32, normalized size = 1.03

$$-\frac{x^3}{3} + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3} - \frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-x**3+1)/(x**6-x**3+1),x)
[Out] -x**3/3 + 2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Giac [A]
time = 3.91, size = 24, normalized size = 0.77

$$-\frac{1}{3}x^3 + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3 - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
[Out] -1/3*x^3 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1))
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.84

$$-\frac{2\,\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^5*(x^3 - 1))/(x^6 - x^3 + 1),x)
[Out] - (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - x^3/3
```

3.22 $\int \frac{x^2(1-x^3)}{1-x^3+x^6} \, dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] -1/6*ln(x^6-x^3+1)-1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1482, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] - Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(1-x^3)}{1-x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right) - \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= -\frac{1}{6}\log\left(1-x^3+x^6\right) - \frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - x^3))/(1 - x^3 + x^6),x]
[Out] ArcTan[(-1 + 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) - Log[1 - x^3 + x^6]/6
```

Maple [A]
time = 0.02, size = 33, normalized size = 0.85

method	result	size
default	$-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	33

risch	$-\frac{\ln(4x^6-4x^3+4)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{9}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/6*ln(x^6-x^3+1)+1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.53, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^3-1)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Fricas [A]
time = 0.44, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^3-1)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Sympy [A]
time = 0.05, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**3+1)/(x**6-x**3+1),x)
[Out] -log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9
```

Giac [A]
time = 3.69, size = 32, normalized size = 0.82

$$\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^3-1)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1)
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.87

$$-\frac{\ln\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9
```

3.23 $\int \frac{1-x^3}{x(1-x^3+x^6)} \, dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} + \log(x) - \frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1488, 814, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6}\log\left(x^6-x^3+1\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/(3*Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x^3}{x(1-x^3+x^6)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1-x}{x(1-x+x^2)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{3}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 44, normalized size = 1.07

$$
\log(x)-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x*(1 - x^3 + x^6)),x]
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^3) & ]/3
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.85

method	result	size
risch	$\ln\left(x\right)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^{3}-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{9}$	33
default	$\ln\left(x\right)-\frac{\ln\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{\left(2x^{3}-1\right)\sqrt{3}}{3}\right)}{9}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.50, size = 38, normalized size = 0.93

$$-\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{3}-1\right)\right)-\frac{1}{6}\,\log\left(x^{6}-x^{3}+1\right)+\frac{1}{3}\,\log\left(x^{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3
*log(x^3)
```

Fricas [A]
time = 0.40, size = 34, normalized size = 0.83

$$-\frac{1}{9}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{3}-1\right)\right)-\frac{1}{6}\,\log\left(x^{6}-x^{3}+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(x)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 1.00

$$\log\left(x\right)-\frac{\log\left(x^{6}-x^{3}+1\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^{3}}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/9
```

Giac [A]
time = 3.71, size = 35, normalized size = 0.85

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log
(abs(x))
```

Mupad [B]
time = 1.86, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/9
```

3.24 $\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{3x^3}+\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -1/3/x^3+2/9*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1488, 814, 632, 210}

$$\frac{2\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/3*1/x^3 + (2*ArcTan[(1 - 2*x^3)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
```

```
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{x^4\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1-x}{x^2\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x^2}+\frac{1}{-1+x-x^2}\right)\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+x-x^2}\,dx,x,x^3\right)\\
&= -\frac{1}{3x^3}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-2x^3\right)\\
&= -\frac{1}{3x^3}+\frac{2\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 45, normalized size = 1.45

$$-\frac{1}{3x^3}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^4*(1 - x^3 + x^6)),x]
```

```
[Out] -1/3*1/x^3 - RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-1 + 2*#1^3) & ]/3
```

Maple [A]
time = 0.03, size = 25, normalized size = 0.81

method	result	size
default	$-\frac{1}{3x^3}-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$	25
risch	$-\frac{1}{3x^3}-\frac{2\sqrt{3}\,\arctan\left(\frac{\left(2x^3-1\right)\sqrt{3}}{3}\right)}{9}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^4/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3/x^3-2/9*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.52, size = 24, normalized size = 0.77

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="maxima")
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

Fricas [A]
time = 0.41, size = 28, normalized size = 0.90

$$-\frac{2\sqrt{3}x^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)+3}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/9*(2*sqrt(3)*x^3*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 3)/x^3
```

Sympy [A]
time = 0.05, size = 36, normalized size = 1.16

$$-\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^3}{3}-\frac{\sqrt{3}}{3}\right)}{9}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**4/(x**6-x**3+1),x)
[Out] -2*sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/9 - 1/(3*x**3)
```

Giac [A]
time = 3.33, size = 24, normalized size = 0.77

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^4/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/3/x^3
```

Mupad [B]
time = 0.04, size = 26, normalized size = 0.84

$$\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{9}-\frac{1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^4*(x^6 - x^3 + 1)),x)
[Out] (2*3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/9 - 1/(3*x^3)
```

3.25 $\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=418

$$-\frac{x^4}{4}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\text{lo}}{9\sqrt[3]{2}}$$

```
[Out] -1/4*x^4+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(
1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(
3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I
*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2
/3)+1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1
/2))^(2/3)-1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^
(2/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

Rubi [A]
time = 0.36, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1516, 12, 1388, 206, 31, 648, 631, 210, 642}

$$-\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{x^4}{4}-\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/4*x^4 - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt
[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/
((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((
3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sq
rt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9
*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3
) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])
^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^
(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^6(1-x^3)}{1-x^3+x^6}\,dx &= -\frac{x^4}{4}-\frac{1}{4}\int -\frac{4x^3}{1-x^3+x^6}\,dx \\
&= -\frac{x^4}{4}+\int \frac{x^3}{1-x^3+x^6}\,dx \\
&= -\frac{x^4}{4}-\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx+\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx \\
&= -\frac{x^4}{4}+\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1+i}}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}}}}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2}} \\
&= -\frac{x^4}{4}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{}\right.}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&= -\frac{x^4}{4}+\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{}\right.}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} \\
&= -\frac{x^4}{4}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\right.}}}{\sqrt{3}}\right.}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 47, normalized size = 0.11

$$-\frac{x^4}{4}+\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/4*x^4 + RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 46, normalized size = 0.11

method	result	size
default	$-\frac{x^4}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	46
risch	$-\frac{x^4}{4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*x^4+1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/4*x^4 + integrate(x^3/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. 2(272) = 544.
time = 0.45, size = 770, normalized size = 1.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] -1/4*x^4 + 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(36*18^(2
/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 324*x^2 + 54*18^(1/3)
*12^(1/3)) + 2/27*18^(2/3)*12^(1/6)*arctan(1/216*(18^(1/3)*12^(5/6)*sqrt(3)
*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) +
18*x^2 + 3*18^(1/3)*12^(1/3)) - 6*18^(1/3)*12^(5/6)*sqrt(3)*x - 216*sin(2/3
*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3)
 + 2)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^
```

```
(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/648*(36*18^(1/3)*12^(
5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) - 108*(18^(1/3)*12^(5/6)*x + 24
*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-72*18^(
2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 216*18^(2/3)*12^(1/6
)*x*cos(2/3*arctan(sqrt(3) + 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3))*(18^(1
/3)*12^(5/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sin
(2/3*arctan(sqrt(3) + 2))) + 648*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) + 2))^2
 - 3)) + 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^
(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/648*(36*18^(1/3)*12^
(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 108*(18^(1/3)*12^(5/6)*x - 2
4*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-72*18^
(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 216*18^(2/3)*12^(1/
6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3))*(18^(
1/3)*12^(5/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*si
n(2/3*arctan(sqrt(3) + 2))) - 648*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) + 2))^
2 - 3)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 1
8^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2)))*log(-72*18^(2/3)*12^(1/6)*sq
rt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 216*18^(2/3)*12^(1/6)*x*cos(2/3*arct
an(sqrt(3) + 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3)) + 1/108*(18^(2/3)*12^(
1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arcta
n(sqrt(3) + 2)))*log(-72*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3)
 + 2)) - 216*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 1296*x^2 +
216*18^(1/3)*12^(1/3))
```

Sympy [**A**]
time = 0.07, size = 31, normalized size = 0.07

$$-\frac{x^4}{4} - \operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 + 9t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**4/4 - RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**
4 + 9*_t + x)))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(272) = 544.
time = 3.73, size = 645, normalized size = 1.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/4*x^4 - 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*
pi)^2 + 2*sqrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*p
```

```
i)*sin(4/9*pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(
3) - 1)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(2*sq
rt(3)*cos(2/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*si
n(2/9*pi)^4 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + s
qrt(3)*cos(2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi)
+ 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^4
- 12*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(
1/9*pi)^3*sin(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) +
 sin(1/9*pi))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt
(3) + 1/2)*sin(1/9*pi))) - 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*sq
rt(3)*cos(4/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/
9*pi)^2 - 2*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*sqrt
(3)*cos(4/9*pi) - cos(4/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(2/9*pi)^3
*sin(2/9*pi) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*c
os(2/9*pi)^2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/
9*pi))*log((-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(8*sq
rt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*c
os(1/9*pi)^4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*s
in(1/9*pi) - cos(1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2
 + 1)
```

Mupad [B]
time = 0.65, size = 332, normalized size = 0.79

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(-36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(-36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{x^4}{4}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(-3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(-3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^6*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*12i -
36)^(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/6)*(3^
(1/2)*12i - 36)^(1/3))/18 - x^4/4 - (2^(2/3)*log(x + (2^(2/3)*3^(1/3)*(- 3^
(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(4/3))/12)*(-
3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3
)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(4/
3))/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x
- (2^(2/3)*3^(1/3)*(- 3^(1/2)*1i - 3)^(1/3))/4 - (2^(2/3)*3^(5/6)*(- 3^(1/2
)*1i - 3)^(1/3)*1i)/12)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
 - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3^(1/2)*1i - 3)^(1/3))/4 + (2^(2/3)*3
^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)/12)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(
5/6)*1i))/36
```

3.26 $\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=382

$$-\frac{x^2}{2}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\,\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\cdots$$

```
[Out] -1/2*x^2+1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/
2))/(1-I*3^(1/2))^(1/3)-1/3*I*2^(1/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2
))^(1/3))*3^(1/2))/(1+I*3^(1/2))^(1/3)+1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1-I*3^(
1/2))^(1/3))/(1-I*3^(1/2))^(1/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1
-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(1/3)/(1-I*3^(1/2))^(1/3)*3^(1/2)-
1/9*I*2^(1/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(1/3)*3^(1/2
)+1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*
2^(1/3)/(1+I*3^(1/2))^(1/3)*3^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$,
Rules used = {1516, 12, 1389, 298, 31, 648, 631, 210, 642}

$$\frac{i\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}-\frac{x^2}{2}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\,\sqrt[3]{1-i\sqrt{3}}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\ 2^{2/3}\sqrt{3}\,\sqrt[3]{1+i\sqrt{3}}}+\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}-\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\,\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/2*x^2 + ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((
1 - I*Sqrt[3])/2)^(1/3) - ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3
))/Sqrt[3]])/((1 + I*Sqrt[3])/2)^(1/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) -
 2^(1/3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1 + I*Sqrt[3
])^(1/3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(1/3)) - ((I/3)*Log[(1
- I*Sqrt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*S
qrt[3]*(1 - I*Sqrt[3])^(1/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 +
I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(2/3)*Sqrt[3]*(1 + I*Sqrt[3])^(1/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1389

```
Int[((d_.)*(x_))^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symb
ol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*
x^n), x], x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[
{a, b, c, d, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(1-x^3)}{1-x^3+x^6}\,dx &= -\frac{x^2}{2} - \frac{1}{2}\int -\frac{2x}{1-x^3+x^6}\,dx \\
&= -\frac{x^2}{2} + \int \frac{x}{1-x^3+x^6}\,dx \\
&= -\frac{x^2}{2} - \frac{i\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} \\
&= -\frac{x^2}{2} + \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \cdots \\
&= -\frac{x^2}{2} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\int \cdots}{\cdots} \\
&= -\frac{x^2}{2} + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} - \frac{i\log\left(\left(1-i\sqrt{\cdots}\right.\right.}{\cdots} \\
&= -\frac{x^2}{2} + \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}} - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}} + \frac{i\log\left(\sqrt[3]{\cdots}\right.}{3\sqrt{3}\cdots}
\end{aligned}$$

<u>Mathematica</u> **[C]** Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 48, normalized size = 0.13

$$-\frac{x^2}{2} + \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)}{-\#1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/2*x^2 + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1 + 2*#1^4) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 44, normalized size = 0.12

method	result	size
default	$-\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44
risch	$-\frac{x^2}{2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*x^2+1/3*sum(_R/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + integrate(x/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 999 vs. 2(246) = 492.
time = 0.50, size = 999, normalized size = 2.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(48*18^(1/3)*12^(1/3
)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 48*
18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 144*x^2 - 24*18^(1/3)*
12^(1/3)*x + 4*18^(2/3)*12^(2/3)) + 2/27*18^(2/3)*12^(1/6)*arctan(-1/108*(1
2*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 - 6*18^(2/3)*1
```

```
2^(2/3)*sqrt(3)*x + 12*(144*cos(2/3*arctan(sqrt(3) - 2))^3 + (18^(2/3)*12^(
2/3)*x - 72)*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2)) - s
qrt(12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arc
tan(sqrt(3) - 2)) + 12*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 +
 36*x^2 - 6*18^(1/3)*12^(1/3)*x + 18^(2/3)*12^(2/3))*(2*18^(2/3)*12^(2/3)*s
qrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(
sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(2/3)*sqrt(3)) + 1
08*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) - 2))^4 - 16*cos(2/3*arctan(sqrt(3)
- 2))^2 + 3))*sin(2/3*arctan(sqrt(3) - 2)) - 1/2*x^2 - 1/27*(18^(2/3)*12^(1
/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan
(sqrt(3) - 2)))*arctan(1/432*(48*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan
(sqrt(3) - 2))^2 - 24*18^(2/3)*12^(2/3)*sqrt(3)*x - 48*(144*cos(2/3*arctan(
sqrt(3) - 2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) - 2)))*
sin(2/3*arctan(sqrt(3) - 2)) - sqrt(-192*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/
3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 192*18^(1/3)*12^(1/3)
*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 576*x^2 - 96*18^(1/3)*12^(1/3)*x + 16*1
8^(2/3)*12^(2/3))*(2*18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))
^2 - 2*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3
) - 2)) - 18^(2/3)*12^(2/3)*sqrt(3)) + 432*sqrt(3))/(16*cos(2/3*arctan(sqrt
(3) - 2))^4 - 16*cos(2/3*arctan(sqrt(3) - 2))^2 + 3)) + 1/27*(18^(2/3)*12^(
1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arcta
n(sqrt(3) - 2)))*arctan(-1/1728*(24*18^(2/3)*12^(2/3)*x - 1728*cos(2/3*arct
an(sqrt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(-384*18^(1/3)*12^(1/3)*x*cos(2/
3*arctan(sqrt(3) - 2))^2 + 576*x^2 + 192*18^(1/3)*12^(1/3)*x + 16*18^(2/3)*
12^(2/3)) + 864)/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2))
)) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/
3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(-192*18^(1/3)*12^(1/3)*sqrt(3
)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 192*18^(1/3
)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 576*x^2 - 96*18^(1/3)*12^(1/3
)*x + 16*18^(2/3)*12^(2/3)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arct
an(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(-384
*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 576*x^2 + 192*18^(1/3
)*12^(1/3)*x + 16*18^(2/3)*12^(2/3))
```

Sympy [A]
time = 0.07, size = 32, normalized size = 0.08

$$-\frac{x^2}{2} - \operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-6561t^5 - 27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -x**2/2 - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-6561*_t**
5 - 27*_t**2 + x)))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 820 vs. 2(246) = 492.
time = 3.83, size = 820, normalized size = 2.15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/2*x^2 - 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi
)^2 + 5*sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 1
0*cos(4/9*pi)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 - sqrt(3)*cos(4/9*pi)^2 + sqr
t(3)*sin(4/9*pi)^2 + 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1
)*cos(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*co
s(2/9*pi)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi
)*sin(2/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi
)^3 - sin(2/9*pi)^5 - sqrt(3)*cos(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^2 + 2*cos
(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2
*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*c
os(1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/
9*pi)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 + sqrt
(3)*cos(1/9*pi)^2 - sqrt(3)*sin(1/9*pi)^2 + 2*cos(1/9*pi)*sin(1/9*pi))*arct
an(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/9
*pi))) - 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi)
^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^3
*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 - 2*sqrt(3)*cos(4/9*pi)*sin(4/
9*pi) - cos(4/9*pi)^2 + sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/
9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*sqrt(3
)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5 - 10*
cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi)*sin(2/9*pi)^4 - 2*sqrt(3)*cos(2
/9*pi)*sin(2/9*pi) - cos(2/9*pi)^2 + sin(2/9*pi)^2)*log((-I*sqrt(3)*cos(2/9
*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(5*sqrt(3)*cos(1/9*pi)^4*sin(1/9*pi
) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5 - cos(1/
9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*pi)^4 + 2*
sqrt(3)*cos(1/9*pi)*sin(1/9*pi) - cos(1/9*pi)^2 + sin(1/9*pi)^2)*log((I*sqr
t(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B]
time = 2.28, size = 309, normalized size = 0.81

$$\frac{\ln\left(x+\left(81\,x-\frac{27\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\left(81\,x-\frac{27\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{x^2}{2}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (81*x - (27*(36 - 3^(1/2)*12i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162
))*(36 - 3^(1/2)*12i)^(1/3))/18 + (log(x - (81*x - (27*(3^(1/2)*12i + 36)^(
2/3))/4)*((3^(1/2)*1i)/486 + 1/162))*(3^(1/2)*12i + 36)^(1/3))/18 - x^2/2 -
 (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 + (2^(1/3)*3^
(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/
6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12 -
 (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3^(
1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)
^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log
(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(3^
(1/3) + 3^(5/6)*1i))/36
```

3.27 $\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=378

$$-x-\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}+\frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\,\mathrm{l}}{}$$

```
[Out] -x-1/3*I*2^(2/3)*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))/(1
-I*3^(1/2))^(2/3)+1/3*I*2^(2/3)*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/
3))*3^(1/2))/(1+I*3^(1/2))^(2/3)+1/9*I*2^(2/3)*ln(-2^(1/3)*x+(1-I*3^(1/2))^
(1/3))/(1-I*3^(1/2))^(2/3)*3^(1/2)-1/18*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(
1/2))^(1/3)+(1-I*3^(1/2))^(2/3))*2^(2/3)/(1-I*3^(1/2))^(2/3)*3^(1/2)-1/9*I*
2^(2/3)*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))/(1+I*3^(1/2))^(2/3)*3^(1/2)+1/18
*I*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*2^(2/3
)/(1+I*3^(1/2))^(2/3)*3^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 378, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {1516, 1361, 206, 31, 648, 631, 210, 642}

$$-\frac{i\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}+\frac{i\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{i\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{3\sqrt[3]{2}\sqrt{3}\left(1+i\sqrt{3}\right)^{2/3}}-x+\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}}-\frac{i\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x - ((I/3)*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/((1 - I*
Sqrt[3])/2)^(2/3) + ((I/3)*ArcTan[(1 + (2*x)/((1 + I*Sqrt[3])/2)^(1/3))/Sqr
t[3]])/((1 + I*Sqrt[3])/2)^(2/3) + ((I/3)*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/
3)*x])/(Sqrt[3]*((1 - I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 + I*Sqrt[3])^(1/
3) - 2^(1/3)*x])/(Sqrt[3]*((1 + I*Sqrt[3])/2)^(2/3)) - ((I/3)*Log[(1 - I*Sq
rt[3])^(2/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]
*(1 - I*Sqrt[3])^(2/3)) + ((I/3)*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt
[3]))^(1/3)*x + 2^(2/3)*x^2])/(2^(1/3)*Sqrt[3]*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1361

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^n), x], x] - Dist[c
/q, Int[1/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*
n] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
```

```
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^3(1-x^3)}{1-x^3+x^6}\,dx = -x + \int \frac{1}{1-x^3+x^6}\,dx$$

$$= -x - \frac{i\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}} + \frac{i\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx}{\sqrt{3}}$$

$$= -x + \frac{i\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}\,x+x^2}\,dx}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \cdots$$

$$= -x + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} - \frac{i\int \frac{\sqrt[3]{\cdots}}{\left(\frac{1}{2}\left(1-i\sqrt{3}\cdots\right.\right.}}{3\sqrt[3]{2}\cdots}$$

$$= -x + \frac{i\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{3\sqrt{3}\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} - \frac{i\log\left(\left(1-i\sqrt{\cdots}\right.\right.}{3\cdots}$$

$$= -x - \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}} + \frac{i\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}} + \frac{i\log\left(\sqrt[3]{\cdots}\right.}{3\sqrt{3}\left(\cdots\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 46, normalized size = 0.12

$$-x + \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{\log(x-\#1)}{-\#1^2+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -x + RootSum[1 - #1^3 + #1^6 & , Log[x - #1]/(-#1^2 + 2*#1^5) & ]/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 41, normalized size = 0.11

method	result	size
default	$-x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	41
risch	$-x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -x+1/3*sum(1/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 770 vs. 2(244) = 488.
time = 0.45, size = 770, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2))*log(72*18^(2/3)*12^(1/6
)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 216*18^(2/3)*12^(1/6)*x*cos(2/3*
arctan(sqrt(3) - 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3)) + 2/27*18^(2/3)*12
^(1/6)*arctan(-1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
- 2)) - sqrt(2)*sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2
```

```
)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 18*x^2 + 3*18^(1/
3)*12^(1/3))*(18^(1/3)*12^(5/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18
^(1/3)*12^(5/6)*sin(2/3*arctan(sqrt(3) - 2))) + 18*(18^(1/3)*12^(5/6)*x + 2
4*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2)) + 108*sqrt(3))
/(4*cos(2/3*arctan(sqrt(3) - 2))^2 - 3))*sin(2/3*arctan(sqrt(3) - 2)) + 1/2
7*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/
6)*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(1/3)*12^(5/6)*sqrt(3)*
x*cos(2/3*arctan(sqrt(3) - 2)) - sqrt(2)*sqrt(18^(2/3)*12^(1/6)*sqrt(3)*x*s
in(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3)
- 2)) + 18*x^2 + 3*18^(1/3)*12^(1/3))*(18^(1/3)*12^(5/6)*sqrt(3)*cos(2/3*ar
ctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sin(2/3*arctan(sqrt(3) - 2))) - 18
*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqr
t(3) - 2)) - 108*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) - 2))^2 - 3)) - 1/27*(1
8^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*s
in(2/3*arctan(sqrt(3) - 2)))*arctan(-1/2592*(72*18^(1/3)*12^(5/6)*sqrt(3)*x
 - 18^(1/3)*12^(5/6)*sqrt(3)*sqrt(-576*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*
arctan(sqrt(3) - 2)) + 5184*x^2 + 864*18^(1/3)*12^(1/3)) - 2592*sin(2/3*arc
tan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2))) + 1/108*(18^(2/3)*12^(1/6)
*sqrt(3)*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sq
rt(3) - 2)))*log(288*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2
)) - 864*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 5184*x^2 + 864*
18^(1/3)*12^(1/3)) - 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3
) - 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) - 2)))*log(-576*18^(2/3)
*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 5184*x^2 + 864*18^(1/3)*
12^(1/3)) - x
```

Sympy [A]
time = 0.07, size = 24, normalized size = 0.06

$$-x - \text{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(729t^4 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**3+1)/(x**6-x**3+1),x)
[Out] -x - RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 + x))
)
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(244) = 488.
time = 4.02, size = 635, normalized size = 1.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(
3)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^
3 - sqrt(3)*cos(4/9*pi) - sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4/9
*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) - 1/9*(sqrt(3)*cos(2/9*pi)
^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*cos(
2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 - sqrt(3)*cos(2/9*pi) -
 sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*sqrt(
3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos(1/9*pi
)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*pi) + 4
*cos(1/9*pi)*sin(1/9*pi)^3 + sqrt(3)*cos(1/9*pi) - sin(1/9*pi))*arctan(-1/2
*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/9*pi)))
- 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*pi)*sin(4/9
*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9*pi)^4 - sq
rt(3)*sin(4/9*pi) + cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi) - cos(4/9*pi))
*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) - 4*sqrt(3)*cos(2
/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2/9*pi)^2 - sin(
2/9*pi)^4 - sqrt(3)*sin(2/9*pi) + cos(2/9*pi))*log((-I*sqrt(3)*cos(2/9*pi)
- cos(2/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(1/9*pi)^3*sin(1/9*pi) - 4
*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*cos(1/9*pi)^2*sin(1/
9*pi)^2 + sin(1/9*pi)^4 + sqrt(3)*sin(1/9*pi) + cos(1/9*pi))*log((I*sqrt(3)
*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - x
```

Mupad [B]
time = 2.38, size = 330, normalized size = 0.87

$$-x+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{12}\right)\,{\left(36-\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{12}\right)\,{\left(36+\sqrt{3}\,12\mathrm{i}\right)}^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{6}\right)\,{\left(3-\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,1\mathrm{i}}{6}\right)\,{\left(3+\sqrt{3}\,1\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 -
 3^(1/2)*1i)^(1/3)*1i)/12)*(36 - 3^(1/2)*12i)^(1/3))/18 - x + (log(x + (2^(
2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^
(1/3)*1i)/12)*(3^(1/2)*12i + 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*3^(1
/3)*(3 - 3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12
)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(
2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^
(4/3))/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log
(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(3 - 3^(1/2)*1i)^(1/3)*
(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i
+ 3)^(1/3)*1i)/6)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.28 $\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{\cdots}\right.}{9\ 2^{2/3}\sqrt[3]{\cdots}}$$

```
[Out] 1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^(
1/3)/(1-I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(1/
2))*2^(1/3)/(1-I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))
^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/18*
ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3
)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3
+I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*
3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1+I*3^(1/2))^(1/3)
```

Rubi [A]
time = 0.19, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1524, 298, 31, 648, 631, 210, 642}

$$\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^
(2/3)*(1 - I*Sqrt[3])^(1/3)) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*Sqr
t[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3 - I*Sqrt[
3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqrt[3])^(1/3
)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1
 + I*Sqrt[3])^(1/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(1 -
 I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + (
(3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x + 2
^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x(1-x^3)}{1-x^3+x^6}\,dx = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\;2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\right.}}}{9\;2^{2/3}\sqrt[3]{1-i\sqrt{3}}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\;2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\;2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\;2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\;2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\;2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\;2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

<u>Mathematica</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 0.13

$$-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^3))/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1 + 2
*#1^4) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 44, normalized size = 0.11

method	result	size
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(\_R^{4}-\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\left(-\_R^{4}+\_R\right)\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] -1/3*sum((_R^4-_R)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -integrate((x^3 - 1)*x/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 992 vs. 2(267) = 534.
time = 0.47, size = 992, normalized size = 2.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(-24*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2 + 12*18^(1/3)*12^(1/3)*x + 18^
(2/3)*12^(2/3)) + 2/27*18^(2/3)*12^(1/6)*arctan(-1/432*(6*18^(2/3)*12^(2/3)
*x - 432*cos(2/3*arctan(sqrt(3) + 2))^2 - 18^(2/3)*12^(2/3)*sqrt(-24*18^(1/
3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2 + 36*x^2 + 12*18^(1/3)*12^(1/3
)*x + 18^(2/3)*12^(2/3)) + 216)/(cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arcta
n(sqrt(3) + 2))))*sin(2/3*arctan(sqrt(3) + 2)) + 1/27*(18^(2/3)*12^(1/6)*sq
rt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(
3) + 2)))*arctan(1/216*(24*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(
```

```
3) + 2))^2 - 12*18^(2/3)*12^(2/3)*sqrt(3)*x - 24*(144*cos(2/3*arctan(sqrt(3
) + 2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/
3*arctan(sqrt(3) + 2)) - sqrt(-48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arcta
n(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 48*18^(1/3)*12^(1/3)*x*cos(2
/3*arctan(sqrt(3) + 2))^2 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(2/3)*1
2^(2/3))*(2*18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 - 2*18
^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) -
 18^(2/3)*12^(2/3)*sqrt(3)) + 216*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) + 2))
^4 - 16*cos(2/3*arctan(sqrt(3) + 2))^2 + 3)) - 1/27*(18^(2/3)*12^(1/6)*sqrt
(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*sin(2/3*arctan(sqrt(3)
 + 2)))*arctan(-1/108*(12*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3
) + 2))^2 - 6*18^(2/3)*12^(2/3)*sqrt(3)*x + 12*(144*cos(2/3*arctan(sqrt(3)
+ 2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/3*
arctan(sqrt(3) + 2)) - sqrt(12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(s
qrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 12*18^(1/3)*12^(1/3)*x*cos(2/3*
arctan(sqrt(3) + 2))^2 + 36*x^2 - 6*18^(1/3)*12^(1/3)*x + 18^(2/3)*12^(2/3)
)*(2*18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2))^2 + 2*18^(2/3)*
12^(2/3)*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/
3)*12^(2/3)*sqrt(3)) + 108*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) + 2))^4 - 16
*cos(2/3*arctan(sqrt(3) + 2))^2 + 3)) + 1/108*(18^(2/3)*12^(1/6)*sqrt(3)*si
n(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))
)*log(48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) + 2))*sin(2/3*a
rctan(sqrt(3) + 2)) + 48*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) + 2))^2
 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(2/3)*12^(2/3)) - 1/108*(18^(2/3
)*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*cos(2/3
*arctan(sqrt(3) + 2)))*log(-48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(s
qrt(3) + 2))*sin(2/3*arctan(sqrt(3) + 2)) + 48*18^(1/3)*12^(1/3)*x*cos(2/3*
arctan(sqrt(3) + 2))^2 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(2/3)*12^(
2/3))
```

Sympy [**A**]
time = 0.07, size = 22, normalized size = 0.05

$$-\operatorname{RootSum}\left(19683t^6 - 243t^3 + 1, \left(t \mapsto t\log\left(-27t^2 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**3+1)/(x**6-x**3+1),x)
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(-27*_t**2 + x)))
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 824 vs. 2(267) = 534.
time = 5.62, size = 824, normalized size = 2.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^5 - 10*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 5*sqr
t(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 5*cos(4/9*pi)^4*sin(4/9*pi) + 10*cos(4/9*p
i)^2*sin(4/9*pi)^3 - sin(4/9*pi)^5 + 2*sqrt(3)*cos(4/9*pi)^2 - 2*sqrt(3)*si
n(4/9*pi)^2 - 4*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(4
/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*p
i)^5 - 10*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*sqrt(3)*cos(2/9*pi)*sin(2
/9*pi)^4 - 5*cos(2/9*pi)^4*sin(2/9*pi) + 10*cos(2/9*pi)^2*sin(2/9*pi)^3 - s
in(2/9*pi)^5 + 2*sqrt(3)*cos(2/9*pi)^2 - 2*sqrt(3)*sin(2/9*pi)^2 - 4*cos(2/
9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*I*
sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(sqrt(3)*cos(1/9*pi)^5 - 10*sqrt(3)*cos(
1/9*pi)^3*sin(1/9*pi)^2 + 5*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 5*cos(1/9*p
i)^4*sin(1/9*pi) - 10*cos(1/9*pi)^2*sin(1/9*pi)^3 + sin(1/9*pi)^5 - 2*sqrt(
3)*cos(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^2 - 4*cos(1/9*pi)*sin(1/9*pi))*arc
tan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(1/
9*pi))) + 1/18*(5*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 10*sqrt(3)*cos(4/9*pi
)^2*sin(4/9*pi)^3 + sqrt(3)*sin(4/9*pi)^5 + cos(4/9*pi)^5 - 10*cos(4/9*pi)^
3*sin(4/9*pi)^2 + 5*cos(4/9*pi)*sin(4/9*pi)^4 + 4*sqrt(3)*cos(4/9*pi)*sin(4
/9*pi) + 2*cos(4/9*pi)^2 - 2*sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9*pi) - c
os(4/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(2/9*pi)^4*sin(2/9*pi) - 10*s
qrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + sqrt(3)*sin(2/9*pi)^5 + cos(2/9*pi)^5
- 10*cos(2/9*pi)^3*sin(2/9*pi)^2 + 5*cos(2/9*pi)*sin(2/9*pi)^4 + 4*sqrt(3)*
cos(2/9*pi)*sin(2/9*pi) + 2*cos(2/9*pi)^2 - 2*sin(2/9*pi)^2)*log((-I*sqrt(3
)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(5*sqrt(3)*cos(1/9*pi)^4*s
in(1/9*pi) - 10*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + sqrt(3)*sin(1/9*pi)^5
 - cos(1/9*pi)^5 + 10*cos(1/9*pi)^3*sin(1/9*pi)^2 - 5*cos(1/9*pi)*sin(1/9*p
i)^4 - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + 2*cos(1/9*pi)^2 - 2*sin(1/9*pi)^
2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B]

time = 2.26, size = 281, normalized size = 0.68

$$\frac{\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}}{6}\right)\,\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x-\frac{\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{2/3}}{12}\right)\,\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,{\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,{\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,{\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{2/3}\,{\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}^2}{24}\right)\,\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\,\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x^3 - 1))/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i - 3)^(2/3))/6)*(3^(1/2)*12i - 36)^(1/
3))/18 + (log(x - (- 3^(1/2)*12i - 36)^(2/3)/12)*(- 3^(1/2)*12i - 36)^(1/3)
)/18 - (2^(2/3)*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6
)*1i)^2)/24)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)
*log(x - (2^(1/3)*(- 3^(1/2)*1i - 3)^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(-
 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/
3)*(3^(1/2)*1i - 3)^(2/3)*(3^(1/3) - 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1
/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*(3^(1/2)*1i - 3)
```

```
^(2/3)*(3^(1/3) + 3^(5/6)*1i)^2)/24)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5
/6)*1i))/36
```

3.29 $\int \frac{1-x^3}{1-x^3+x^6}\,dx$

Optimal. Leaf size=411

$$-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\cdots\right.}{9\sqrt[3]{2}\left(1\cdots\right.}$$

```
[Out] -1/6*arctan(1/3*(1+2*2^(1/3)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2))*2^
(2/3)/(1-I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3-I*3^(1
/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2)
)^(1/3)+(1-I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)-1/18
*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/
3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(
3+I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I
*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1+I*3^(1/2))^(2/3)
```

Rubi [A]
time = 0.18, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$,
Rules used = {1436, 206, 31, 648, 631, 210, 642}

$$-\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*((I - Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/
(2^(1/3)*(1 - I*Sqrt[3])^(2/3)) + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 + I*
Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) - ((3 - I*Sq
rt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*Sqrt[3])^(
2/3)) - ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)
*(1 + I*Sqrt[3])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3) + (2*(
1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3])^(2/3))
+ ((3 + I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(1/3)*x
+ 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{1-x^3}{1-x^3+x^6}\,dx = \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx$$

$$= -\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1-i\sqrt{3}}-x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}}\left(1-i\right.}}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} +$$

$$= -\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}} +$$

$$= -\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}} + \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 57, normalized size = 0.14

$$-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^3}{-\#1^2+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(1 - x^3 + x^6),x]
```

```
[Out] -1/3*RootSum[1 - #1^3 + #1^6 & , (-Log[x - #1] + Log[x - #1]*#1^3)/(-#1^2 +
 2*#1^5) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 44, normalized size = 0.11

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\left(-\_R^3+1\right)\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*sum((-_R^3+1)/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^3 - 1)/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 765 vs. 2(267) = 534.
time = 0.40, size = 765, normalized size = 1.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/54*18^(2/3)*12^(1/6)*cos(2/3*arctan(sqrt(3) + 2))*log(-72*18^(2/3)*12^(1/
6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) + 216*18^(2/3)*12^(1/6)*x*cos(2/3
*arctan(sqrt(3) + 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3)) + 2/27*18^(2/3)*1
2^(1/6)*arctan(1/648*(36*18^(1/3)*12^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3)
 + 2)) - 108*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) + 2)))*sin(2/
3*arctan(sqrt(3) + 2)) - sqrt(-72*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arcta
n(sqrt(3) + 2)) + 216*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 12
96*x^2 + 216*18^(1/3)*12^(1/3))*(18^(1/3)*12^(5/6)*sqrt(3)*cos(2/3*arctan(s
qrt(3) + 2)) - 3*18^(1/3)*12^(5/6)*sin(2/3*arctan(sqrt(3) + 2))) + 648*sqrt
```

```
(3))/(4*cos(2/3*arctan(sqrt(3) + 2))^2 - 3))*sin(2/3*arctan(sqrt(3) + 2)) -
 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12
^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(-1/1296*(72*18^(1/3)*12^(5/6)*s
qrt(3)*x*cos(2/3*arctan(sqrt(3) + 2)) + 216*(18^(1/3)*12^(5/6)*x - 24*cos(2
/3*arctan(sqrt(3) + 2)))*sin(2/3*arctan(sqrt(3) + 2)) - sqrt(-288*18^(2/3)*
12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2)) - 864*18^(2/3)*12^(1/6)*x*c
os(2/3*arctan(sqrt(3) + 2)) + 5184*x^2 + 864*18^(1/3)*12^(1/3))*(18^(1/3)*1
2^(5/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 3*18^(1/3)*12^(5/6)*sin(2/3*
arctan(sqrt(3) + 2))) - 1296*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) + 2))^2 - 3
)) - 1/27*(18^(2/3)*12^(1/6)*sqrt(3)*cos(2/3*arctan(sqrt(3) + 2)) + 18^(2/3
)*12^(1/6)*sin(2/3*arctan(sqrt(3) + 2)))*arctan(1/216*(18^(1/3)*12^(5/6)*sq
rt(3)*sqrt(2)*sqrt(2*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2
)) + 18*x^2 + 3*18^(1/3)*12^(1/3)) - 6*18^(1/3)*12^(5/6)*sqrt(3)*x - 216*si
n(2/3*arctan(sqrt(3) + 2)))/cos(2/3*arctan(sqrt(3) + 2))) + 1/108*(18^(2/3)
*12^(1/6)*sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) - 18^(2/3)*12^(1/6)*cos(2/3*
arctan(sqrt(3) + 2)))*log(576*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sq
rt(3) + 2)) + 5184*x^2 + 864*18^(1/3)*12^(1/3)) - 1/108*(18^(2/3)*12^(1/6)*
sqrt(3)*sin(2/3*arctan(sqrt(3) + 2)) + 18^(2/3)*12^(1/6)*cos(2/3*arctan(sqr
t(3) + 2)))*log(-288*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) + 2
)) - 864*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) + 2)) + 5184*x^2 + 864*
18^(1/3)*12^(1/3))
```

Sympy [A]
time = 0.07, size = 26, normalized size = 0.06

$$-\operatorname{RootSum}\left(19683t^{6}-243t^{3}+1,\left(t\mapsto t\log\left(729t^{4}-9t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 - 243*_t**3 + 1, Lambda(_t, _t*log(729*_t**4 - 9*_t +
x)))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(267) = 534.
time = 4.71, size = 640, normalized size = 1.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(sqrt(3)*cos(4/9*pi)^4 - 6*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + sqrt(3
)*sin(4/9*pi)^4 + 4*cos(4/9*pi)^3*sin(4/9*pi) - 4*cos(4/9*pi)*sin(4/9*pi)^3
 + 2*sqrt(3)*cos(4/9*pi) + 2*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(
4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(sqrt(3)*cos(2/9*
```

```
pi)^4 - 6*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + sqrt(3)*sin(2/9*pi)^4 + 4*c
os(2/9*pi)^3*sin(2/9*pi) - 4*cos(2/9*pi)*sin(2/9*pi)^3 + 2*sqrt(3)*cos(2/9*
pi) + 2*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/2*
I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(sqrt(3)*cos(1/9*pi)^4 - 6*sqrt(3)*cos
(1/9*pi)^2*sin(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^4 - 4*cos(1/9*pi)^3*sin(1/9*
pi) + 4*cos(1/9*pi)*sin(1/9*pi)^3 - 2*sqrt(3)*cos(1/9*pi) + 2*sin(1/9*pi))*
arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*sin
(1/9*pi))) + 1/18*(4*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 4*sqrt(3)*cos(4/9*
pi)*sin(4/9*pi)^3 - cos(4/9*pi)^4 + 6*cos(4/9*pi)^2*sin(4/9*pi)^2 - sin(4/9
*pi)^4 + 2*sqrt(3)*sin(4/9*pi) - 2*cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9*pi)
 - cos(4/9*pi))*x + x^2 + 1) + 1/18*(4*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi) -
4*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - cos(2/9*pi)^4 + 6*cos(2/9*pi)^2*sin(2
/9*pi)^2 - sin(2/9*pi)^4 + 2*sqrt(3)*sin(2/9*pi) - 2*cos(2/9*pi))*log((-I*s
qrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(4*sqrt(3)*cos(1/9*pi
)^3*sin(1/9*pi) - 4*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + cos(1/9*pi)^4 - 6*c
os(1/9*pi)^2*sin(1/9*pi)^2 + sin(1/9*pi)^4 - 2*sqrt(3)*sin(1/9*pi) - 2*cos(
1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1)
```

Mupad [B]
time = 2.30, size = 319, normalized size = 0.78

$$\frac{\ln\left(x-\frac{\left(-\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36-\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}+\frac{\ln\left(x+\frac{\left(\frac{27}{2}+\frac{\sqrt{3}\,9\mathrm{i}}{2}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{54}\right)\left(-36+\sqrt{3}\,12\mathrm{i}\right)^{1/3}}{18}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)^3}{16}+27\right)}{108}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)\left(\frac{3\left(-3+\sqrt{3}\,1\mathrm{i}\right)\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)^3}{16}-27\right)}{108}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x+\frac{2^{2/3}\,3^{5/6}\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-3-\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}-3^{5/6}\,1\mathrm{i}\right)}{36}-\frac{2^{2/3}\ln\left(x-\frac{2^{2/3}\,3^{5/6}\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}1\mathrm{i}}{6}\right)\left(-3+\sqrt{3}\,1\mathrm{i}\right)^{1/3}\left(3^{1/3}+3^{5/6}\,1\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^6 - x^3 + 1),x)
```

```
[Out] (log(x - (((3^(1/2)*9i)/2 - 27/2)*(- 3^(1/2)*12i - 36)^(1/3))/54)*(- 3^(1/2
)*12i - 36)^(1/3))/18 + (log(x + (((3^(1/2)*9i)/2 + 27/2)*(3^(1/2)*12i - 36
)^(1/3))/54)*(3^(1/2)*12i - 36)^(1/3))/18 - (2^(2/3)*log(x - (2^(2/3)*(- 3^
(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i)*((3*(3^(1/2)*1i + 3)*(3^(1/3) +
3^(5/6)*1i)^3)/16 + 27))/108)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1
i))/36 - (2^(2/3)*log(x + (2^(2/3)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6
)*1i)*((3*(3^(1/2)*1i - 3)*(3^(1/3) - 3^(5/6)*1i)^3)/16 - 27))/108)*(3^(1/2
)*1i - 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(2/3)*3^(5
/6)*(- 3^(1/2)*1i - 3)^(1/3)*1i)/6)*(- 3^(1/2)*1i - 3)^(1/3)*(3^(1/3) - 3^(
5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i - 3)^(1/3)*1i)
/6)*(3^(1/2)*1i - 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36
```

3.30 $\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx$

Optimal. Leaf size=416

$$-\frac{1}{x}-\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{\left(3+i\sqrt{3}\right)\log}{9\ 2^{2}}$$

```
[Out] -1/x+1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(1/2)
)*2^(1/3)/(1+I*3^(1/2))^(1/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(3-I*
3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I*3^(
1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(1/3)/(1+I*3^(1/2))^(1/3)-
1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))
^(1/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^(2/3
))*(3+I*3^(1/2))*2^(1/3)/(1-I*3^(1/2))^(1/3)-1/6*arctan(1/3*(1+2*2^(1/3)*x/
(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(1/3)/(1-I*3^(1/2))^(1/3)
```

Rubi [A]
time = 0.18, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1518, 1388, 298, 31, 648, 631, 210, 642}

$$-\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}\,x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}\,x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}-\frac{1}{x}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1-i\sqrt{3}}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}\,x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\ 2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sqrt[
3]])/(3*2^(2/3)*(1 - I*Sqrt[3])^(1/3)) + ((I - Sqrt[3])*ArcTan[(1 + (2*x)/(
(1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(2/3)*(1 + I*Sqrt[3])^(1/3)) - ((3
 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(2/3)*(1 - I*Sqr
t[3])^(1/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*
2^(2/3)*(1 + I*Sqrt[3])^(1/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2/3)
 + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 - I*Sqrt[3])^
(1/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3]))^(
1/3)*x + 2^(2/3)*x^2])/(18*2^(2/3)*(1 + I*Sqrt[3])^(1/3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Dist[-(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
```

```
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^3}{x^2(1-x^3+x^6)}\,dx = -\frac{1}{x} - \int \frac{x^4}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{x} + \frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx - \frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{x}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^3}\,d$$

$$= -\frac{1}{x} + -\frac{\left(-3-i\sqrt{3}\right)\int \frac{-\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}+x}{\left(\frac{1}{2}\left(1-i\sqrt{3}\right)\right)^{2/3}+\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}x+x^2}\,dx}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\int}{9\,2}$$

$$= -\frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\right.}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{1}{x} - \frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}} - \frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}-\right.}{9\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

$$= -\frac{1}{x} - \frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\,2^{2/3}\sqrt[3]{1-i\sqrt{3}}} + \frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+\right.}}}{\sqrt{3}}\right.}{3\,2^{2/3}\sqrt[3]{1+i\sqrt{3}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 47, normalized size = 0.11

$$-\frac{1}{x} - \frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1^2}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^2*(1 - x^3 + x^6)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^3) & ]/
3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 46, normalized size = 0.11

method	result	size
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^{6}-9\_Z^{3}+1\right)}\_R\ln\left(-27\_R^{5}+6\_R^{2}+x\right)\right)}{3}$	40
default	$-\frac{1}{x}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{6}-\_Z^{3}+1\right)}\frac{\_R^{4}\ln\left(x-\_R\right)}{2\_R^{5}-\_R^{2}}\right)}{3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^2/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x-1/3*sum(_R^4/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^4/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1009 vs. 2(272) = 544.
time = 0.44, size = 1009, normalized size = 2.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2))*log(-48*18^(1/3)*
12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)
) + 48*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 144*x^2 - 24*18
^(1/3)*12^(1/3)*x + 4*18^(2/3)*12^(2/3)) + 8*18^(2/3)*12^(1/6)*x*arctan(1/2
```

```
16*(24*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 - 12*18^(
2/3)*12^(2/3)*sqrt(3)*x - 24*(144*cos(2/3*arctan(sqrt(3) - 2))^3 + (18^(2/3
)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) - 2
)) - sqrt(-48*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2))*sin(
2/3*arctan(sqrt(3) - 2)) + 48*18^(1/3)*12^(1/3)*x*cos(2/3*arctan(sqrt(3) -
2))^2 + 144*x^2 - 24*18^(1/3)*12^(1/3)*x + 4*18^(2/3)*12^(2/3))*(2*18^(2/3)
*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 - 2*18^(2/3)*12^(2/3)*cos(
2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(2/3)*s
qrt(3)) + 216*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) - 2))^4 - 16*cos(2/3*arct
an(sqrt(3) - 2))^2 + 3))*sin(2/3*arctan(sqrt(3) - 2)) + 4*(18^(2/3)*12^(1/6
)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - 18^(2/3)*12^(1/6)*x*sin(2/3*arct
an(sqrt(3) - 2)))*arctan(-1/108*(12*18^(2/3)*12^(2/3)*sqrt(3)*x*cos(2/3*arc
tan(sqrt(3) - 2))^2 - 6*18^(2/3)*12^(2/3)*sqrt(3)*x + 12*(144*cos(2/3*arcta
n(sqrt(3) - 2))^3 + (18^(2/3)*12^(2/3)*x - 72)*cos(2/3*arctan(sqrt(3) - 2))
)*sin(2/3*arctan(sqrt(3) - 2)) - sqrt(12*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(2/
3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 12*18^(1/3)*12^(1/3)*
x*cos(2/3*arctan(sqrt(3) - 2))^2 + 36*x^2 - 6*18^(1/3)*12^(1/3)*x + 18^(2/3
)*12^(2/3))*(2*18^(2/3)*12^(2/3)*sqrt(3)*cos(2/3*arctan(sqrt(3) - 2))^2 + 2
*18^(2/3)*12^(2/3)*cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)
) - 18^(2/3)*12^(2/3)*sqrt(3)) + 108*sqrt(3))/(16*cos(2/3*arctan(sqrt(3) -
2))^4 - 16*cos(2/3*arctan(sqrt(3) - 2))^2 + 3)) - 4*(18^(2/3)*12^(1/6)*sqrt
(3)*x*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*x*sin(2/3*arctan(sqr
t(3) - 2)))*arctan(-1/1728*(24*18^(2/3)*12^(2/3)*x - 1728*cos(2/3*arctan(sq
rt(3) - 2))^2 - 18^(2/3)*12^(2/3)*sqrt(-384*18^(1/3)*12^(1/3)*x*cos(2/3*arc
tan(sqrt(3) - 2))^2 + 576*x^2 + 192*18^(1/3)*12^(1/3)*x + 16*18^(2/3)*12^(2
/3)) + 864)/(cos(2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)))) -
(18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/
6)*x*cos(2/3*arctan(sqrt(3) - 2)))*log(192*18^(1/3)*12^(1/3)*sqrt(3)*x*cos(
2/3*arctan(sqrt(3) - 2))*sin(2/3*arctan(sqrt(3) - 2)) + 192*18^(1/3)*12^(1/
3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 576*x^2 - 96*18^(1/3)*12^(1/3)*x + 16
*18^(2/3)*12^(2/3)) + (18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) -
 2)) - 18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)))*log(-384*18^(1/3)*
12^(1/3)*x*cos(2/3*arctan(sqrt(3) - 2))^2 + 576*x^2 + 192*18^(1/3)*12^(1/3)
*x + 16*18^(2/3)*12^(2/3)) - 108)/x
```

Sympy [A]
time = 0.08, size = 31, normalized size = 0.07

$$-\operatorname{RootSum}\left(19683t^6+243t^3+1,\left(t\mapsto t\log\left(6561t^5+54t^2+x\right)\right)\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**2/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(6561*_t**5 + 54*_t*
*2 + x))) - 1/x
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 832 vs. $2(272) = 544$.
time = 4.55, size = 832, normalized size = 2.00

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^2/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^5 - 20*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi)^2 + 10*
sqrt(3)*cos(4/9*pi)*sin(4/9*pi)^4 - 10*cos(4/9*pi)^4*sin(4/9*pi) + 20*cos(4
/9*pi)^2*sin(4/9*pi)^3 - 2*sin(4/9*pi)^5 + sqrt(3)*cos(4/9*pi)^2 - sqrt(3)*
sin(4/9*pi)^2 - 2*cos(4/9*pi)*sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos
(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2
/9*pi)^5 - 20*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*sqrt(3)*cos(2/9*pi)*
sin(2/9*pi)^4 - 10*cos(2/9*pi)^4*sin(2/9*pi) + 20*cos(2/9*pi)^2*sin(2/9*pi)
^3 - 2*sin(2/9*pi)^5 + sqrt(3)*cos(2/9*pi)^2 - sqrt(3)*sin(2/9*pi)^2 - 2*co
s(2/9*pi)*sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/
2*I*sqrt(3) + 1/2)*sin(2/9*pi))) - 1/9*(2*sqrt(3)*cos(1/9*pi)^5 - 20*sqrt(3
)*cos(1/9*pi)^3*sin(1/9*pi)^2 + 10*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^4 + 10*c
os(1/9*pi)^4*sin(1/9*pi) - 20*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sin(1/9*pi)^5
 - sqrt(3)*cos(1/9*pi)^2 + sqrt(3)*sin(1/9*pi)^2 - 2*cos(1/9*pi)*sin(1/9*pi
))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*
sin(1/9*pi))) + 1/18*(10*sqrt(3)*cos(4/9*pi)^4*sin(4/9*pi) - 20*sqrt(3)*cos
(4/9*pi)^2*sin(4/9*pi)^3 + 2*sqrt(3)*sin(4/9*pi)^5 + 2*cos(4/9*pi)^5 - 20*c
os(4/9*pi)^3*sin(4/9*pi)^2 + 10*cos(4/9*pi)*sin(4/9*pi)^4 + 2*sqrt(3)*cos(4
/9*pi)*sin(4/9*pi) + cos(4/9*pi)^2 - sin(4/9*pi)^2)*log((-I*sqrt(3)*cos(4/9
*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(2/9*pi)^4*sin(2/9*p
i) - 20*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^3 + 2*sqrt(3)*sin(2/9*pi)^5 + 2*c
os(2/9*pi)^5 - 20*cos(2/9*pi)^3*sin(2/9*pi)^2 + 10*cos(2/9*pi)*sin(2/9*pi)^
4 + 2*sqrt(3)*cos(2/9*pi)*sin(2/9*pi) + cos(2/9*pi)^2 - sin(2/9*pi)^2)*log(
(-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) + 1/18*(10*sqrt(3)*cos(
1/9*pi)^4*sin(1/9*pi) - 20*sqrt(3)*cos(1/9*pi)^2*sin(1/9*pi)^3 + 2*sqrt(3)*
sin(1/9*pi)^5 - 2*cos(1/9*pi)^5 + 20*cos(1/9*pi)^3*sin(1/9*pi)^2 - 10*cos(1
/9*pi)*sin(1/9*pi)^4 - 2*sqrt(3)*cos(1/9*pi)*sin(1/9*pi) + cos(1/9*pi)^2 -
sin(1/9*pi)^2)*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/x
```

Mupad [B]
time = 0.40, size = 313, normalized size = 0.75

$$\frac{\ln\left(-x+\left(162\,x+\frac{27\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(-x-\left(162\,x+\frac{27\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{2/3}}{4}\right)\,\left(-\frac{1}{162}+\frac{\sqrt{3}\,1{}\mathrm{i}}{486}\right)\right)\,\left(36-\sqrt{3}\,12{}\mathrm{i}\right)^{1/3}}{18}-\frac{1}{x}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}-\frac{2^{1/3}\,3^{1/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{12}+\frac{2^{1/3}\,3^{1/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}\,1{}\mathrm{i}}{4}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{1/3}\,3^{2/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{2/3}}{6}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^2*(x^6 - x^3 + 1)),x)
```

```
[Out] (log((162*x + (27*(3^(1/2)*12i + 36)^(2/3))/4)*((3^(1/2)*1i)/486 + 1/162) -
 x)*(3^(1/2)*12i + 36)^(1/3))/18 + (log(- x - (162*x + (27*(36 - 3^(1/2)*12
i)^(2/3))/4)*((3^(1/2)*1i)/486 - 1/162))*(36 - 3^(1/2)*12i)^(1/3))/18 - 1/x
 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1i)^(2/3))/12 - (2^(1/3)*
3^(1/6)*(3 - 3^(1/2)*1i)^(2/3)*1i)/4)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) - 3^(
5/6)*1i))/36 - (2^(2/3)*log(x + (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/12
 + (2^(1/3)*3^(1/6)*(3^(1/2)*1i + 3)^(2/3)*1i)/4)*(3^(1/2)*1i + 3)^(1/3)*(3
^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(1/3)*3^(2/3)*(3 - 3^(1/2)*1
i)^(2/3))/6)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*l
og(x - (2^(1/3)*3^(2/3)*(3^(1/2)*1i + 3)^(2/3))/6)*(3^(1/2)*1i + 3)^(1/3)*(
3^(1/3) - 3^(5/6)*1i))/36
```

3.31 $\int \frac{1-x^3}{x^3(1-x^3+x^6)}\,dx$

Optimal. Leaf size=418

$$-\frac{1}{2x^2}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)}{9}$$

```
[Out] -1/2/x^2-1/6*arctan(1/3*(1+2*2^(1/3)*x/(1+I*3^(1/2))^(1/3))*3^(1/2))*(I-3^(
1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)-1/18*ln(-2^(1/3)*x+(1+I*3^(1/2))^(1/3))*(
3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1+I
*3^(1/2))^(1/3)+(1+I*3^(1/2))^(2/3))*(3-I*3^(1/2))*2^(2/3)/(1+I*3^(1/2))^(2
/3)-1/18*ln(-2^(1/3)*x+(1-I*3^(1/2))^(1/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1
/2))^(2/3)+1/36*ln(2^(2/3)*x^2+2^(1/3)*x*(1-I*3^(1/2))^(1/3)+(1-I*3^(1/2))^
(2/3))*(3+I*3^(1/2))*2^(2/3)/(1-I*3^(1/2))^(2/3)+1/6*arctan(1/3*(1+2*2^(1/3
)*x/(1-I*3^(1/2))^(1/3))*3^(1/2))*(3^(1/2)+I)*2^(2/3)/(1-I*3^(1/2))^(2/3)
```

Rubi [A]

time = 0.23, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1518, 12, 1388, 206, 31, 648, 631, 210, 642}

$$\frac{\left(\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(-\sqrt{3}+i\right)\text{ArcTan}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{1}{2x^2}+\frac{\left(3+i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1-i\sqrt{3}\right)}x+\left(1-i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}+\frac{\left(3-i\sqrt{3}\right)\log\left(2^{2/3}x^2+\sqrt[3]{2\left(1+i\sqrt{3}\right)}x+\left(1+i\sqrt{3}\right)^{2/3}\right)}{18\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3+i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1-i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(-\sqrt[3]{2}x+\sqrt[3]{1+i\sqrt{3}}\right)}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 + ((I + Sqrt[3])*ArcTan[(1 + (2*x)/((1 - I*Sqrt[3])/2)^(1/3))/Sq
rt[3]])/(3*2^(1/3)*(1 - I*Sqrt[3])^(2/3)) - ((I - Sqrt[3])*ArcTan[(1 + (2*x
)/((1 + I*Sqrt[3])/2)^(1/3))/Sqrt[3]])/(3*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) -
((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(1/3) - 2^(1/3)*x])/(9*2^(1/3)*(1 - I*
Sqrt[3])^(2/3)) - ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(1/3) - 2^(1/3)*x])/
(9*2^(1/3)*(1 + I*Sqrt[3])^(2/3)) + ((3 + I*Sqrt[3])*Log[(1 - I*Sqrt[3])^(2
/3) + (2*(1 - I*Sqrt[3]))^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 - I*Sqrt[3
])^(2/3)) + ((3 - I*Sqrt[3])*Log[(1 + I*Sqrt[3])^(2/3) + (2*(1 + I*Sqrt[3])
)^(1/3)*x + 2^(2/3)*x^2])/(18*2^(1/3)*(1 + I*Sqrt[3])^(2/3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1388

```
Int[((d_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^n/2)*(b/q + 1), Int[(d*x)^(m -
 n)/(b/2 + q/2 + c*x^n), x], x] - Dist[(d^n/2)*(b/q - 1), Int[(d*x)^(m - n)
/(b/2 - q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[n2, 2*n] &&
NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && GeQ[m, n]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^3}{x^3\left(1-x^3+x^6\right)}\,dx = -\frac{1}{2x^2}-\frac{1}{2}\int \frac{2x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{2x^2}-\int \frac{x^3}{1-x^3+x^6}\,dx$$

$$= -\frac{1}{2x^2}+\frac{1}{6}\left(-3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}-\frac{i\sqrt{3}}{2}+x^3}\,dx-\frac{1}{6}\left(3+i\sqrt{3}\right)\int \frac{1}{-\frac{1}{2}+\frac{i\sqrt{3}}{2}+x^{3}}$$

$$= -\frac{1}{2x^2}-\frac{\left(3-i\sqrt{3}\right)\int \frac{1}{-\sqrt[3]{\frac{1}{2}\left(1+i\sqrt{3}\right)}+x}\,dx}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\int \frac{-2^{2/3}\sqrt[3]{1}}{\left(\frac{1}{2}\left(1+i\sqrt{3}\right)\right)^{2/3}+}}{9\sqrt[3]{2}\left(1+i\sqrt{}\right.}$$

$$= -\frac{1}{2x^2}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}\right.}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{1}{2x^2}-\frac{\left(3+i\sqrt{3}\right)\log\left(\sqrt[3]{1-i\sqrt{3}}-\sqrt[3]{2}\,x\right)}{9\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(3-i\sqrt{3}\right)\log\left(\sqrt[3]{1+i\sqrt{3}}\right.}{9\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{2/3}}$$

$$= -\frac{1}{2x^2}+\frac{\left(i+\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{\frac{1}{2}\left(1-i\sqrt{3}\right)}}}{\sqrt{3}}\right)}{3\sqrt[3]{2}\left(1-i\sqrt{3}\right)^{2/3}}-\frac{\left(i-\sqrt{3}\right)\tan^{-1}\left(\frac{1+\frac{}{\sqrt[3]{\frac{1}{2}\left(1\right.}}}{\sqrt{}}\right.}{3\sqrt[3]{2}\left(1+i\sqrt{3}\right)^{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 47, normalized size = 0.11

$$-\frac{1}{2x^2}-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{\log(x-\#1)\#1}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x^3*(1 - x^3 + x^6)),x]
```

```
[Out] -1/2*1/x^2 - RootSum[1 - #1^3 + #1^6 & , (Log[x - #1]*#1)/(-1 + 2*#1^3) & ]
/3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 46, normalized size = 0.11

method	result	size
risch	$-\frac{1}{2x^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(27\_Z^6-9\_Z^3+1\right)}\_R\ln\left(-18\_R^4+3\_R+x\right)\right)}{3}$	38
default	$-\frac{1}{2x^2}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^6-\_Z^3+1\right)}\frac{\_R^3\ln\left(x-\_R\right)}{2\_R^5-\_R^2}\right)}{3}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x^3/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2-1/3*sum(_R^3/(2*_R^5-_R^2)*ln(x-_R),_R=RootOf(_Z^6-_Z^3+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^3/(x^6 - x^3 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 802 vs. 2(272) = 544.
time = 0.42, size = 802, normalized size = 1.92

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/108*(2*18^(2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) - 2))*log(-36*18^(2/3
)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 324*x^2 + 54*18^(1/3)*1
2^(1/3)) - 8*18^(2/3)*12^(1/6)*x^2*arctan(-1/648*(18*18^(1/3)*12^(5/6)*sqrt
(3)*x - 18^(1/3)*12^(5/6)*sqrt(3)*sqrt(-36*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(
2/3*arctan(sqrt(3) - 2)) + 324*x^2 + 54*18^(1/3)*12^(1/3)) - 648*sin(2/3*ar
ctan(sqrt(3) - 2)))/cos(2/3*arctan(sqrt(3) - 2)))*sin(2/3*arctan(sqrt(3) -
```

```
2)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3) - 2)) + 18^(2
/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) - 2)))*arctan(-1/108*(6*18^(1/3)*12
^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - sqrt(2)*sqrt(18^(2/3)*12^(1
/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 3*18^(2/3)*12^(1/6)*x*cos(2/3*
arctan(sqrt(3) - 2)) + 18*x^2 + 3*18^(1/3)*12^(1/3))*(18^(1/3)*12^(5/6)*sqr
t(3)*cos(2/3*arctan(sqrt(3) - 2)) + 3*18^(1/3)*12^(5/6)*sin(2/3*arctan(sqrt
(3) - 2))) + 18*(18^(1/3)*12^(5/6)*x + 24*cos(2/3*arctan(sqrt(3) - 2)))*sin
(2/3*arctan(sqrt(3) - 2)) + 108*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) - 2))^2
- 3)) - 4*(18^(2/3)*12^(1/6)*sqrt(3)*x^2*cos(2/3*arctan(sqrt(3) - 2)) - 18^
(2/3)*12^(1/6)*x^2*sin(2/3*arctan(sqrt(3) - 2)))*arctan(1/108*(6*18^(1/3)*1
2^(5/6)*sqrt(3)*x*cos(2/3*arctan(sqrt(3) - 2)) - sqrt(2)*sqrt(18^(2/3)*12^(
1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) - 3*18^(2/3)*12^(1/6)*x*cos(2/3
*arctan(sqrt(3) - 2)) + 18*x^2 + 3*18^(1/3)*12^(1/3))*(18^(1/3)*12^(5/6)*sq
rt(3)*cos(2/3*arctan(sqrt(3) - 2)) - 3*18^(1/3)*12^(5/6)*sin(2/3*arctan(sqr
t(3) - 2))) - 18*(18^(1/3)*12^(5/6)*x - 24*cos(2/3*arctan(sqrt(3) - 2)))*si
n(2/3*arctan(sqrt(3) - 2)) - 108*sqrt(3))/(4*cos(2/3*arctan(sqrt(3) - 2))^2
 - 3)) + (18^(2/3)*12^(1/6)*sqrt(3)*x^2*sin(2/3*arctan(sqrt(3) - 2)) - 18^(
2/3)*12^(1/6)*x^2*cos(2/3*arctan(sqrt(3) - 2)))*log(72*18^(2/3)*12^(1/6)*sq
rt(3)*x*sin(2/3*arctan(sqrt(3) - 2)) + 216*18^(2/3)*12^(1/6)*x*cos(2/3*arct
an(sqrt(3) - 2)) + 1296*x^2 + 216*18^(1/3)*12^(1/3)) - (18^(2/3)*12^(1/6)*s
qrt(3)*x^2*sin(2/3*arctan(sqrt(3) - 2)) + 18^(2/3)*12^(1/6)*x^2*cos(2/3*arc
tan(sqrt(3) - 2)))*log(72*18^(2/3)*12^(1/6)*sqrt(3)*x*sin(2/3*arctan(sqrt(3
) - 2)) - 216*18^(2/3)*12^(1/6)*x*cos(2/3*arctan(sqrt(3) - 2)) + 1296*x^2 +
 216*18^(1/3)*12^(1/3)) - 54)/x^2
```

Sympy [**A**]
time = 0.08, size = 32, normalized size = 0.08

$$- \operatorname{RootSum}\left(19683t^6 + 243t^3 + 1, \left(t \mapsto t\log\left(-1458t^4 - 9t + x\right)\right)\right) - \frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x**3/(x**6-x**3+1),x)
```

```
[Out] -RootSum(19683*_t**6 + 243*_t**3 + 1, Lambda(_t, _t*log(-1458*_t**4 - 9*_t
+ x))) - 1/(2*x**2)
```

Giac [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 645 vs. 2(272) = 544.
time = 4.19, size = 645, normalized size = 1.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x^3/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/9*(2*sqrt(3)*cos(4/9*pi)^4 - 12*sqrt(3)*cos(4/9*pi)^2*sin(4/9*pi)^2 + 2*s
qrt(3)*sin(4/9*pi)^4 + 8*cos(4/9*pi)^3*sin(4/9*pi) - 8*cos(4/9*pi)*sin(4/9*
pi)^3 + sqrt(3)*cos(4/9*pi) + sin(4/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos
(4/9*pi) + 2*x)/((1/2*I*sqrt(3) + 1/2)*sin(4/9*pi))) + 1/9*(2*sqrt(3)*cos(2
/9*pi)^4 - 12*sqrt(3)*cos(2/9*pi)^2*sin(2/9*pi)^2 + 2*sqrt(3)*sin(2/9*pi)^4
 + 8*cos(2/9*pi)^3*sin(2/9*pi) - 8*cos(2/9*pi)*sin(2/9*pi)^3 + sqrt(3)*cos(
2/9*pi) + sin(2/9*pi))*arctan(1/2*((-I*sqrt(3) - 1)*cos(2/9*pi) + 2*x)/((1/
2*I*sqrt(3) + 1/2)*sin(2/9*pi))) + 1/9*(2*sqrt(3)*cos(1/9*pi)^4 - 12*sqrt(3
)*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sqrt(3)*sin(1/9*pi)^4 - 8*cos(1/9*pi)^3*s
in(1/9*pi) + 8*cos(1/9*pi)*sin(1/9*pi)^3 - sqrt(3)*cos(1/9*pi) + sin(1/9*pi
))*arctan(-1/2*((-I*sqrt(3) - 1)*cos(1/9*pi) - 2*x)/((1/2*I*sqrt(3) + 1/2)*
sin(1/9*pi))) + 1/18*(8*sqrt(3)*cos(4/9*pi)^3*sin(4/9*pi) - 8*sqrt(3)*cos(4
/9*pi)*sin(4/9*pi)^3 - 2*cos(4/9*pi)^4 + 12*cos(4/9*pi)^2*sin(4/9*pi)^2 - 2
*sin(4/9*pi)^4 + sqrt(3)*sin(4/9*pi) - cos(4/9*pi))*log((-I*sqrt(3)*cos(4/9
*pi) - cos(4/9*pi))*x + x^2 + 1) + 1/18*(8*sqrt(3)*cos(2/9*pi)^3*sin(2/9*pi
) - 8*sqrt(3)*cos(2/9*pi)*sin(2/9*pi)^3 - 2*cos(2/9*pi)^4 + 12*cos(2/9*pi)^
2*sin(2/9*pi)^2 - 2*sin(2/9*pi)^4 + sqrt(3)*sin(2/9*pi) - cos(2/9*pi))*log(
(-I*sqrt(3)*cos(2/9*pi) - cos(2/9*pi))*x + x^2 + 1) - 1/18*(8*sqrt(3)*cos(1
/9*pi)^3*sin(1/9*pi) - 8*sqrt(3)*cos(1/9*pi)*sin(1/9*pi)^3 + 2*cos(1/9*pi)^
4 - 12*cos(1/9*pi)^2*sin(1/9*pi)^2 + 2*sin(1/9*pi)^4 - sqrt(3)*sin(1/9*pi)
- cos(1/9*pi))*log((I*sqrt(3)*cos(1/9*pi) + cos(1/9*pi))*x + x^2 + 1) - 1/2
/x^2
```

Mupad [B]
time = 2.40, size = 332, normalized size = 0.79

$$\frac{\ln\left(x+\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(36-\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}+\frac{\ln\left(x-\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{6}\right)\,{\left(36+\sqrt{3}\,12{}\mathrm{i}\right)}^{1/3}}{18}-\frac{1}{2\,x^2}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x-\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{2}+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{4/3}}{12}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}-\frac{2^{2/3}\,3^{5/6}\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(3-\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}+3^{5/6}\,1{}\mathrm{i}\right)}{36}-\frac{2^{2/3}\,\ln\left(x+\frac{2^{2/3}\,3^{1/3}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}}{4}+\frac{2^{2/3}\,3^{5/6}\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,1{}\mathrm{i}}{12}\right)\,{\left(3+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/3}\,\left(3^{1/3}-3^{5/6}\,1{}\mathrm{i}\right)}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3 - 1)/(x^3*(x^6 - x^3 + 1)),x)
```

```
[Out] (log(x + (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^(1/3)*1i)/6)*(36 - 3^(1/2)*12i)^
(1/3))/18 + (log(x - (2^(2/3)*3^(5/6)*(3^(1/2)*1i + 3)^(1/3)*1i)/6)*(3^(1/2
)*12i + 36)^(1/3))/18 - 1/(2*x^2) - (2^(2/3)*log(x - (2^(2/3)*3^(1/3)*(3 -
3^(1/2)*1i)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(4/3))/12)*(3 - 3^
(1/2)*1i)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36 - (2^(2/3)*log(x - (2^(2/3)*3^(1
/3)*(3^(1/2)*1i + 3)^(1/3))/2 + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(4/3))/12
)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*log(x + (2^(
2/3)*3^(1/3)*(3 - 3^(1/2)*1i)^(1/3))/4 - (2^(2/3)*3^(5/6)*(3 - 3^(1/2)*1i)^
(1/3)*1i)/12)*(3 - 3^(1/2)*1i)^(1/3)*(3^(1/3) + 3^(5/6)*1i))/36 - (2^(2/3)*
log(x + (2^(2/3)*3^(1/3)*(3^(1/2)*1i + 3)^(1/3))/4 + (2^(2/3)*3^(5/6)*(3^(1
/2)*1i + 3)^(1/3)*1i)/12)*(3^(1/2)*1i + 3)^(1/3)*(3^(1/3) - 3^(5/6)*1i))/36
```

3.32 $\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] 1/6*ln(x^6-x^3+1)+1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1482, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(x^6-x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
```

```
[Out] ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3] + Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(-2+x^3)}{1-x^3+x^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{-2+x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)\\
&= \frac{1}{6}\log\left(1-x^3+x^6\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= -\frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 37, normalized size = 1.03

$$-\frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{6}\log\left(1-x^3+x^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(-2 + x^3))/(1 - x^3 + x^6),x]
[Out] -(ArcTan[(-1 + 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[1 - x^3 + x^6]/6
```

Maple [A]
time = 0.04, size = 33, normalized size = 0.92

method	result	size
default	$\frac{\ln(x^6-x^3+1)}{6}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}$	33

risch	$\frac{\ln(4x^6-4x^3+4)}{6} - \frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3-2)/(x^6-x^3+1),x,method=_RETURNVERBOSE)
[Out] 1/6*ln(x^6-x^3+1)-1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="maxima")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Fricas [A]
time = 0.35, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="fricas")
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Sympy [A]
time = 0.04, size = 37, normalized size = 1.03

$$\frac{\log\left(x^6-x^3+1\right)}{6}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^3}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3-2)/(x**6-x**3+1),x)
[Out] log(x**6 - x**3 + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3)/3
```

Giac [A]
time = 4.76, size = 32, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\,(2\,x^3-1)\right)+\frac{1}{6}\,\log\left(x^6-x^3+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3-2)/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) + 1/6*log(x^6 - x^3 + 1)
```

Mupad [B]
time = 1.84, size = 34, normalized size = 0.94

$$\frac{\ln\left(x^6-x^3+1\right)}{6}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^3 - 2))/(x^6 - x^3 + 1),x)
```

```
[Out] log(x^6 - x^3 + 1)/6 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3))/3
```

3.33 $\int \frac{1+x^3}{x(1-x^3+x^6)}\,dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1488, 814, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{x\left(1-x+x^2\right)}\,dx, x, x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)\,dx, x, x^3\right)\\
&= \log(x)+\frac{1}{3}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^3\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&, \frac{-2\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x*(1 - x^3 + x^6)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

Maple [A]
time = 0.02, size = 35, normalized size = 0.90

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/x/(x^6-x^3+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 38, normalized size = 0.97

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + 1/3*
log(x^3)
```

Fricas [A]
time = 0.34, size = 34, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 1.05

$$\log\left(x\right) - \frac{\log\left(x^{6} - x^{3} + 1\right)}{6} + \frac{\sqrt{3}\ \text{atan}\left(\frac{2\sqrt{3}\ x^{3}}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/x/(x**6-x**3+1),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

Giac [A]
time = 4.01, size = 35, normalized size = 0.90

$$\frac{1}{3}\sqrt{3}\ \arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^{3} - 1\right)\right) - \frac{1}{6}\log\left(x^{6} - x^{3} + 1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/x/(x^6-x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

Mupad [B]
time = 1.85, size = 36, normalized size = 0.92

$$\ln\left(x\right) - \frac{\ln\left(x^{6} - x^{3} + 1\right)}{6} - \frac{\sqrt{3}\ \text{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\ x^{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x*(x^6 - x^3 + 1)),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/3
```

3.34 $\int \frac{1+x^3}{x-x^4+x^7} \, dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)$$

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)-1/3*arctan(1/3*(-2*x^3+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {1608, 1488, 814, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^3}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] -(ArcTan[(1 - 2*x^3)/Sqrt[3]]/Sqrt[3]) + Log[x] - Log[1 - x^3 + x^6]/6
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^3}{x-x^4+x^7}\,dx &= \int \frac{1+x^3}{x\left(1-x^3+x^6\right)}\,dx\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1+x}{x\left(1-x+x^2\right)}\,dx,x,x^3\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{2-x}{1-x+x^2}\right)\,dx,x,x^3\right)\\
&= \log(x)+\frac{1}{3}\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^3\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^3\right)\\
&= \log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^3\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x^3}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{6}\log\left(1-x^3+x^6\right)
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 1.41

$$\log(x)-\frac{1}{3}\text{RootSum}\left[1-\#1^3+\#1^6\&,\frac{-2\log(x-\#1)+\log(x-\#1)\#1^3}{-1+2\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(x - x^4 + x^7),x]
```

```
[Out] Log[x] - RootSum[1 - #1^3 + #1^6 & , (-2*Log[x - #1] + Log[x - #1]*#1^3)/(-
1 + 2*#1^3) & ]/3
```

Maple [A]
time = 0.04, size = 35, normalized size = 0.90

method	result	size
risch	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^3-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{3}$	33
default	$\ln(x)-\frac{\ln(x^6-x^3+1)}{6}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^3-1)\sqrt{3}}{3}\right)}{3}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^7-x^4+x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/6*ln(x^6-x^3+1)+1/3*3^(1/2)*arctan(1/3*(2*x^3-1)*3^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="maxima")
```

```
[Out] -integrate((x^5 - 2*x^2)/(x^6 - x^3 + 1), x) + log(x)
```

Fricas [A]
time = 0.33, size = 34, normalized size = 0.87

$$\frac{1}{3}\sqrt{3}\,\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^3-1\right)\right)-\frac{1}{6}\log\left(x^6-x^3+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
x)
```

Sympy [A]
time = 0.05, size = 41, normalized size = 1.05

$$\log(x) - \frac{\log(x^6 - x^3 + 1)}{6} + \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^3}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**7-x**4+x),x)
```

```
[Out] log(x) - log(x**6 - x**3 + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x**3/3 - sqrt(3)/3
)/3
```

Giac [A]
time = 4.83, size = 35, normalized size = 0.90

$$\frac{1}{3}\,\sqrt{3}\,\arctan\left(\frac{1}{3}\,\sqrt{3}\,\left(2\,x^3 - 1\right)\right) - \frac{1}{6}\,\log\left(x^6 - x^3 + 1\right) + \log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^7-x^4+x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^3 - 1)) - 1/6*log(x^6 - x^3 + 1) + log(
abs(x))
```

Mupad [B]
time = 0.04, size = 36, normalized size = 0.92

$$\ln(x) - \frac{\ln(x^6 - x^3 + 1)}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3} - \frac{2\sqrt{3}\,x^3}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3 + 1)/(x - x^4 + x^7),x)
```

```
[Out] log(x) - log(x^6 - x^3 + 1)/6 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^3)/3
))/3
```

3.35 $\int (d+ex^3)^{5/2} (a+bx^3+cx^6)\, dx$

Optimal. Leaf size=396

$$\frac{54d^2(16cd^2-58bde+667ae^2)\,x\sqrt{d+ex^3}}{124729e^2}+\frac{30d(16cd^2-58bde+667ae^2)\,x(d+ex^3)^{3/2}}{124729e^2}+\frac{2(16cd^2-58bde+ \cdots}{11 \cdots}$$

```
[Out] 30/124729*d*(667*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(3/2)/e^2+2/11339*(66
7*a*e^2-58*b*d*e+16*c*d^2)*x*(e*x^3+d)^(5/2)/e^2-2/667*(-29*b*e+8*c*d)*x*(e
*x^3+d)^(7/2)/e^2+2/29*c*x^4*(e*x^3+d)^(7/2)/e+54/124729*d^2*(667*a*e^2-58*
b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+54/124729*3^(3/4)*d^3*(667*a*e^2-58*b
*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2))
)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*
((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)
^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1
/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1425, 396, 201, 224}

$$\frac{54\, 3^{3/4}\sqrt{2+\sqrt{3}}\, d^3\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,(667ae^2-58bde+16cd^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{e}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{124729e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\sqrt{d+ex^3}}+\frac{2x(d+ex^3)^{5/2}(667ae^2-58bde+16cd^2)}{11339e^2}+\frac{30dx(d+ex^3)^{3/2}(667ae^2-58bde+16cd^2)}{124729e^2}+\frac{54d^2x\sqrt{d+ex^3}\,(667ae^2-58bde+16cd^2)}{124729e^2}-\frac{2x(d+ex^3)^{7/2}(8cd-29be)}{667e^2}+\frac{2cx^4(d+ex^3)^{7/2}}{29e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (54*d^2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*Sqrt[d + e*x^3])/(124729*e^2) +
 (30*d*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(3/2))/(124729*e^2)
+ (2*(16*c*d^2 - 58*b*d*e + 667*a*e^2)*x*(d + e*x^3)^(5/2))/(11339*e^2) - (
2*(8*c*d - 29*b*e)*x*(d + e*x^3)^(7/2))/(667*e^2) + (2*c*x^4*(d + e*x^3)^(7
/2))/(29*e) + (54*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^3*(16*c*d^2 - 58*b*d*e + 667*
a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^
2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d
^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/
(124729*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3)
 + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^{5/2}\left(a+bx^3+cx^6\right)\,dx = \frac{2cx^4(d+ex^3)^{7/2}}{29e} + \frac{2\int (d+ex^3)^{5/2}\left(\frac{29ae}{2}-\left(4cd-\frac{29be}{2}\right)x^3\right)\,dx}{29e}$$

$$= -\frac{2(8cd-29be)x(d+ex^3)^{7/2}}{667e^2} + \frac{2cx^4(d+ex^3)^{7/2}}{29e} - \frac{1}{667}\Bigg(-667a - \frac{2d(}{}$$

$$= \frac{2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x(d+ex^3)^{5/2}}{11339} - \frac{2(8cd-29be)x(d+ex^3)^{7/2}}{667e^2} + \frac{2}{}$$

$$= \frac{30d\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x(d+ex^3)^{3/2}}{124729} + \frac{2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x(d+}{11339}$$

$$= \frac{54d^2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\sqrt{d+ex^3}}{124729} + \frac{30d\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x(d}{124729}$$

$$= \frac{54d^2\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x\sqrt{d+ex^3}}{124729} + \frac{30d\left(667a+\frac{2d(8cd-29be)}{e^2}\right)x(d}{124729}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.11, size = 103, normalized size = 0.26

$$\frac{x\sqrt{d+ex^3}\left(-2\left(d+ex^3\right)^3\left(8cd-29be-23cex^3\right)+\frac{\left(16cd^4+29d^2e(-2bd+23ae)\right)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{3};\frac{4}{3};-\frac{ex^3}{d}\right)}{\sqrt{1+\frac{ex^3}{d}}}\right)}{667e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^3*(8*c*d - 29*b*e - 23*c*e*x^3) + ((16*c
*d^4 + 29*d^2*e*(-2*b*d + 23*a*e))*Hypergeometric2F1[-5/2, 1/3, 4/3, -((e*x
^3)/d)])/Sqrt[1 + (e*x^3)/d]))/(667*e^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1069 vs. 2(321) = 642.
time = 0.61, size = 1070, normalized size = 2.70

method	result
risch	$\frac{2x(4301e^4cx^{12}+5423e^4bx^9+11407de^3cx^9+7337ae^4x^6+15631bde^3x^6+8591cd^2e^2x^6+24679de^3ax^3+14123d^2e^2bx^3+405d^3ecx^3-}{124729e^2}$
elliptic	$\frac{2ce^2x^{13}\sqrt{ex^3+d}}{29}+\frac{2(e^3b+\frac{61}{29}de^2c)x^{10}\sqrt{ex^3+d}}{23e}+\frac{2\left(ae^3+3de^2b+3cd^2e-\frac{20d\left(e^3b+\frac{61}{29}de^2c\right)}{23e}\right)x^7\sqrt{ex^3+d}}{17e}+$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*(2/29*e^2*x^13*(e*x^3+d)^(1/2)+122/667*d*e*x^10*(e*x^3+d)^(1/2)+1562/1133
9*d^2*x^7*(e*x^3+d)^(1/2)+810/124729*d^3/e*x^4*(e*x^3+d)^(1/2)-1296/124729/
e^2*d^4*x*(e*x^3+d)^(1/2)-864/124729*I/e^3*d^5*3^(1/2)*(-d*e^2)^(1/3)*(I*(x
+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1
/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(
-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2
)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*
e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*
3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(2/23*e^2*x^10*(e*x^3+d)^(1/2)+98/391*
d*e*x^7*(e*x^3+d)^(1/2)+974/4301*d^2*x^4*(e*x^3+d)^(1/2)+162/4301*d^3/e*x*(
e*x^3+d)^(1/2)+108/4301*I/e^2*d^4*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^
2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((
x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)
))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2
)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e
```

```
*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(
1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d
*e^2)^(1/3)))^(1/2)))+a*(2/17*e^2*x^7*(e*x^3+d)^(1/2)+74/187*d*e*x^4*(e*x^3
+d)^(1/2)+106/187*d^2*x*(e*x^3+d)^(1/2)-54/187*I*d^3*3^(1/2)/e*(-d*e^2)^(1/
3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d
*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(
1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(
-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/
3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2
)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3
)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(x^3*e + d)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.10, size = 162, normalized size = 0.41

$$\frac{2}{124729}\left(81\left(16\,cd^5-58\,bd^4e+667\,ad^3e^2\right)e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(0,-4\,de^{(-1)},x\right)-\left(648\,cd^4xe-11\left(391\,cx^{13}+493\,bx^{10}+667\,ax^7\right)e^5-\left(11407\,cdx^{10}+15631\,bdx^7+24679\,adx^4\right)e^4-\left(8591\,cd^2x^7+14123\,bd^2x^4+35351\,ad^2x\right)e^3-81\left(5\,cd^3x^4+29\,bd^3x\right)e^2\right)\sqrt{x^3e+d}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/124729*(81*(16*c*d^5 - 58*b*d^4*e + 667*a*d^3*e^2)*e^(1/2)*weierstrassPIn
verse(0, -4*d*e^(-1), x) - (648*c*d^4*x*e - 11*(391*c*x^13 + 493*b*x^10 + 6
67*a*x^7)*e^5 - (11407*c*d*x^10 + 15631*b*d*x^7 + 24679*a*d*x^4)*e^4 - (859
1*c*d^2*x^7 + 14123*b*d^2*x^4 + 35351*a*d^2*x)*e^3 - 81*(5*c*d^3*x^4 + 29*b
*d^3*x)*e^2)*sqrt(x^3*e + d))*e^(-3)
```

Sympy [A]
time = 4.30, size = 400, normalized size = 1.01

$$\frac{ad^{\frac{5}{2}}x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{2ad^{\frac{3}{2}}ex^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{a\sqrt{d}\,e^2x^7\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{bd^{\frac{5}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{2bd^{\frac{3}{2}}ex^7\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{b\sqrt{d}\,e^2x^{10}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{13}{3}\right)}+\frac{cd^{\frac{5}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{2cd^{\frac{3}{2}}ex^{10}\Gamma\left(\frac{10}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{13}{3}\right)}+\frac{c\sqrt{d}\,e^2x^{13}\Gamma\left(\frac{13}{3}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{13}{3}\\\frac{16}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{16}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**(5/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(5/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*gamma(4/3)) + 2*a*d**(3/2)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,)
```

```
, e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + a*sqrt(d)*e**2*x**7*gamma(7/3)
*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*
d**(5/2)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/
d)/(3*gamma(7/3)) + 2*b*d**(3/2)*e*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3
,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3)) + b*sqrt(d)*e**2*x**10*gamma(
10/3)*hyper((-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)
) + c*d**(5/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar
(I*pi)/d)/(3*gamma(10/3)) + 2*c*d**(3/2)*e*x**10*gamma(10/3)*hyper((-1/2, 1
0/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3)) + c*sqrt(d)*e**2*x
**13*gamma(13/3)*hyper((-1/2, 13/3), (16/3,), e*x**3*exp_polar(I*pi)/d)/(3*
gamma(16/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(5/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)*(x^3*e + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^3+d\right)^{5/2}\,\left(c\,x^6+b\,x^3+a\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6),x)
[Out] int((d + e*x^3)^(5/2)*(a + b*x^3 + c*x^6), x)
```

3.36 $\int (d+ex^3)^{3/2} (a+bx^3+cx^6) \, dx$

Optimal. Leaf size=356

$$\frac{18d(16cd^2-46bde+391ae^2)\,x\sqrt{d+ex^3}}{21505e^2}+\frac{2(16cd^2-46bde+391ae^2)\,x(d+ex^3)^{3/2}}{4301e^2}-\frac{2(8cd-23be)x(d+e}{391e^2}$$

```
[Out] 2/4301*(391*a*e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^(3/2)/e^2-2/391*(-23*b*e+8
*c*d)*x*(e*x^3+d)^(5/2)/e^2+2/23*c*x^4*(e*x^3+d)^(5/2)/e+18/21505*d*(391*a*
e^2-46*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+18/21505*3^(3/4)*d^2*(391*a*e^
2-46*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^
(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(
1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2
)))^2)^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*
x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1425, 396, 201, 224}

$$\frac{18\, 3^{3/4}\sqrt{2+\sqrt{3}}\, d^2\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\,(391ae^2-46bde+16cd^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{e}x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{21505e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}x\right)^2}}\sqrt{d+ex^3}}+\frac{2x(d+ex^3)^{3/2}(391ae^2-46bde+16cd^2)}{4301e^2}+\frac{18dx\sqrt{d+ex^3}(391ae^2-46bde+16cd^2)}{21505e^2}-\frac{2x(d+ex^3)^{5/2}(8cd-23be)}{391e^2}+\frac{2cx^4(d+ex^3)^{5/2}}{23e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (18*d*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*Sqrt[d + e*x^3])/(21505*e^2) + (2
*(16*c*d^2 - 46*b*d*e + 391*a*e^2)*x*(d + e*x^3)^(3/2))/(4301*e^2) - (2*(8*
c*d - 23*b*e)*x*(d + e*x^3)^(5/2))/(391*e^2) + (2*c*x^4*(d + e*x^3)^(5/2))/
(23*e) + (18*3^(3/4)*Sqrt[2 + Sqrt[3]]*d^2*(16*c*d^2 - 46*b*d*e + 391*a*e^2
)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((
1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3
) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(2150
5*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(
1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \left(d+ex^3\right)^{3/2}\left(a+bx^3+cx^6\right)\,dx = \frac{2cx^4(d+ex^3)^{5/2}}{23e}+\frac{2\int (d+ex^3)^{3/2}\left(\frac{23ae}{2}-\left(4cd-\frac{23be}{2}\right)x^3\right)\,dx}{23e}$$

$$= -\frac{2(8cd-23be)x(d+ex^3)^{5/2}}{391e^2}+\frac{2cx^4(d+ex^3)^{5/2}}{23e}-\frac{1}{391}\Bigg(-391a-\frac{2d(}{}$$

$$= \frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x(d+ex^3)^{3/2}}{4301}-\frac{2(8cd-23be)x(d+ex^3)^{5/2}}{391e^2}+$$

$$= \frac{18d\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\sqrt{d+ex^3}}{21505}+\frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x(d+e}{4301}$$

$$= \frac{18d\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x\sqrt{d+ex^3}}{21505}+\frac{2\left(391a+\frac{2d(8cd-23be)}{e^2}\right)x(d+e}{4301}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.96, size = 101, normalized size = 0.28

$$\frac{x\sqrt{d+ex^3}\left(-2(d+ex^3)^2\left(8cd-23be-17cex^3\right)+\frac{(16cd^3+23de(-2bd+17ae))\,{}_2F_1\left(-\frac{3}{2},\frac{1}{3};\frac{4}{3};-\frac{ex^3}{d}\right)}{\sqrt{1+\frac{ex^3}{d}}}\right)}{391e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)^2*(8*c*d - 23*b*e - 17*c*e*x^3) + ((16*c
*d^3 + 23*d*e*(-2*b*d + 17*a*e))*Hypergeometric2F1[-3/2, 1/3, 4/3, -((e*x^3
)/d)])/Sqrt[1 + (e*x^3)/d]))/(391*e^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1009 vs. 2(285) = 570.
time = 0.22, size = 1010, normalized size = 2.84

method	result

risch	$\frac{2x(935e^3cx^9+1265e^3bx^6+1430de^2cx^6+1955ae^3x^3+2300bde^2x^3+135cd^2ex^3+5474de^2a+621d^2eb-216cd^3)\sqrt{ex^3+d}}{21505e^2} -$
elliptic	$\frac{2cex^{10}\sqrt{ex^3+d}}{23}+\frac{2(e^2b+\frac{26}{23}cde)x^7\sqrt{ex^3+d}}{17e}+\frac{2\left(ae^2+2deb+cd^2-\frac{14d\left(e^2b+\frac{26}{23}cde\right)}{17e}\right)x^4\sqrt{ex^3+d}}{11e}+\frac{2\left(2ade\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*(2/23*e*x^10*(e*x^3+d)^(1/2)+52/391*d*x^7*(e*x^3+d)^(1/2)+54/4301*d^2/e*x
^4*(e*x^3+d)^(1/2)-432/21505*d^3/e^2*x*(e*x^3+d)^(1/2)-288/21505*I*d^4/e^3*
3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*
e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/
3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-
d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(
-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(2/17*e*x^
7*(e*x^3+d)^(1/2)+40/187*d*x^4*(e*x^3+d)^(1/2)+54/935*d^2/e*x*(e*x^3+d)^(1/
2)+36/935*I*d^3/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I
*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)
^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(
x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(
1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3
)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2
)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^
(1/2)))+a*(2/11*e*x^4*(e*x^3+d)^(1/2)+28/55*d*x*(e*x^3+d)^(1/2)-18/55*I*d^2
*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^
2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(
-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^
(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^
```

```
3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e
*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3
)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(x^3*e + d)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 130, normalized size = 0.37

$$\frac{2}{21505}\left(27\left(16\,cd^4-46\,bd^3e+391\,ad^2e^2\right)e^{\frac{1}{2}}\text{weierstrassPInverse}\left(0,-4\,de^{(-1)},x\right)-\left(216\,cd^3xe-5\left(187\,cx^{10}+253\,bx^7+391\,ax^4\right)e^4-2\left(715\,cdx^7+1150\,bdx^4+2737\,adx\right)e^3-27\left(5\,cd^2x^4+23\,bd^2x\right)e^2\right)\sqrt{x^3e+d}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/21505*(27*(16*c*d^4 - 46*b*d^3*e + 391*a*d^2*e^2)*e^(1/2)*weierstrassPInv
erse(0, -4*d*e^(-1), x) - (216*c*d^3*x*e - 5*(187*c*x^10 + 253*b*x^7 + 391*
a*x^4)*e^4 - 2*(715*c*d*x^7 + 1150*b*d*x^4 + 2737*a*d*x)*e^3 - 27*(5*c*d^2*
x^4 + 23*b*d^2*x)*e^2)*sqrt(x^3*e + d))*e^(-3)
```

Sympy [A]
time = 2.87, size = 257, normalized size = 0.72

$$\frac{ad^{\frac{3}{2}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{a\sqrt{d}\,ex^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{bd^{\frac{3}{2}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{b\sqrt{d}\,ex^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{cd^{\frac{3}{2}}x^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}+\frac{c\sqrt{d}\,ex^{10}\Gamma\left(\frac{10}{3}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{10}{3}\\\frac{13}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**(3/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*d**(3/2)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d
)/(3*gamma(4/3)) + a*sqrt(d)*e*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e
*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*d**(3/2)*x**4*gamma(4/3)*hyper(
(-1/2, 4/3), (7/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + b*sqrt(d)*e
*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*g
amma(10/3)) + c*d**(3/2)*x**7*gamma(7/3)*hyper((-1/2, 7/3), (10/3,), e*x**3
*exp_polar(I*pi)/d)/(3*gamma(10/3)) + c*sqrt(d)*e*x**10*gamma(10/3)*hyper((
-1/2, 10/3), (13/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(13/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(3/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*(x^3*e + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(e\,x^3+d\right)^{3/2}\,\left(c\,x^6+b\,x^3+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6),x)
```

```
[Out] int((d + e*x^3)^(3/2)*(a + b*x^3 + c*x^6), x)
```

3.37 $\int \sqrt{d+ex^3}\,(a+bx^3+cx^6)\,dx$

Optimal. Leaf size=316

$$\frac{2(16cd^2-34bde+187ae^2)\,x\sqrt{d+ex^3}}{935e^2}-\frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2}+\frac{2cx^4(d+ex^3)^{3/2}}{17e}+\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,d}{}$$

```
[Out] -2/187*(-17*b*e+8*c*d)*x*(e*x^3+d)^(3/2)/e^2+2/17*c*x^4*(e*x^3+d)^(3/2)/e+2
/935*(187*a*e^2-34*b*d*e+16*c*d^2)*x*(e*x^3+d)^(1/2)/e^2+2/935*3^(3/4)*d*(1
87*a*e^2-34*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3
)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+
1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1
+3^(1/2)))^2)^(1/2)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e
^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1425, 396, 201, 224}

$$\frac{2\,3^{3/4}\sqrt{2+\sqrt{3}}\,d\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,(187ae^2-34bde+16cd^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{e}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)\middle|-7-4\sqrt{3}\right)}{935e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\sqrt{d+ex^3}}+\frac{2x\sqrt{d+ex^3}\,(187ae^2-34bde+16cd^2)}{935e^2}-\frac{2x(d+ex^3)^{3/2}(8cd-17be)}{187e^2}+\frac{2cx^4(d+ex^3)^{3/2}}{17e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

```
[Out] (2*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*x*Sqrt[d + e*x^3])/(935*e^2) - (2*(8*c
*d - 17*b*e)*x*(d + e*x^3)^(3/2))/(187*e^2) + (2*c*x^4*(d + e*x^3)^(3/2))/(
17*e) + (2*3^(3/4)*Sqrt[2 + Sqrt[3]]*d*(16*c*d^2 - 34*b*d*e + 187*a*e^2)*(d
^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 +
Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) +
e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(935*e^(7
/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x
)^2]*Sqrt[d + e*x^3])
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
```

```
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \sqrt{d+ex^3}\left(a+bx^3+cx^6\right)\,dx = \frac{2cx^4(d+ex^3)^{3/2}}{17e} + \frac{2\int \sqrt{d+ex^3}\left(\frac{17ae}{2}-\left(4cd-\frac{17be}{2}\right)x^3\right)\,dx}{17e}$$

$$= -\frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2} + \frac{2cx^4(d+ex^3)^{3/2}}{17e} - \frac{1}{187}\left(-187a - \frac{2d(8c\right.$$

$$= \frac{2}{935}\left(187a+\frac{2d(8cd-17be)}{e^2}\right)x\sqrt{d+ex^3} - \frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2}$$

$$= \frac{2}{935}\left(187a+\frac{2d(8cd-17be)}{e^2}\right)x\sqrt{d+ex^3} - \frac{2(8cd-17be)x(d+ex^3)^{3/2}}{187e^2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.22, size = 98, normalized size = 0.31

$$\frac{x\sqrt{d+ex^3}\left(-2(d+ex^3)\left(8cd-17be-11cex^3\right)+\frac{\left(16cd^2+17e(-2bd+11ae)\right){}_2F_1\left(-\frac{1}{2},\frac{1}{3};\frac{4}{3};-\frac{ex^3}{d}\right)}{\sqrt{1+\frac{ex^3}{d}}}\right)}{187e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^3]*(a + b*x^3 + c*x^6),x]
```

```
[Out] (x*Sqrt[d + e*x^3]*(-2*(d + e*x^3)*(8*c*d - 17*b*e - 11*c*e*x^3) + ((16*c*d
^2 + 17*e*(-2*b*d + 11*a*e))*Hypergeometric2F1[-1/2, 1/3, 4/3, -((e*x^3)/d)
])/Sqrt[1 + (e*x^3)/d]))/(187*e^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 955 vs. 2(249) = 498.
time = 0.21, size = 956, normalized size = 3.03

method	result

risch	$\frac{2x\left(55c\,e^{2}x^{6}+85b\,e^{2}x^{3}+15cde\,x^{3}+187a\,e^{2}+51deb-24c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{935e^{2}}-\frac{2id\left(187a\,e^{2}-34deb+16c\,d^{2}\right)\sqrt{3}\,\left(-d\,e^{2}\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\right.}{}}}{}$
elliptic	$\frac{2c\,x^{7}\sqrt{e\,x^{3}+d}}{17}+\frac{2\left(eb+\frac{3cd}{17}\right)x^{4}\sqrt{e\,x^{3}+d}}{11e}+\frac{2\left(ae+bd-\frac{8d\left(eb+\frac{3cd}{17}\right)}{11e}\right)x\sqrt{e\,x^{3}+d}}{5e}-\frac{2i\left(ad-\frac{2d\left(ae+bd-\frac{8d\left(eb+\right.}{11e}\right.}{5e}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] c*(2/17*x^7*(e*x^3+d)^(1/2)+6/187*d/e*x^4*(e*x^3+d)^(1/2)-48/935/e^2*d^2*x*
(e*x^3+d)^(1/2)-32/935*I/e^3*d^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2
)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x
-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))
)^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)
*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*
(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1
/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*
e^2)^(1/3)))^(1/2)))+b*(2/11*x^4*(e*x^3+d)^(1/2)+6/55*d/e*x*(e*x^3+d)^(1/2)
+4/55*I*d^2/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(
1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/
3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/
2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3)
)^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/
2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*
(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2
)))+a*(2/5*x*(e*x^3+d)^(1/2)-2/5*I*d*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(
-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/
2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3
^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+
1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/
```

```
3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/
e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(x^3*e + d), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 98, normalized size = 0.31

$$\frac{2}{935}\left(3\left(16\,cd^{3}-34\,bd^{2}e+187\,ade^{2}\right)e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(0,-4\,de^{(-1)},x\right)-\left(24\,cd^{2}xe-\left(55\,cx^{7}+85\,bx^{4}+187\,ax\right)e^{3}-3\left(5\,cdx^{4}+17\,bdx\right)e^{2}\right)\sqrt{x^{3}e+d}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 2/935*(3*(16*c*d^3 - 34*b*d^2*e + 187*a*d*e^2)*e^(1/2)*weierstrassPInverse(
0, -4*d*e^(-1), x) - (24*c*d^2*x*e - (55*c*x^7 + 85*b*x^4 + 187*a*x)*e^3 -
3*(5*c*d*x^4 + 17*b*d*x)*e^2)*sqrt(x^3*e + d))*e^(-3)
```

Sympy [A]
time = 1.64, size = 124, normalized size = 0.39

$$\frac{a\sqrt{d}\,x\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{1}{3}\\\frac{4}{3}\end{matrix}\right|\frac{ex^{3}e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{4}{3}\right)}+\frac{b\sqrt{d}\,x^{4}\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\right|\frac{ex^{3}e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{c\sqrt{d}\,x^{7}\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{ex^{3}e^{i\pi}}{d}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**3+d)**(1/2)*(c*x**6+b*x**3+a),x)
```

```
[Out] a*sqrt(d)*x*gamma(1/3)*hyper((-1/2, 1/3), (4/3,), e*x**3*exp_polar(I*pi)/d)
/(3*gamma(4/3)) + b*sqrt(d)*x**4*gamma(4/3)*hyper((-1/2, 4/3), (7/3,), e*x*
*3*exp_polar(I*pi)/d)/(3*gamma(7/3)) + c*sqrt(d)*x**7*gamma(7/3)*hyper((-1/
2, 7/3), (10/3,), e*x**3*exp_polar(I*pi)/d)/(3*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^3+d)^(1/2)*(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)*sqrt(x^3*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{e\,x^3+d}\,\left(c\,x^6+b\,x^3+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6),x)
```

```
[Out] int((d + e*x^3)^(1/2)*(a + b*x^3 + c*x^6), x)
```

3.38 $\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx$

Optimal. Leaf size=278

$$-\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2}+\frac{2cx^4\sqrt{d+ex^3}}{11e}+\frac{2\sqrt{2+\sqrt{3}}\,(16cd^2-22bde+55ae^2)\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}\cdots}{\left(\left(1\cdots\right.\right.}}}{55\sqrt[4]{3}\,e^{7/3}\sqrt{\frac{\sqrt[3]{d}\cdots}{\left(\left(1+\cdots\right.\right.}}}$$

```
[Out] -2/55*(-11*b*e+8*c*d)*x*(e*x^3+d)^(1/2)/e^2+2/11*c*x^4*(e*x^3+d)^(1/2)/e+2/
165*(55*a*e^2-22*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d
^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+
e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1425, 396, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,(55ae^2-22bde+16cd^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{e}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)\middle|-7-4\sqrt{3}\right)}{55\sqrt[4]{3}\,e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,\sqrt{d+ex^3}}-\frac{2x\sqrt{d+ex^3}\,(8cd-11be)}{55e^2}+\frac{2cx^4\sqrt{d+ex^3}}{11e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

```
[Out] (-2*(8*c*d - 11*b*e)*x*Sqrt[d + e*x^3])/(55*e^2) + (2*c*x^4*Sqrt[d + e*x^3]
)/(11*e) + (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 22*b*d*e + 55*a*e^2)*(d^(1/3) +
 e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])
*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*
x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(55*3^(1/4)*e^(7/
3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)
^2]*Sqrt[d + e*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1425

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[c*x^(n + 1)*((d + e*x^n)^(q + 1)/(e*(n*(q + 2) + 1)))
, x] + Dist[1/(e*(n*(q + 2) + 1)), Int[(d + e*x^n)^q*(a*e*(n*(q + 2) + 1) -
 (c*d*(n + 1) - b*e*(n*(q + 2) + 1))*x^n), x], x] /; FreeQ[{a, b, c, d, e,
n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e
^2, 0]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{\sqrt{d+ex^3}}\,dx = \frac{2cx^4\sqrt{d+ex^3}}{11e} + \frac{2\int \frac{\frac{11ae}{2}-\left(4cd-\frac{11be}{2}\right)x^3}{\sqrt{d+ex^3}}\,dx}{11e}$$

$$= -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2} + \frac{2cx^4\sqrt{d+ex^3}}{11e} - \frac{1}{55}\left(-55a-\frac{2d(8cd-11be)}{e^2}\right)\int$$

$$= -\frac{2(8cd-11be)x\sqrt{d+ex^3}}{55e^2} + \frac{2cx^4\sqrt{d+ex^3}}{11e} + \frac{2\sqrt{2+\sqrt{3}}\,(16cd^2-22bde+55a}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.07, size = 98, normalized size = 0.35

$$\frac{x\left(-2(d+ex^3)(8cd-11be-5cex^3)+(16cd^2+11e(-2bd+5ae))\sqrt{1+\frac{ex^3}{d}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)\right)}{55e^2\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/Sqrt[d + e*x^3],x]
```

```
[Out] (x*(-2*(d + e*x^3)*(8*c*d - 11*b*e - 5*c*e*x^3) + (16*c*d^2 + 11*e*(-2*b*d
+ 5*a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)
]))/(55*e^2*Sqrt[d + e*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(215) = 430$.
time = 0.20, size = 907, normalized size = 3.26

method	result
risch	$\frac{2x(5ce\,x^3+11eb-8cd)\sqrt{e\,x^3+d}}{55e^2} - \frac{2i(55a\,e^2-22deb+16c\,d^2)\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-d\,e^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}}{2e}\right)\sqrt{3}\,e}{(-d\,e^2)^{\frac{1}{3}}}}}{}$
elliptic	$\frac{2c\,x^4\sqrt{e\,x^3+d}}{11e} + \frac{2\left(b-\frac{8cd}{11e}\right)x\sqrt{e\,x^3+d}}{5e} - \frac{2i\left(a-\frac{2d\left(b-\frac{8cd}{11e}\right)}{5e}\right)\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-d\,e^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-d\,e^2)^{\frac{1}{3}}}{2e}\right)\sqrt{3}}{(-d\,e^2)^{\frac{1}{3}}}}}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(2/11/e*x^4*(e*x^3+d)^(1/2)-16/55*d/e^2*x*(e*x^3+d)^(1/2)-32/165*I/e^3*d^
2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2
)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-
d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(
1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3
+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*
(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)
/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(2/5/e*x
*(e*x^3+d)^(1/2)+4/15*I*d/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(
1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/
e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(
```

```
1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/
(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d
*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)
,(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2
)^(1/3)))^(1/2)))-2/3*I*a*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/
3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*
(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/
2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-
d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e
^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(
I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3)))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(x^3*e + d), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 69, normalized size = 0.25

$$\frac{2}{55}\left(\left(16\,cd^2-22\,bde+55\,ae^2\right)e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(0,-4\,de^{(-1)},x\right)-\left(8\,cdxe-\left(5\,cx^4+11\,bx\right)e^2\right)\sqrt{x^3e+d}\right)e^{(-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/55*((16*c*d^2 - 22*b*d*e + 55*a*e^2)*e^(1/2)*weierstrassPInverse(0, -4*d*
e^(-1), x) - (8*c*d*x*e - (5*c*x^4 + 11*b*x)*e^2)*sqrt(x^3*e + d))*e^(-3)
```

Sympy [A]
time = 1.39, size = 119, normalized size = 0.43

$$\frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{4}{3}\right)}+\frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{7}{3}\right)}+\frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3\sqrt{d}\,\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(1/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 1/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(
d)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), e*x**3*exp_pol
ar(I*pi)/d)/(3*sqrt(d)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((1/2, 7/3), (1
0/3,), e*x**3*exp_polar(I*pi)/d)/(3*sqrt(d)*gamma(10/3))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/sqrt(x^3*e + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6+b\,x^3+a}{\sqrt{e\,x^3+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2),x)
```

```
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(1/2), x)
```

3.39 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2(cd^2-bde+ae^2)\,x}{3de^2\sqrt{d+ex^3}}+\frac{2cx\sqrt{d+ex^3}}{5e^2}-\frac{2\sqrt{2+\sqrt{3}}\,(16cd^2-5e(2bd+ae))\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\ldots}{\left(\left(1+\sqrt{3}\right)\ldots\right.}}}{15\sqrt[4]{3}\,de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}\ldots\right.}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{\ldots}\right.}}}$$

```
[Out] 2/3*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(1/2)+2/5*c*x*(e*x^3+d)^(1/2)/e^2
-2/45*(16*c*d^2-5*e*(a*e+2*b*d))*(d^(1/3)+e^(1/3)*x)*EllipticF((e^(1/3)*x+d
^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2)+2*I)*(1/2*6^(
1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/
3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d/e^(7/3)/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3
)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1423, 396, 224}

$$-\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,(16cd^2-5e(ae+2bd))\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{e}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{15\sqrt[4]{3}\,de^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\sqrt{d+ex^3}}+\frac{2x(ae^2-bde+cd^2)}{3de^2\sqrt{d+ex^3}}+\frac{2cx\sqrt{d+ex^3}}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(3*d*e^2*Sqrt[d + e*x^3]) + (2*c*x*Sqrt[d + e
*x^3])/(5*e^2) - (2*Sqrt[2 + Sqrt[3]]*(16*c*d^2 - 5*e*(2*b*d + a*e))*(d^(1/
3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt
[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1
/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)], -7 - 4*Sqrt[3]])/(15*3^(1/4)*d
*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
```

```
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{3/2}}\,dx = \frac{2(cd^2-bde+ae^2)\,x}{3de^2\sqrt{d+ex^3}} - \frac{2\int \frac{\frac{1}{2}(2cd^2-e(2bd+ae))-\frac{3}{2}cdex^3}{\sqrt{d+ex^3}}\,dx}{3de^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{3de^2\sqrt{d+ex^3}} + \frac{2cx\sqrt{d+ex^3}}{5e^2} - \frac{(16cd^2-5e(2bd+ae))\int \frac{1}{\sqrt{d+ex^3}}\,dx}{15de^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{3de^2\sqrt{d+ex^3}} + \frac{2cx\sqrt{d+ex^3}}{5e^2} - \frac{2\sqrt{2+\sqrt{3}}\,(16cd^2-5e(2bd+ae))\left(\sqrt[3]{d}+\right.}{15\sqrt[4]{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.08, size = 102, normalized size = 0.35

$$\frac{x\left(2(5e(-bd+ae)+cd(8d+3ex^3))+(-16cd^2+5e(2bd+ae))\sqrt{1+\frac{ex^3}{d}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)\right)}{15de^2\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x]
```

```
[Out] (x*(2*(5*e*(-(b*d) + a*e) + c*d*(8*d + 3*e*x^3)) + (-16*c*d^2 + 5*e*(2*b*d
+ a*e))*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])
)/(15*d*e^2*Sqrt[d + e*x^3])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(226) = 452$.
time = 0.24, size = 934, normalized size = 3.23

method	result
elliptic	$\frac{2x(a e^2-deb+c d^2)}{3e^2 d\sqrt{\left(x^3+\frac{d}{e}\right)e}}+\frac{2cx\sqrt{e x^3+d}}{5e^2}-\frac{2i\left(\frac{eb-cd}{e^2}+\frac{a e^2-deb+c d^2}{3d e^2}-\frac{2cd}{5e^2}\right)\sqrt{3}\,(-d e^2)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{(-d e^2)^{\frac{1}{3}}}{2e}-\frac{i\sqrt{3}\,(-d}{2e}\right.}{(-d e^2)^{\frac{1}{3}}}}}{}$
default	Expression too large to display
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(2/3/e^2*d*x/((x^3+d/e)*e)^(1/2)+2/5/e^2*x*(e*x^3+d)^(1/2)+32/45*I*d/e^3*
3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^
(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*
e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/
3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d
)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-
d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(
-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(-2/3/e*x/
((x^3+d/e)*e)^(1/2)-4/9*I/e^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(
1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/
e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(
1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/
(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d
*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)
,(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2
)^(1/3)))^(1/2)))+a*(2/3*x/d/((x^3+d/e)*e)^(1/2)-2/9*I/d*3^(1/2)/e*(-d*e^2)
^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e
```

```
/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I
*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)
/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^
(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^
(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(x^3*e + d)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 123, normalized size = 0.43

$$\frac{2\left(\left(5\,ax^3e^3-16\,cd^3+5\,(2\,bdx^3+ad)e^2-2\,(8\,cd^2x^3-5\,bd^2)e\right)e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(0,-4\,de^{(-1)},x\right)+\left(8\,cd^2xe+5\,axe^3+(3\,cdx^4-5\,bdx)e^2\right)\sqrt{x^3e+d}\right)}{15\,(dx^3e^4+d^2e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/15*((5*a*x^3*e^3 - 16*c*d^3 + 5*(2*b*d*x^3 + a*d)*e^2 - 2*(8*c*d^2*x^3 -
5*b*d^2)*e)*e^(1/2)*weierstrassPInverse(0, -4*d*e^(-1), x) + (8*c*d^2*x*e +
 5*a*x*e^3 + (3*c*d*x^4 - 5*b*d*x)*e^2)*sqrt(x^3*e + d))/(d*x^3*e^4 + d^2*e
^3)
```

Sympy [A]
time = 6.10, size = 119, normalized size = 0.41

$$\frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{3},\frac{3}{2}\\\frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{4}{3},\frac{3}{2}\\\frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{2},\frac{7}{3}\\\frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{3}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(3/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 3/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3
/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 3/2), (7/3,), e*x**3*exp_po
lar(I*pi)/d)/(3*d**(3/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((3/2, 7/3),
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(3/2)*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(x^3*e + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6+b\,x^3+a}{{\left(e\,x^3+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2),x)
```

```
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(3/2), x)
```

3.40 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{5/2}}\,dx$

Optimal. Leaf size=309

$$\frac{2(cd^2-bde+ae^2)\,x}{9de^2\,(d+ex^3)^{3/2}}-\frac{2(11cd^2-2bde-7ae^2)\,x}{27d^2e^2\sqrt{d+ex^3}}+\frac{2\sqrt{2+\sqrt{3}}\,(16cd^2+e(2bd+7ae))\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^2\cdots}{((\cdots}}}{27\sqrt[4]{3}\,d^2e^{7/3}\sqrt{\frac{\cdots}{((1\cdots}}}$$

```
[Out] 2/9*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(3/2)-2/27*(-7*a*e^2-2*b*d*e+11*c
*d^2)*x/d^2/e^2/(e*x^3+d)^(1/2)+2/81*(16*c*d^2+e*(7*a*e+2*b*d))*(d^(1/3)+e^
(1/3)*x)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^
(1/2))),I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*
x+e^(2/3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^2/e^(7/3)
/(e*x^3+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)
))^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1423, 393, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,(e(7ae+2bd)+16cd^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{e}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{27\sqrt[4]{3}\,d^2e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\sqrt{d+ex^3}}-\frac{2x(-7ae^2-2bde+11cd^2)}{27d^2e^2\sqrt{d+ex^3}}+\frac{2x(ae^2-bde+cd^2)}{9de^2\,(d+ex^3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(9*d*e^2*(d + e*x^3)^(3/2)) - (2*(11*c*d^2 -
2*b*d*e - 7*a*e^2)*x)/(27*d^2*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*(
16*c*d^2 + e*(2*b*d + 7*a*e))*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1/3)
*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*EllipticF[
ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)
*x)], -7 - 4*Sqrt[3]])/(27*3^(1/4)*d^2*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) + e^(
1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
```

```
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{5/2}}\,dx = \frac{2(cd^2-bde+ae^2)\,x}{9de^2\,(d+ex^3)^{3/2}} - \frac{2\int \frac{\frac{1}{2}(2cd^2-e(2bd+7ae))-\frac{9}{2}cdex^3}{(d+ex^3)^{3/2}}\,dx}{9de^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{9de^2\,(d+ex^3)^{3/2}} - \frac{2(11cd^2-2bde-7ae^2)\,x}{27d^2e^2\sqrt{d+ex^3}} - \frac{\left(4\left(-\frac{9}{2}cd^2e+\frac{1}{4}e(2cd^2-e(2bd+\right.\right.}{27d^2e^3}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{9de^2\,(d+ex^3)^{3/2}} - \frac{2(11cd^2-2bde-7ae^2)\,x}{27d^2e^2\sqrt{d+ex^3}} + \frac{2\sqrt{2+\sqrt{3}}\,(16cd^2+e(2bd+7}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 10.10, size = 129, normalized size = 0.42

$$\frac{-2x(cd^2(8d+11ex^3)+e(bd(d-2ex^3)-ae(10d+7ex^3)))+(16cd^2+e(2bd+7ae))\,x(d+ex^3)\sqrt{1+\frac{ex^3}{d}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)}{27d^2e^2\,(d+ex^3)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x]

[Out] (-2*x*(c*d^2*(8*d + 11*e*x^3) + e*(b*d*(d - 2*e*x^3) - a*e*(10*d + 7*e*x^3))) + (16*c*d^2 + e*(2*b*d + 7*a*e))*x*(d + e*x^3)*Sqrt[1 + (e*x^3)/d]*Hyper geometric2F1[1/3, 1/2, 4/3, -((e*x^3)/d)])/(27*d^2*e^2*(d + e*x^3)^(3/2))

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1004 vs. 2(246) = 492.
time = 0.20, size = 1005, normalized size = 3.25

method	result
elliptic	$\frac{2x\left(a\,e^2-deb+c\,d^2\right)\sqrt{e\,x^3+d}}{9d\,e^4\left(x^3+\frac{d}{e}\right)^2}+\frac{2x\left(7a\,e^2+2deb-11c\,d^2\right)}{27e^2d^2\sqrt{\left(x^3+\frac{d}{e}\right)e}}-\frac{2i\left(\frac{c}{e^2}+\frac{7a\,e^2+2deb-11c\,d^2}{27e^2d^2}\right)\sqrt{3}\,\left(-d\,e^2\right)^{\frac{1}{3}}\sqrt{\frac{i\left(x+\frac{\left(-d\,e^2\right)^{\frac{1}{3}}}{2e}-\cdots\right.}{\cdots}}}{\cdots}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] c*(2/9/e^4*d*x*(e*x^3+d)^(1/2)/(x^3+d/e)^2-22/27/e^2*x/((x^3+d/e)*e)^(1/2)- 32/81*I/e^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2) /e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/ (-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e* (-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1 /2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I* 3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d* e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2))+ b*(-2/9/e^3*x*(e*x^3+d)^(1/2)/(x^3+d/e)^2+4/27/e*x/d/((x^3+d/e)*e)^(1/2)-4/ 81*I/e^2/d*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/ e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/

```
2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3
^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e
^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+a
*(2/9*x/d/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+14/27*x/d^2/((x^3+d/e)*e)^(1/2)-1
4/81*I/d^2*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2
)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))
/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e
*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(
1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I
*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d
*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(x^3*e + d)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 187, normalized size = 0.61

$$\frac{2\left(\left(7\,ax^6e^4+16\,cd^4+2\,(bdx^6+7\,adx^3)e^3+(16\,cd^2x^6+4\,bd^2x^3+7\,ad^2)e^2+2\,(16\,cd^3x^3+bd^3)e\right)e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(0,-4\,de^{(-1)},x\right)+\left(7\,ax^4e^4-8\,cd^3xe+2\,(bdx^4+5\,adx)e^3-(11\,cd^2x^4+bd^2x)e^2\right)\sqrt{x^3e+d}\right)}{27\,(d^2x^6e^5+2\,d^3x^3e^4+d^4e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/27*((7*a*x^6*e^4 + 16*c*d^4 + 2*(b*d*x^6 + 7*a*d*x^3)*e^3 + (16*c*d^2*x^6
 + 4*b*d^2*x^3 + 7*a*d^2)*e^2 + 2*(16*c*d^3*x^3 + b*d^3)*e)*e^(1/2)*weierst
rassPInverse(0, -4*d*e^(-1), x) + (7*a*x^4*e^4 - 8*c*d^3*x*e + 2*(b*d*x^4 +
 5*a*d*x)*e^3 - (11*c*d^2*x^4 + b*d^2*x)*e^2)*sqrt(x^3*e + d))/(d^2*x^6*e^5
 + 2*d^3*x^3*e^4 + d^4*e^3)
```

Sympy [A]
time = 33.55, size = 119, normalized size = 0.39

$$\frac{ax\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{5}{2}\\\frac{4}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{bx^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{4}{3},\frac{5}{2}\\\frac{7}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{cx^7\Gamma\left(\frac{7}{3}\right){}_2F_1\left(\begin{matrix}\frac{7}{3},\frac{5}{2}\\\frac{10}{3}\end{matrix}\middle|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{5}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(5/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 5/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(5
/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 5/2), (7/3,), e*x**3*exp_po
lar(I*pi)/d)/(3*d**(5/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((7/3, 5/2),
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(5/2)*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(x^3*e + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6+b\,x^3+a}{{\left(e\,x^3+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2),x)
```

```
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(5/2), x)
```

3.41 $\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx$

Optimal. Leaf size=349

$$\frac{2(cd^2-bde+ae^2)\,x}{15de^2\,(d+ex^3)^{5/2}}-\frac{2(17cd^2-2bde-13ae^2)\,x}{135d^2e^2\,(d+ex^3)^{3/2}}+\frac{2(16cd^2+14bde+91ae^2)\,x}{405d^3e^2\sqrt{d+ex^3}}+\frac{2\sqrt{2+\sqrt{3}}\,(16cd^2+14b\ldots}{\ldots}$$

```
[Out] 2/15*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(5/2)-2/135*(-13*a*e^2-2*b*d*e+1
7*c*d^2)*x/d^2/e^2/(e*x^3+d)^(3/2)+2/405*(91*a*e^2+14*b*d*e+16*c*d^2)*x/d^3
/e^2/(e*x^3+d)^(1/2)+2/1215*(91*a*e^2+14*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x
)*EllipticF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))
,I*3^(1/2)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/
3)*x^2)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^3/e^(7/3)/(e*x^3
+d)^(1/2)/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1423, 393, 205, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\sqrt[3]{e}\,x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,(91ae^2+14bde+16cd^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{e}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{405\sqrt[4]{3}\,d^3e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\sqrt{d+ex^3}}-\frac{2x(-13ae^2-2bde+17cd^2)}{135d^2e^2\,(d+ex^3)^{3/2}}+\frac{2x(ae^2-bde+cd^2)}{15de^2\,(d+ex^3)^{5/2}}+\frac{2x(91ae^2+14bde+16cd^2)}{405d^3e^2\sqrt{d+ex^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(15*d*e^2*(d + e*x^3)^(5/2)) - (2*(17*c*d^2 -
 2*b*d*e - 13*a*e^2)*x)/(135*d^2*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 14
*b*d*e + 91*a*e^2)*x)/(405*d^3*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]]*
(16*c*d^2 + 14*b*d*e + 91*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d^(1
/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Ellipti
cF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e^(1
/3)*x)], -7 - 4*Sqrt[3]])/(405*3^(1/4)*d^3*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/3) +
 e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{7/2}}\,dx = \frac{2(cd^2-bde+ae^2)\,x}{15de^2\,(d+ex^3)^{5/2}} - \frac{2\int \frac{\frac{1}{2}(2cd^2-e(2bd+13ae))-\frac{15}{2}cdex^3}{(d+ex^3)^{5/2}}\,dx}{15de^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{15de^2\,(d+ex^3)^{5/2}} - \frac{2(17cd^2-2bde-13ae^2)\,x}{135d^2e^2\,(d+ex^3)^{3/2}} + \frac{(16cd^2+14bde+91ae^2)\int \frac{}{(d+e}}{135d^2e^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{15de^2\,(d+ex^3)^{5/2}} - \frac{2(17cd^2-2bde-13ae^2)\,x}{135d^2e^2\,(d+ex^3)^{3/2}} + \frac{2(16cd^2+14bde+91ae^2)\,x}{405d^3e^2\sqrt{d+ex^3}} +$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{15de^2\,(d+ex^3)^{5/2}} - \frac{2(17cd^2-2bde-13ae^2)\,x}{135d^2e^2\,(d+ex^3)^{3/2}} + \frac{2(16cd^2+14bde+91ae^2)\,x}{405d^3e^2\sqrt{d+ex^3}} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.14, size = 166, normalized size = 0.48

$$\frac{2x(cd^2(-8d^2-19dex^3+16e^2x^6)+e(bd(-7d^2+34dex^3+14e^2x^6)+ae(157d^2+221dex^3+91e^2x^6)))+(16cd^2+7e(2bd+13ae))\,x(d+ex^3)^2\sqrt{1+\frac{ex^3}{d}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)}{405d^3e^2\,(d+ex^3)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x]
```

```
[Out] (2*x*(c*d^2*(-8*d^2 - 19*d*e*x^3 + 16*e^2*x^6) + e*(b*d*(-7*d^2 + 34*d*e*x^
3 + 14*e^2*x^6) + a*e*(157*d^2 + 221*d*e*x^3 + 91*e^2*x^6))) + (16*c*d^2 +
7*e*(2*b*d + 13*a*e))*x*(d + e*x^3)^2*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1
[1/3, 1/2, 4/3, -((e*x^3)/d)])/(405*d^3*e^2*(d + e*x^3)^(5/2))
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1094 vs. 2(282) = 564.
time = 0.22, size = 1095, normalized size = 3.14

method	result

elliptic	$\frac{2x(a\,e^2-deb+c\,d^2)\sqrt{e\,x^3+d}}{15d\,e^5\left(x^3+\frac{d}{e}\right)^3}+\frac{2x(13a\,e^2+2deb-17c\,d^2)\sqrt{e\,x^3+d}}{135d^2e^4\left(x^3+\frac{d}{e}\right)^2}+\frac{2x(91a\,e^2+14deb+16c\,d^2)}{405e^2d^3\sqrt{\left(x^3+\frac{d}{e}\right)e}}-\frac{2i(91a\,e^2+14deb+}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] c*(2/15*x*d/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^3-34/135*x/e^4*(e*x^3+d)^(1/2)/(x
^3+d/e)^2+32/405/e^2*x/d/((x^3+d/e)*e)^(1/2)-32/1215*I/e^3/d*3^(1/2)*(-d*e^
2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)
*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2
*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/
2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*
3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2
)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(-2/15*x/e^4*(e*x^3+d)^(
1/2)/(x^3+d/e)^3+4/135*x/d/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+28/405/e*x/d^2/(
(x^3+d/e)*e)^(1/2)-28/1215*I/e^2/d^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d
*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)
*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1
/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(
1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/
2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3)
)^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*
(-d*e^2)^(1/3)))^(1/2)))+a*(2/15*x/d/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^3+26/135
*x/d^2/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+182/405*x/d^3/((x^3+d/e)*e)^(1/2)-18
2/1215*I/d^3*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1
/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3
))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2
/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))
^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2
*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(
-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)
))
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(x^3*e + d)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.08, size = 262, normalized size = 0.75

$$\frac{2\left((91\,ax^9e^5+16\,cd^5+7\,(2\,bdx^9+39\,adx^6)e^4+(16\,cd^2x^9+42\,bd^2x^6+273\,ad^2x^3)e^3+(48\,cd^3x^6+42\,bd^3x^3+91\,ad^3)e^2+2\,(24\,cd^4x^3+7\,bd^4)e)e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(0,-4\,de^{(-1)},x\right)+(91\,ax^7e^5-8\,cd^4xe+(14\,bdx^7+221\,adx^4)e^4+(16\,cd^2x^7+34\,bd^2x^4+157\,ad^2x)e^3-(19\,cd^3x^4+7\,bd^3x)e^2)\sqrt{x^3e+d}\right)}{405\,(d^3x^9e^6+3\,d^4x^6e^5+3\,d^5x^3e^4+d^6e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/405*((91*a*x^9*e^5 + 16*c*d^5 + 7*(2*b*d*x^9 + 39*a*d*x^6)*e^4 + (16*c*d^
2*x^9 + 42*b*d^2*x^6 + 273*a*d^2*x^3)*e^3 + (48*c*d^3*x^6 + 42*b*d^3*x^3 +
91*a*d^3)*e^2 + 2*(24*c*d^4*x^3 + 7*b*d^4)*e)*e^(1/2)*weierstrassPInverse(0
, -4*d*e^(-1), x) + (91*a*x^7*e^5 - 8*c*d^4*x*e + (14*b*d*x^7 + 221*a*d*x^4
)*e^4 + (16*c*d^2*x^7 + 34*b*d^2*x^4 + 157*a*d^2*x)*e^3 - (19*c*d^3*x^4 + 7
*b*d^3*x)*e^2)*sqrt(x^3*e + d))/(d^3*x^9*e^6 + 3*d^4*x^6*e^5 + 3*d^5*x^3*e^
4 + d^6*e^3)
```

Sympy [A]
time = 152.64, size = 119, normalized size = 0.34

$$\frac{ax\Gamma\left(\frac{1}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{1}{3},\frac{7}{2}\\\frac{4}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{7}{2}}\Gamma\left(\frac{4}{3}\right)}+\frac{bx^4\Gamma\left(\frac{4}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{4}{3},\frac{7}{2}\\\frac{7}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{7}{2}}\Gamma\left(\frac{7}{3}\right)}+\frac{cx^7\Gamma\left(\frac{7}{3}\right){}_{2}F_{1}\left(\left.\begin{matrix}\frac{7}{3},\frac{7}{2}\\\frac{10}{3}\end{matrix}\right|\frac{ex^3e^{i\pi}}{d}\right)}{3d^{\frac{7}{2}}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(7/2),x)
```

```
[Out] a*x*gamma(1/3)*hyper((1/3, 7/2), (4/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(7
/2)*gamma(4/3)) + b*x**4*gamma(4/3)*hyper((4/3, 7/2), (7/3,), e*x**3*exp_po
lar(I*pi)/d)/(3*d**(7/2)*gamma(7/3)) + c*x**7*gamma(7/3)*hyper((7/3, 7/2),
(10/3,), e*x**3*exp_polar(I*pi)/d)/(3*d**(7/2)*gamma(10/3))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(7/2),x, algorithm="giac")
[Out] integrate((c*x^6 + b*x^3 + a)/(x^3*e + d)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6+b\,x^3+a}{{\left(e\,x^3+d\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2),x)
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(7/2), x)
```

3.42 $\int \frac{a+bx^3+cx^6}{\left(d+ex^3\right)^{9/2}}\,dx$

Optimal. Leaf size=389

$$\frac{2(cd^2-bde+ae^2)\,x}{21de^2\left(d+ex^3\right)^{7/2}}-\frac{2(23cd^2-2bde-19ae^2)\,x}{315d^2e^2\left(d+ex^3\right)^{5/2}}+\frac{2(16cd^2+26bde+247ae^2)\,x}{2835d^3e^2\left(d+ex^3\right)^{3/2}}+\frac{2(16cd^2+26bde+247ae}{1215d^4e^2\sqrt{d+ex^3}}$$

```
[Out] 2/21*(a*e^2-b*d*e+c*d^2)*x/d/e^2/(e*x^3+d)^(7/2)-2/315*(-19*a*e^2-2*b*d*e+2
3*c*d^2)*x/d^2/e^2/(e*x^3+d)^(5/2)+2/2835*(247*a*e^2+26*b*d*e+16*c*d^2)*x/d
^3/e^2/(e*x^3+d)^(3/2)+2/1215*(247*a*e^2+26*b*d*e+16*c*d^2)*x/d^4/e^2/(e*x^
3+d)^(1/2)+2/3645*(247*a*e^2+26*b*d*e+16*c*d^2)*(d^(1/3)+e^(1/3)*x)*Ellipti
cF((e^(1/3)*x+d^(1/3)*(1-3^(1/2)))/(e^(1/3)*x+d^(1/3)*(1+3^(1/2))),I*3^(1/2
)+2*I)*(1/2*6^(1/2)+1/2*2^(1/2))*((d^(2/3)-d^(1/3)*e^(1/3)*x+e^(2/3)*x^2)/(
e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(3/4)/d^4/e^(7/3)/(e*x^3+d)^(1/2)
/(d^(1/3)*(d^(1/3)+e^(1/3)*x)/(e^(1/3)*x+d^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1423, 393, 205, 224}

$$\frac{2\sqrt{2+\sqrt{3}}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)\sqrt{\frac{d^{2/3}-\sqrt[3]{d}\,\sqrt[3]{e}\,x+e^{2/3}x^2}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\,(247ae^2+26bde+16cd^2)\,F\left(\text{ArcSin}\left(\frac{\sqrt[3]{e}\,x+\left(1-\sqrt{3}\right)\sqrt[3]{d}}{\sqrt[3]{e}\,x+\left(1+\sqrt{3}\right)\sqrt[3]{d}}\right)|-7-4\sqrt{3}\right)}{1215\sqrt[4]{3}\,d^4e^{7/3}\sqrt{\frac{\sqrt[3]{d}\left(\sqrt[3]{d}+\sqrt[3]{e}\,x\right)}{\left(\left(1+\sqrt{3}\right)\sqrt[3]{d}+\sqrt[3]{e}\,x\right)^2}}\sqrt{d+ex^3}}-\frac{2x(-19ae^2-2bde+23cd^2)}{315d^2e^2\left(d+ex^3\right)^{5/2}}+\frac{2x(ae^2-bde+cd^2)}{21de^2\left(d+ex^3\right)^{7/2}}+\frac{2x(247ae^2+26bde+16cd^2)}{1215d^4e^2\sqrt{d+ex^3}}+\frac{2x(247ae^2+26bde+16cd^2)}{2835d^3e^2\left(d+ex^3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

```
[Out] (2*(c*d^2 - b*d*e + a*e^2)*x)/(21*d*e^2*(d + e*x^3)^(7/2)) - (2*(23*c*d^2 -
 2*b*d*e - 19*a*e^2)*x)/(315*d^2*e^2*(d + e*x^3)^(5/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(2835*d^3*e^2*(d + e*x^3)^(3/2)) + (2*(16*c*d^2 + 26
*b*d*e + 247*a*e^2)*x)/(1215*d^4*e^2*Sqrt[d + e*x^3]) + (2*Sqrt[2 + Sqrt[3]
]*(16*c*d^2 + 26*b*d*e + 247*a*e^2)*(d^(1/3) + e^(1/3)*x)*Sqrt[(d^(2/3) - d
^(1/3)*e^(1/3)*x + e^(2/3)*x^2)/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Elli
pticF[ArcSin[((1 - Sqrt[3])*d^(1/3) + e^(1/3)*x)/((1 + Sqrt[3])*d^(1/3) + e
^(1/3)*x)], -7 - 4*Sqrt[3]])/(1215*3^(1/4)*d^4*e^(7/3)*Sqrt[(d^(1/3)*(d^(1/
3) + e^(1/3)*x))/((1 + Sqrt[3])*d^(1/3) + e^(1/3)*x)^2]*Sqrt[d + e*x^3])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 + Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[s*
((s + r*x)/((1 + Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1423

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Simp[(-(c*d^2 - b*d*e + a*e^2))*x*((d + e*x^n)^(q + 1)/(d*
e^2*n*(q + 1))), x] + Dist[1/(n*(q + 1)*d*e^2), Int[(d + e*x^n)^(q + 1)*Sim
p[c*d^2 - b*d*e + a*e^2*(n*(q + 1) + 1) + c*d*e*n*(q + 1)*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - b*d*e + a*e^2, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^3+cx^6}{(d+ex^3)^{9/2}}\,dx = \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2\int \frac{\frac{1}{2}(2cd^2-e(2bd+19ae))-\frac{21}{2}cdex^3}{(d+ex^3)^{7/2}}\,dx}{21de^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2(23cd^2-2bde-19ae^2)\,x}{315d^2e^2\,(d+ex^3)^{5/2}} + \frac{(16cd^2+26bde+247ae^2)\int \frac{}{(d\text{-}}}{315d^2e^2}$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2(23cd^2-2bde-19ae^2)\,x}{315d^2e^2\,(d+ex^3)^{5/2}} + \frac{2(16cd^2+26bde+247ae^2)\,x}{2835d^3e^2\,(d+ex^3)^{3/2}} +$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2(23cd^2-2bde-19ae^2)\,x}{315d^2e^2\,(d+ex^3)^{5/2}} + \frac{2(16cd^2+26bde+247ae^2)\,x}{2835d^3e^2\,(d+ex^3)^{3/2}} +$$

$$= \frac{2(cd^2-bde+ae^2)\,x}{21de^2\,(d+ex^3)^{7/2}} - \frac{2(23cd^2-2bde-19ae^2)\,x}{315d^2e^2\,(d+ex^3)^{5/2}} + \frac{2(16cd^2+26bde+247ae^2)\,x}{2835d^3e^2\,(d+ex^3)^{3/2}} +$$

Mathematica [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.18, size = 200, normalized size = 0.51

$$\frac{2x(cd^2(-56d^3-189d^2ex^3+384de^2x^6+112e^3x^9)+e(bd(-91d^3+756d^2ex^3+624de^2x^6+182e^3x^9)+ae(3388d^3+7182d^2ex^3+5928de^2x^6+1729e^3x^9)))+7(16cd^2+13e(2bd+19ae))\,x(d+ex^3)^3\sqrt{1+\frac{ex^3}{d}}\;{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};-\frac{ex^3}{d}\right)}{8505d^4e^2\,(d+ex^3)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x]
```

```
[Out] (2*x*(c*d^2*(-56*d^3 - 189*d^2*e*x^3 + 384*d*e^2*x^6 + 112*e^3*x^9) + e*(b*
d*(-91*d^3 + 756*d^2*e*x^3 + 624*d*e^2*x^6 + 182*e^3*x^9) + a*e*(3388*d^3 +
 7182*d^2*e*x^3 + 5928*d*e^2*x^6 + 1729*e^3*x^9))) + 7*(16*c*d^2 + 13*e*(2*
b*d + 19*a*e))*x*(d + e*x^3)^3*Sqrt[1 + (e*x^3)/d]*Hypergeometric2F1[1/3, 1
/2, 4/3, -((e*x^3)/d)])/(8505*d^4*e^2*(d + e*x^3)^(7/2))
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1181 vs. 2(318) = 636.
time = 0.22, size = 1182, normalized size = 3.04

method	result

elliptic	$\frac{2x\left(a\,e^{2}-deb+c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{21d\,e^{6}\left(x^{3}+\frac{d}{e}\right)^{4}}+\frac{2x\left(19a\,e^{2}+2deb-23c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{315d^{2}e^{5}\left(x^{3}+\frac{d}{e}\right)^{3}}+\frac{2x\left(247a\,e^{2}+26deb+16c\,d^{2}\right)\sqrt{e\,x^{3}+d}}{2835d^{3}e^{4}\left(x^{3}+\frac{d}{e}\right)^{2}}+\frac{2x}{121}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x,method=_RETURNVERBOSE)

[Out] c*(2/21*x*d/e^6*(e*x^3+d)^(1/2)/(x^3+d/e)^4-46/315*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^3+32/2835*x/d/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^2+32/1215/e^2*x/d^2/((x^3+d/e)*e)^(1/2)-32/3645*I/e^3/d^2*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+b*(-2/21*x/e^5*(e*x^3+d)^(1/2)/(x^3+d/e)^4+4/315*x/d/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^3+52/2835*x/d^2/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^2+52/1215/e*x/d^3/((x^3+d/e)*e)^(1/2)-52/3645*I/e^2/d^3*3^(1/2)*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))+a*(2/21*x/d/e^4*(e*x^3+d)^(1/2)/(x^3+d/e)^4+38/315*x/d^2/e^3*(e*x^3+d)^(1/2)/(x^3+d/e)^3+494/2835*x/d^3/e^2*(e*x^3+d)^(1/2)/(x^3+d/e)^2+494/1215*x/d^4/((x^3+d/e)*e)^(1/2)-494/3645*I/d^4*3^(1/2)/e*(-d*e^2)^(1/3)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)*((x-1/e*(-d*e^2)^(1/3))/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)*(-I*(x+1/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2)/(e*x^3+d)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2/e*(-d*e^2)^(1/3)-1/2*I*3^(1/2)/e*(-d*e^2)^(1/3))*3^(1/2)*e/(-d*e^2)^(1/3))^(1/2),(I*3^(1/2)/e*(-d*e^2)^(1/3)/(-3/2/e*(-d*e^2)^(1/3)+1/2*I*3^(1/2)/e*(-d*e^2)^(1/3)))^(1/2)))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(x^3*e + d)^(9/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.09, size = 339, normalized size = 0.87

$$\frac{2\left(7\left(247\,ax^{12}e^{6}+16\,cd^{6}+26\,(bdx^{12}+38\,adx^{9})e^{5}+2\,(8\,cd^{2}x^{12}+52\,bd^{2}x^{9}+741\,ad^{2}x^{6})e^{4}+4\,(16\,cd^{3}x^{9}+39\,bd^{3}x^{6}+247\,ad^{3}x^{3})e^{3}+(96\,cd^{4}x^{6}+104\,bd^{4}x^{3}+247\,ad^{4})e^{2}+2\,(32\,cd^{5}x^{3}+13\,bd^{5})e\right)e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(0,-4\,de^{(-1)},x\right)+(1729\,ax^{10}e^{6}-56\,cd^{5}xe+26\,(7\,bdx^{10}+228\,adx^{7})e^{5}+2\,(56\,cd^{2}x^{10}+312\,bd^{2}x^{7}+3591\,ad^{2}x^{4})e^{4}+4\,(96\,cd^{3}x^{7}+189\,bd^{3}x^{4}+847\,ad^{3}x)e^{3}-7\,(27\,cd^{4}x^{4}+13\,bd^{4}x)e^{2})\sqrt{x^{3}e+d}\right)}{8505\,(d^{4}x^{12}e^{7}+4\,d^{5}x^{9}e^{6}+6\,d^{6}x^{6}e^{5}+4\,d^{7}x^{3}e^{4}+d^{8}e^{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="fricas")
```

```
[Out] 2/8505*(7*(247*a*x^12*e^6 + 16*c*d^6 + 26*(b*d*x^12 + 38*a*d*x^9)*e^5 + 2*(
8*c*d^2*x^12 + 52*b*d^2*x^9 + 741*a*d^2*x^6)*e^4 + 4*(16*c*d^3*x^9 + 39*b*d
^3*x^6 + 247*a*d^3*x^3)*e^3 + (96*c*d^4*x^6 + 104*b*d^4*x^3 + 247*a*d^4)*e^
2 + 2*(32*c*d^5*x^3 + 13*b*d^5)*e)*e^(1/2)*weierstrassPInverse(0, -4*d*e^(-
1), x) + (1729*a*x^10*e^6 - 56*c*d^5*x*e + 26*(7*b*d*x^10 + 228*a*d*x^7)*e^
5 + 2*(56*c*d^2*x^10 + 312*b*d^2*x^7 + 3591*a*d^2*x^4)*e^4 + 4*(96*c*d^3*x^
7 + 189*b*d^3*x^4 + 847*a*d^3*x)*e^3 - 7*(27*c*d^4*x^4 + 13*b*d^4*x)*e^2)*s
qrt(x^3*e + d))/(d^4*x^12*e^7 + 4*d^5*x^9*e^6 + 6*d^6*x^6*e^5 + 4*d^7*x^3*e
^4 + d^8*e^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**6+b*x**3+a)/(e*x**3+d)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^6+b*x^3+a)/(e*x^3+d)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^6 + b*x^3 + a)/(x^3*e + d)^(9/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c\,x^6+b\,x^3+a}{{\left(e\,x^3+d\right)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2),x)
[Out] int((a + b*x^3 + c*x^6)/(d + e*x^3)^(9/2), x)
```

3.43 $\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=433

$$\frac{ex}{c}-\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\ldots}{\sqrt[4]{-b+\ldots}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b+\sqrt{b^2-4ac}\right)^{3\ldots}}$$

```
[Out] e*x/c-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(c*d-b*e+
(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a*c+b^2)^
(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(
c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(-b-(-4*a
*c+b^2)^(1/2))^(3/4)-1/4*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(
1/4))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(5/4)/(
-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)
^(1/2))^(1/4))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/
c^(5/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A]

time = 0.77, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1516, 1436, 218, 214, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{ex}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])
/(2*2^(1/4)*c^(5/4)*(-b + Sqrt[b^2 - 4*a*c])^(3/4)) - ((c*d - b*e + (b*c*d
- b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4))
- ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/
4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(5/4)*(-b + Sqr
t[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^4(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{ex}{c} - \frac{\int \frac{ae-(cd-be)x^4}{a+bx^4+cx^8}\,dx}{c}$$

$$= \frac{ex}{c} + \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right.}{2}$$

$$= \frac{ex}{c} - \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b+\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2c\sqrt{-b+\sqrt{b^2-4ac}}} - \frac{\left(cd-be-\right.}{}$$

$$= \frac{ex}{c} - \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,c^{5/4}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} - \frac{\left(cd-be-\frac{bcd-b^2}{\sqrt{b^2-}}\right.}{2\sqrt[4]{2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 88, normalized size = 0.20

$$\frac{ex}{c} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{ae\log(x-\#1)-cd\log(x-\#1)\#1^4+be\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (e*x)/c - RootSum[a + b*#1^4 + c*#1^8 & , (a*e*Log[x - #1] - c*d*Log[x - #1
]*#1^4 + b*e*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ]/(4*c)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 67, normalized size = 0.15

method	result	size
default	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4 b+a\right)}\frac{\left((-eb+cd)\_R^4-ae\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}}{4c}$	67
risch	$\frac{ex}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4 b+a\right)}\frac{\left((-eb+cd)\_R^4-ae\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}}{4c}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e*x/c+1/4/c*sum(((-b*e+c*d)*_R^4-a*e)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(
_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] x*e/c + integrate(((c*d - b*e)*x^4 - a*e)/(c*x^8 + b*x^4 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 21980 vs. 2(366) = 732.
time = 78.74, size = 21980, normalized size = 50.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -1/4*(4*c*sqrt(sqrt(1/2)*sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)*d^3*e + 6
*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3
+ (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7
)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^
3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4
*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 -
 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^
2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12
 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*arctan(1/2*(sqrt(1
/2)*sqrt(c^10*d^12*x^2 - 10*b*c^9*d^11*x^2*e + 5*(9*b^2*c^8 - 2*a*c^9)*d^10
*x^2*e^2 - 10*(12*b^3*c^7 - 7*a*b*c^8)*d^9*x^2*e^3 + 15*(14*b^4*c^6 - 14*a*
b^2*c^7 + a^2*c^8)*d^8*x^2*e^4 - 12*(21*b^5*c^5 - 29*a*b^3*c^6 + 3*a^2*b*c^
7)*d^7*x^2*e^5 + 2*(105*b^6*c^4 - 169*a*b^4*c^5 - 13*a^2*b^2*c^6 + 26*a^3*c
^7)*d^6*x^2*e^6 - 60*(2*b^7*c^3 - 3*a*b^5*c^4 - 3*a^2*b^3*c^5 + 3*a^3*b*c^6
)*d^5*x^2*e^7 + 15*(3*b^8*c^2 - 2*a*b^6*c^3 - 17*a^2*b^4*c^4 + 16*a^3*b^2*c
^5 + a^4*c^6)*d^4*x^2*e^8 - 10*(b^9*c + 2*a*b^7*c^2 - 17*a^2*b^5*c^3 + 14*a
^3*b^3*c^4 + 5*a^4*b*c^5)*d^3*x^2*e^9 + (b^10 + 12*a*b^8*c - 53*a^2*b^6*c^2
```

+ 16*a^3*b^4*c^3 + 69*a^4*b^2*c^4 - 10*a^5*c^5)*d^2*x^2*e^10 - 2*(a*b^9 - 2*a^2*b^7*c - 9*a^3*b^5*c^2 + 22*a^4*b^3*c^3 - 7*a^5*b*c^4)*d*x^2*e^11 + (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*x^2*e^12 + 1/2*sqrt(1/2)*(2*(b^2*c^10 - 4*a*c^11)*d^10 - 18*(b^3*c^9 - 4*a*b*c^10)*d^9*e + (73*b^4*c^8 - 318*a*b^2*c^9 + 104*a^2*c^10)*d^8*e^2 - 8*(22*b^5*c^7 - 109*a*b^3*c^8 + 84*a^2*b*c^9)*d^7*e^3 + 20*(14*b^6*c^6 - 80*a*b^4*c^7 + 101*a^2*b^2*c^8 - 20*a^3*c^9)*d^6*e^4 - 4*(77*b^7*c^5 - 507*a*b^5*c^6 + 899*a^2*b^3*c^7 - 412*a^3*b*c^8)*d^5*e^5 + 2*(119*b^8*c^4 - 897*a*b^6*c^5 + 2061*a^2*b^4*c^6 - 1558*a^3*b^2*c^7 + 200*a^4*c^8)*d^4*e^6 - 8*(16*b^9*c^3 - 137*a*b^7*c^4 + 389*a^2*b^5*c^5 - 421*a^3*b^3*c^6 + 132*a^4*b*c^7)*d^3*e^7 + 2*(23*b^10*c^2 - 222*a*b^8*c^3 + 755*a^2*b^6*c^4 - 1080*a^3*b^4*c^5 + 573*a^4*b^2*c^6 - 52*a^5*c^7)*d^2*e^8 - 2*(5*b^11*c - 54*a*b^9*c^2 + 215*a^2*b^7*c^3 - 386*a^3*b^5*c^4 + 297*a^4*b^3*c^5 - 68*a^5*b*c^6)*d*e^9 + (b^12 - 12*a*b^10*c + 55*a^2*b^8*c^2 - 120*a^3*b^6*c^3 + 125*a^4*b^4*c^4 - 54*a^5*b^2*c^5 + 8*a^6*c^6)*e^10 - (2*(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)*d^5*e - 9*(b^7*c^9 - 12*a*b^5*c^10 + 48*a^2*b^3*c^11 - 64*a^3*b*c^12)*d^4*e^2 + 4*(4*b^8*c^8 - 51*a*b^6*c^9 + 228*a^2*b^4*c^10 - 400*a^3*b^2*c^11 + 192*a^4*c^12)*d^3*e^3 - 2*(7*b^9*c^7 - 95*a*b^7*c^8 + 468*a^2*b^5*c^9 - 976*a^3*b^3*c^10 + 704*a^4*b*c^11)*d^2*e^4 + 2*(3*b^10*c^6 - 43*a*b^8*c^7 + 229*a^2*b^6*c^8 - 540*a^3*b^4*c^9 + 496*a^4*b^2*c^10 - 64*a^5*c^11)*d*e^5 - (b^11*c^5 - 15*a*b^9*c^6 + 85*a^2*b^7*c^7 - 220*a^3*b^5*c^8 + 240*a^4*b^3*c^9 - 64*a^5*b*c^10)*e^6)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(b*c^4*d^4 - 4*(b^2*c^3 - 2*a*c^4)*d^3*e + 6*(b^3*c^2 - 3*a*b*c^3)*d^2*e^2 - 4*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^3 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^4 - (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*sqrt((c^8*d^8 - 8*b*c^7*d^7*e + 4*(7*b^2*c^6 - 3*a*c^7)*d^6*e^2 - 8*(7*b^3*c^5 - 8*a*b*c^6)*d^5*e^3 + 2*(35*b^4*c^4 - 71*a*b^2*c^5 + 19*a^2*c^6)*d^4*e^4 - 8*(7*b^5*c^3 - 21*a*b^3*c^4 + 13*a^2*b*c^5)*d^3*e^5 + 4*(7*b^6*c^2 - 28*a*b^4*c^3 + 28*a^2*b^2*c^4 - 3*a^3*c^5)*d^2*e^6 - 8*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^7 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^8)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)))*((b^4*c^7 - 8*a*b^2*c^8 + 16*a^2*c^9)*d^7 - 7*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*d^6*e + 3*(7*b^6*c^5 - 59*a*b^4*c^6 + 136*a^2*b^2*c^7 - 48*a^3*c^8)*d^5*e^2 - 5*(7*b^7*c^4 - 64*a*b^5*c^5 + 176*a^2*b^3*c^6 - 128*a^3*b*c^7)*d^4*e^3 + (35*b^8*c^3 - 351*a*b^6*c^4 + 1147*a^2*b^4*c^5 - 1288*a^3*b^2*c^6 + 304*a^4*c^7)*d^3*e^4 - 3*(7*b^9*c^2 - 77*a*b^7*c^3 + 293*a^2*b^5*c^4 - 440*a^3*b^3*c^5 + 208*a^4*b*c^6)*d^2*e^5 + (7*b^10*c - 84*a*b^8*c^2 + 364*a^2*b^6*c^3 - 675*a^3*b^4*c^4 + 472*a^4*b^2*c^5 - 48*a^5*c^6)*d*e^6 - (b^11 - 13*a*b^9*c + 63*a^2*b^7*c^2 - 138*a^3*b^5*c^3 + 128*a^4*b^3*c^4 -

```
32*a^5*b*c^5)*e^7 + ((b^7*c^8 - 12*a*b^5*c^9 + 48*a^2*b^3*c^10 - 64*a^3*b*c
^11)*d^3 - 3*(b^8*c^7 - 14*a*b^6*c^8 + 72*a^2*b^4*c^9 - 160*a^3*b^2*c^10 +
128*a^4*c^11)*d^2*e + 3*(b^9*c^6 - 15*a*b^7*c^7 + 84*a^2*b^5*c^8 - 208*a^3*
b^3*c^9 + 192*a^4*b*c^10)*d*e^2 - (b^10*c^5 - 1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.63, size = 2500, normalized size = 5.77

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((((((4*x*(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^
2 - 2048*a^3*b^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 163
84*a^5*c^7*d*e - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c - (16*(-(b
^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c
- b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 1
28*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4
 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d
^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c
^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2
) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)
^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 -
 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 32
0*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*
```

```
d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6
+ 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*(16384*a^5*c^8*d - 256*a^2*b^6*
c^5*d + 3072*a^3*b^4*c^6*d - 12288*a^4*b^2*c^7*d))/c)*(-(b^9*e^4 + b^5*c^4*
d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) -
 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e
- 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^
3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7
*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*
d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3
*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*
d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e
^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7
- 256*a^3*b^2*c^8)))^(3/4) - (16*(a^3*b^6*e^5 - 4*a^6*c^3*e^5 + 4*a^3*b*c^5
*d^5 - 7*a^4*b^4*c*e^5 - a^2*b^7*d*e^4 + 12*a^4*c^5*d^4*e - a^2*b^3*c^4*d^5
 + 13*a^5*b^2*c^2*e^5 + 8*a^5*c^4*d^2*e^3 - 6*a^2*b^5*c^2*d^3*e^2 + 32*a^3*
b^3*c^3*d^3*e^2 - 22*a^3*b^4*c^2*d^2*e^3 + 22*a^4*b^2*c^3*d^2*e^3 + 4*a^3*b
^5*c*d*e^4 - 20*a^5*b*c^3*d*e^4 + 4*a^2*b^4*c^3*d^4*e + 4*a^2*b^6*c*d^2*e^3
 - 19*a^3*b^2*c^4*d^4*e - 32*a^4*b*c^4*d^3*e^2 + 5*a^4*b^3*c^2*d*e^4))/c)*(
-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4*b*c^4*e^4
+ 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e + 61*a^2*b^5*c^2*
e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 6*b^7*c^
2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4*d^2*e^2 + 6*b^
2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3*e*(-(4*a*c - b
^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a*b^5*c^3*d^2*e^
2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*b*c^5*d^2*e^2 +
 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a*b*c
^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c^5 - 16*a*b^6*c
^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4) + (4*x*(a^4*b^4*e^6 - 2*a^3*
c^5*d^6 + 2*a^6*c^2*e^6 - 4*a^5*b^2*c*e^6 - 2*a^3*b^5*d*e^5 + a^2*b^2*c^4*d
^6 + a^2*b^6*d^2*e^4 - 2*a^4*c^4*d^4*e^2 + 2*a^5*c^3*d^2*e^4 + 6*a^2*b^4*c^
2*d^4*e^2 - 16*a^3*b^2*c^3*d^4*e^2 + 8*a^3*b^3*c^2*d^3*e^3 - 17*a^4*b^2*c^2
*d^2*e^4 + 10*a^3*b*c^4*d^5*e + 6*a^4*b^3*c*d*e^5 + 2*a^5*b*c^2*d*e^5 - 4*a
^2*b^3*c^3*d^5*e - 4*a^2*b^5*c*d^3*e^3 + 2*a^3*b^4*c*d^2*e^4 + 12*a^4*b*c^3
*d^3*e^3))/c)*(-(b^9*e^4 + b^5*c^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
 c^4*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80
*a^4*b*c^4*e^4 + 128*a^3*c^6*d^3*e - 128*a^4*c^5*d*e^3 - 4*b^6*c^3*d^3*e +
61*a^2*b^5*c^2*e^4 - 120*a^3*b^3*c^3*e^4 + a^2*c^2*e^4*(-(4*a*c - b^2)^5)^(
1/2) + 6*b^7*c^2*d^2*e^2 - 13*a*b^7*c*e^4 - 4*b^8*c*d*e^3 + 240*a^2*b^3*c^4
*d^2*e^2 + 6*b^2*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 3*a*b^2*c*e^4*(-(4*
a*c - b^2)^5)^(1/2) + 40*a*b^4*c^4*d^3*e + 48*a*b^6*c^2*d*e^3 - 4*b*c^3*d^3
*e*(-(4*a*c - b^2)^5)^(1/2) - 4*b^3*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 66*a
```

```
*b^5*c^3*d^2*e^2 - 128*a^2*b^2*c^5*d^3*e - 200*a^2*b^4*c^3*d*e^3 - 288*a^3*
b*c^5*d^2*e^2 + 320*a^3*b^2*c^4*d*e^3 - 6*a*c^3*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 8*a*b*c^2*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^4*c^9 + b^8*c
^5 - 16*a*b^6*c^6 + 96*a^2*b^4*c^7 - 256*a^3*b^2*c^8)))^(1/4)*1i + ((((4*x*
(4096*a^4*b*c^7*d^2 + 4096*a^5*b*c^6*e^2 + 256*a^2*b^5*c^5*d^2 - 2048*a^3*b
^3*c^6*d^2 + 256*a^3*b^5*c^4*e^2 - 2048*a^4*b^3*c^5*e^2 - 16384*a^5*c^7*d*e
 - 1024*a^3*b^4*c^5*d*e + 8192*a^4*b^2*c^6*d*e))/c + (16*(-(b^9*e^4 + b^5*c
^4*d^4 + b^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + c^4*d^4*(-(4*a*c - b^2)^5)^(1/2
) - 8*a*b^3*c^5*d^4 + 16*a^2*b*c^6*d^4 + 80*a^4...
```

3.44 $\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=72

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{e\log\left(a+bx^4+cx^8\right)}{8c}$$

```
[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c-1/4*(-b*e+2*c*d)*arctanh((2*c*x^4+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1482, 648, 632, 212, 642}

$$\frac{e\log\left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] -1/4*((2*c*d - b*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b^2 -
 4*a*c]) + (e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(d+ex^4)}{a+bx^4+cx^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{d+ex}{a+bx+cx^2}\,dx,x,x^4\right)\\ &= \frac{e\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8c}+\frac{(2cd-be)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x^4\right)}{8c}\\ &= \frac{e\log\left(a+bx^4+cx^8\right)}{8c}-\frac{(2cd-be)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right)}{4c}\\ &= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4c\sqrt{b^2-4ac}}+\frac{e\log\left(a+bx^4+cx^8\right)}{8c}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 71, normalized size = 0.99

$$\frac{-\frac{2(-2cd+be)\tan^{-1}\left(\frac{b+2cx^4}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+e\log\left(a+bx^4+cx^8\right)}{8c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((-2*(-2*c*d + b*e)*ArcTan[(b + 2*c*x^4)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4
*a*c] + e*Log[a + b*x^4 + c*x^8])/(8*c)
```

Maple [A]
time = 0.07, size = 66, normalized size = 0.92

method	result

default	$\frac{e \ln \left(c \,x^{8}+b \,x^{4}+a \right)}{8 c}+\frac{\left(d -\frac{b e}{2 c}\right) \arctan \left(\frac{2 c \,x^{4}+b}{\sqrt{4 a c -b^{2}}}\right)}{2 \sqrt{4 a c -b^{2}}}$
risch	$\frac{\ln \left(\left(\left(-4 a b c e +8 a \,c^{2} d +b^{3} e -2 b^{2} c d +\sqrt{-\left(e b -2 c d \right)^{2} \left(4 a c -b^{2}\right)}\, b \right) x^{4}+2 \sqrt{-\left(e b -2 c d \right)^{2} \left(4 a c -b^{2}\right)}\, a \right) a e}{8 a c -2 b^{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*e*ln(c*x^8+b*x^4+a)/c+1/2*(d-1/2/c*b*e)/(4*a*c-b^2)^(1/2)*arctan((2*c*x
^4+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.52, size = 220, normalized size = 3.06

$$\left[\frac{(b^2-4\,ac)e\log\left(cx^8+bx^4+a\right)-\sqrt{b^2-4\,ac}\,(2\,cd-be)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac+(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{8\,(b^2c-4\,ac^2)}, \frac{(b^2-4\,ac)e\log\left(cx^8+bx^4+a\right)-2\,\sqrt{-b^2+4\,ac}\,(2\,cd-be)\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{8\,(b^2c-4\,ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/8*((b^2 - 4*a*c)*e*log(c*x^8 + b*x^4 + a) - sqrt(b^2 - 4*a*c)*(2*c*d - b
*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c + (2*c*x^4 + b)*sqrt(b^2 - 4*a
*c))/(c*x^8 + b*x^4 + a)))/(b^2*c - 4*a*c^2), 1/8*((b^2 - 4*a*c)*e*log(c*x^
8 + b*x^4 + a) - 2*sqrt(-b^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^4 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)))/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(63) = 126$.
time = 113.15, size = 287, normalized size = 3.99

$$\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\,(4ac-b^2)}\right)\log\left(x^4+\frac{-16ac\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c(4ac-b^2)}\right)+2ae+4b^2\left(\frac{e}{8c}-\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c(4ac-b^2)}\right)-bd}{be-2cd}\right)+\left(\frac{e}{8c}+\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c\,(4ac-b^2)}\right)\log\left(x^4+\frac{-16ac\left(\frac{e}{8c}+\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c(4ac-b^2)}\right)+2ae+4b^2\left(\frac{e}{8c}+\frac{\sqrt{-4ac+b^2}\,(be-2cd)}{8c(4ac-b^2)}\right)-bd}{be-2cd}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] (e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4
 + (-16*a*c*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2
))) + 2*a*e + 4*b**2*(e/(8*c) - sqrt(-4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a
*c - b**2))) - b*d)/(b*e - 2*c*d)) + (e/(8*c) + sqrt(-4*a*c + b**2)*(b*e -
2*c*d)/(8*c*(4*a*c - b**2)))*log(x**4 + (-16*a*c*(e/(8*c) + sqrt(-4*a*c + b
**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) + 2*a*e + 4*b**2*(e/(8*c) + sqrt(-
4*a*c + b**2)*(b*e - 2*c*d)/(8*c*(4*a*c - b**2))) - b*d)/(b*e - 2*c*d))
```

Giac [A]
time = 5.79, size = 70, normalized size = 0.97

$$\frac{e\log\left(cx^8+bx^4+a\right)}{8\,c}+\frac{(2\,cd-be)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*e*log(c*x^8 + b*x^4 + a)/c + 1/4*(2*c*d - b*e)*arctan((2*c*x^4 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

Mupad [B]
time = 4.21, size = 2500, normalized size = 34.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] - (log(a + b*x^4 + c*x^8)*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) -
 (atan((8*x^4*(((a*c - b^2)*(((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448*b
^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 1
6*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*
e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16
*b^2*c)) + ((b*e - 2*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^
3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2
*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(8*c*
(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)) - (((
(b*e - 2*c*d)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4
*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) +
(32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a
*c - b^2)^(1/2))))/(8*c*(4*a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c
```

```
*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(b*e - 2*c*d))/
(8*c*(4*a*c - b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4
*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*
(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 1
44*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 +
20*b^3*c*e^3 - 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(8*c*(4*a*c - b^2)^(1/2)
) - (b^3*c*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/(2*(64*a*c^2 - 16*b^2*c)*(
4*a*c - b^2)^(3/2))))/(8*a^3*c^2) + ((b^3 - 3*a*b*c)*(b^3*e^4 + (b^3*(b*e -
 2*c*d)^4)/(8*(4*a*c - b^2)^2) - c^3*d^3*e - ((((b*e - 2*c*d)*(((b*e - 2*c*
d)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*
a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a
*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(8*c*(4*
a*c - b^2)^(1/2)) + (4*b^3*c^2*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d)^2)/((64*a
*c^2 - 16*b^2*c)*(4*a*c - b^2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^
2*c)) + ((4*b^2*e - 16*a*c*e)*(((4*b^2*e - 16*a*c*e)*(96*b*c^4*d^2 + ((4*b^
2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d + (256*b^3*c^4*(4*b^2*e - 16
*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 144*b^3*c^2*e^
2 - 240*b^2*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) - 8*c^4*d^3 + 20*b^3*c*e^3
- 48*b^2*c^2*d*e^2 + 36*b*c^3*d^2*e))/(2*(64*a*c^2 - 16*b^2*c)) + 3*b*c^2*d
^2*e^2 - ((((4*b^2*e - 16*a*c*e)*(((b*e - 2*c*d)*(448*b^3*c^3*e - 384*b^2*c
^4*d + (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a
*c - b^2)^(1/2)) + (32*b^3*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c
^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) + ((b*e - 2
*c*d)*(96*b*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(448*b^3*c^3*e - 384*b^2*c^4*d
+ (256*b^3*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 -
 16*b^2*c)) + 144*b^3*c^2*e^2 - 240*b^2*c^3*d*e))/(8*c*(4*a*c - b^2)^(1/2))
)*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) - 3*b^2*c*d*e^3))/(8*a^3*c^2*(4*
a*c - b^2)^(1/2)))*(4*a*c - b^2)^2)/(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*
e^2 - 32*b*c^3*d^3*e - 8*b^3*c*d*e^3) + ((a*c - b^2)*(4*a*c - b^2)^2*(((4*b
^2*e - 16*a*c*e)*(((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*
a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^
(1/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*
b^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*e - 16*a*c*e))/(2*(64*a*c^2 - 16*b^2*c)
) + ((b*e - 2*c*d)*(64*a*c^4*d^2 + ((4*b^2*e - 16*a*c*e)*(768*a*b^2*c^3*e -
 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)
))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c^2*e^2 - 256*a*b*c^3*d*e))/(8*c*(
4*a*c - b^2)^(1/2))))/(2*(64*a*c^2 - 16*b^2*c)) - ((b*e - 2*c*d)*(((((b*e -
 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e - 16*a*c
*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (64*a*b^2*c^3*(4*b
^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(1/2))
)*(b*e - 2*c*d))/(8*c*(4*a*c - b^2)^(1/2)) + (8*a*b^2*c^2*(4*b^2*e - 16*a*c
*e)*(b*e - 2*c*d)^2)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2))))/(8*c*(4*a*c -
b^2)^(1/2)) + ((b*e - 2*c*d)*(((4*b^2*e - 16*a*c*e)*(64*a*c^4*d^2 + ((4*b^2
*e - 16*a*c*e)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*b^2*c^4*(4*b^2*e -
 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(2*(64*a*c^2 - 16*b^2*c)) + 208*a*b^2*c
```

```
^2*e^2 - 256*a*b*c^3*d*e))/(2*(64*a*c^2 - 16*b^2*c)) + 24*a*b^2*c*e^3 + 16*
a*c^3*d^2*e - 40*a*b*c^2*d*e^2))/(8*c*(4*a*c - b^2)^(1/2)) - (a*b^2*c*(4*b^
2*e - 16*a*c*e)*(b*e - 2*c*d)^3)/((64*a*c^2 - 16*b^2*c)*(4*a*c - b^2)^(3/2)
)))/(a^3*c^2*(b^4*e^4 + 16*c^4*d^4 + 24*b^2*c^2*d^2*e^2 - 32*b*c^3*d^3*e -
8*b^3*c*d*e^3)) + ((4*a*c - b^2)^(3/2)*(b^3 - 3*a*b*c)*(a*b^2*e^4 - ((4*b^2
*e - 16*a*c*e)*(((((b*e - 2*c*d)*(768*a*b^2*c^3*e - 512*a*b*c^4*d + (512*a*
b^2*c^4*(4*b^2*e - 16*a*c*e))/(64*a*c^2 - 16*b^2*c)))/(8*c*(4*a*c - b^2)^(1
/2)) + (64*a*b^2*c^3*(4*b^2*e - 16*a*c*e)*(b*e - 2*c*d))/((64*a*c^2 - 16*b^
2*c)*(4*a*c - b^2)^(1/2)))*(b*e - 2*c*d))/(8*c*...
```

3.45 $\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b^2-4ac}}}-\cdots$$

```
[Out] 1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/
(-4*a*c+b^2)^(1/2))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*arcta
nh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/(-4*a*c+
b^2)^(1/2))*2^(1/4)/c^(3/4)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+1/4*arctan(2^(1/4
)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/
2))*2^(1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)-1/4*arctanh(2^(1/4)*c^(1/
4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^(
1/4)/c^(3/4)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]
time = 0.29, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1524, 304, 211, 214}

$$\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{\sqrt{b^2-4ac}-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b - Sqr
t[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1/4))
+ ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b + S
qrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])^(1/4)
) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b - Sqrt[b^2 - 4*a*c])^(1
/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*c^(3/4)*(-b + Sqrt[b^2 - 4*a*c])
^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}-\frac{1}{2}\sqrt{}}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\,\sqrt{c}\,x^2}\,dx}{2\sqrt{2}\,\sqrt{c}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int -}{}$$

$$= \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b-\sqrt{b^2-4ac}}}+\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{2\,2^{3/4}c^{3/4}\sqrt[4]{-b+\sqrt{b}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 59, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1
+ 2*c*#1^5) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.05, size = 51, normalized size = 0.14

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6e+d\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	51
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(\_R^6e+d\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}\right)}{4}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^6*e+_R^2*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+
a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((x^4*e + d)*x^2/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20777 vs. 2(305) = 610.
time = 195.92, size = 20777, normalized size = 55.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] -sqrt(sqrt(1/2)*sqrt(-(b*c^3*d^4 - 8*a*c^3*d^3*e + 6*a*b*c^2*d^2*e^2 - 4*(a
*b^2*c - 2*a^2*c^2)*d*e^3 + (a*b^3 - 3*a^2*b*c)*e^4 - (a*b^4*c^3 - 8*a^2*b^
2*c^4 + 16*a^3*c^5)*sqrt((c^6*d^8 - 12*a*c^5*d^6*e^2 + 8*a*b*c^4*d^5*e^3 -
48*a^2*b*c^3*d^3*e^5 - 2*(a*b^2*c^3 - 19*a^2*c^4)*d^4*e^4 + 4*(7*a^2*b^2*c^
2 - 3*a^3*c^3)*d^2*e^6 - 8*(a^2*b^3*c - a^3*b*c^2)*d*e^7 + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*e^8)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*
```

a^5*c^9)))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5)))*arctan(1/2*((b^2*c^9 - 4*a*c^10)*d^14*x*e - 3*(b^3*c^8 - 4*a*b*c^9)*d^13*x*e^2 + 3*(b^4*c^7 - 7*a*b^2*c^8 + 12*a^2*c^9)*d^12*x*e^3 - (b^5*c^6 - 42*a*b^3*c^7 + 152*a^2*b*c^8)*d^11*x*e^4 - (59*a*b^4*c^6 - 241*a^2*b^2*c^7 + 20*a^3*c^8)*d^10*x*e^5 + 3*(14*a*b^5*c^5 - 79*a^2*b^3*c^6 + 92*a^3*b*c^7)*d^9*x*e^6 - (14*a*b^6*c^4 - 252*a^2*b^4*c^5 + 717*a^3*b^2*c^6 + 268*a^4*c^7)*d^8*x*e^7 + 2*(a*b^7*c^3 - 123*a^2*b^5*c^4 + 342*a^3*b^3*c^5 + 536*a^4*b*c^6)*d^7*x*e^8 + (151*a^2*b^6*c^3 - 210*a^3*b^4*c^4 - 1509*a^4*b^2*c^5 - 268*a^5*c^6)*d^6*x*e^9 - (55*a^2*b^7*c^2 + 68*a^3*b^5*c^3 - 995*a^4*b^3*c^4 - 628*a^5*b*c^5)*d^5*x*e^10 + (11*a^2*b^8*c + 77*a^3*b^6*c^2 - 345*a^4*b^4*c^3 - 551*a^5*b^2*c^4 - 20*a^6*c^5)*d^4*x*e^11 - (a^2*b^9 + 24*a^3*b^7*c - 39*a^4*b^5*c^2 - 298*a^5*b^3*c^3 + 24*a^6*b*c^4)*d^3*x*e^12 + (3*a^3*b^8 + 9*a^4*b^6*c - 83*a^5*b^4*c^2 - 13*a^6*b^2*c^3 + 36*a^7*c^4)*d^2*x*e^13 - (3*a^4*b^7 - 10*a^5*b^5*c - 13*a^6*b^3*c^2 + 20*a^7*b*c^3)*d*x*e^14 + (a^5*b^6 - 6*a^6*b^4*c + 9*a^7*b^2*c^2 - 4*a^8*c^3)*x*e^15 - sqrt(c^12*d^20*x^2 - 6*b*c^11*d^19*x^2*e + 3*(5*b^2*c^10 - 2*a*c^11)*d^18*x^2*e^2 - 10*(2*b^3*c^9 - 5*a*b*c^10)*d^17*x^2*e^3 + (15*b^4*c^8 - 170*a*b^2*c^9 - 19*a^2*c^10)*d^16*x^2*e^4 - 6*(b^5*c^7 - 52*a*b^3*c^8 - 12*a^2*b*c^9)*d^15*x^2*e^5 + (b^6*c^6 - 340*a*b^4*c^7 + 4*a^2*b^2*c^8 + 56*a^3*c^9)*d^14*x^2*e^6 + 2*(113*a*b^5*c^6 - 228*a^2*b^3*c^7 - 284*a^3*b*c^8)*d^13*x^2*e^7 - 2*(45*a*b^6*c^5 - 527*a^2*b^4*c^6 - 1064*a^3*b^2*c^7 - 137*a^4*c^8)*d^12*x^2*e^8 + 2*(10*a*b^7*c^4 - 601*a^2*b^5*c^5 - 2068*a^3*b^3*c^6 - 874*a^4*b*c^7)*d^11*x^2*e^9 - (2*a*b^8*c^3 - 813*a^2*b^6*c^4 - 4736*a^3*b^4*c^5 - 4538*a^4*b^2*c^6 - 412*a^5*c^7)*d^10*x^2*e^10 - 2*(172*a^2*b^7*c^3 + 1693*a^3*b^5*c^4 + 3160*a^4*b^3*c^5 + 978*a^5*b*c^6)*d^9*x^2*e^11 + (91*a^2*b^8*c^2 + 1548*a^3*b^6*c^3 + 5240*a^4*b^4*c^4 + 3740*a^5*b^2*c^5 + 274*a^6*c^6)*d^8*x^2*e^12 - 2*(7*a^2*b^9*c + 224*a^3*b^7*c^2 + 1357*a^4*b^5*c^3 + 1900*a^5*b^3*c^4 + 460*a^6*b*c^5)*d^7*x^2*e^13 + (a^2*b^10 + 76*a^3*b^8*c + 883*a^4*b^6*c^2 + 2300*a^5*b^4*c^3 + 1220*a^6*b^2*c^4 + 56*a^7*c^5)*d^6*x^2*e^14 - 2*(3*a^3*b^9 + 84*a^4*b^7*c + 433*a^5*b^5*c^2 + 444*a^6*b^3*c^3 + 44*a^7*b*c^4)*d^5*x^2*e^15 + (15*a^4*b^8 + 190*a^5*b^6*c + 410*a^6*b^4*c^2 + 64*a^7*b^2*c^3 - 19*a^8*c^4)*d^4*x^2*e^16 - 2*(10*a^5*b^7 + 55*a^6*b^5*c + 28*a^7*b^3*c^2 - 21*a^8*b*c^3)*d^3*x^2*e^17 + (15*a^6*b^6 + 24*a^7*b^4*c - 17*a^8*b^2*c^2 - 6*a^9*c^3)*d^2*x^2*e^18 - 2*(3*a^7*b^5 - 2*a^8*b^3*c - a^9*b*c^2)*d*x^2*e^19 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*x^2*e^20 - 1/2*sqrt(1/2)*((b^3*c^11 - 4*a*b*c^12)*d^18 - 4*(b^4*c^10 - 3*a*b^2*c^11 - 4*a^2*c^12)*d^17*e + 3*(2*b^5*c^9 - 5*a*b^3*c^10 - 12*a^2*b*c^11)*d^16*e^2 - 4*(b^6*c^8 - 8*a*b^4*c^9 + 8*a^2*b^2*c^10 + 32*a^3*c^11)*d^15*e^3 + (b^7*c^7 - 76*a*b^5*c^8 + 100*a^2*b^3*c^9 + 752*a^3*b*c^10)*d^14*e^4 + 4*(23*a*b^6*c^7 + 4*a^2*b^4*c^8 - 380*a^3*b^2*c^9 - 16*a^4*c^10)*d^13*e^5 - (55*a*b^7*c^6 + 68*a^2*b^5*c^7 - 1364*a^3*b^3*c^8 + 848*a^4*b*c^9)*d^12*e^6 + 4*(4*a*b^8*c^5 - 33*a^2*b^6*c^6 - 248*a^3*b^4*c^7 + 1192*a^4*b^2*c^8 + 288*a^5*c^9)*d^11*e^7 - (2*a*b^9*c^4 - 263*a^2*b^7*c^5 - 1612*a^3*b^5*c^6 + 8706*a^4*b^3*c^7 + 7288*a^5*b*c^8)*d^10*e^8 - 4*(45*a^2*b^8*c^4 + 523*a^3*b^6*c^5 - 1730*a^4*b^4*c^6 - 4194*a^5*b^2*c^7 - 536*a^6*c^8)*d^9*e^9 + (64*a^2*b^9*c^3 + 1495*a^3*b^7*c^4 - 1776*a^4*b^5*c^5 - 18738*a^5*b^3*c^6

```
- 8696*a^6*b*c^7)*d^8*e^10 - 12*(a^2*b^10*c^2 + 52*a^3*b^8*c^3 + 81*a^4*b^6
*c^4 - 936*a^5*b^4*c^5 - 1112*a^6*b^2*c^6 - 96*a^7*c^7)*d^7*e^11 + (a^2*b^1
1*c + 152*a^3*b^9*c^2 + 967*a^4*b^7*c^3 - 3540*a^5*b^5*c^4 - 10604*a^6*b^3*
c^5 - 2768*a^7*b*c^6)*d^6*e^12 - 4*(5*a^3*b^10*c + 86*a^4*b^8*c^2 - 65*a^5*
b^6*c^3 - 1268*a^6*b^4*c^4 - 676*a^7*b^2*c^5 + 16*a^8*c^6)*d^5*e^13 + (a^3*
b^11 + 60*a^4*b^9*c + 191*a^5*b^7*c^2 - 1356*a^6*b^5*c^3 - 1820*a^7*b^3*c^4
 + 368*a^8*b*c^5)*d^4*e^14 - 4*(a^4*b^10 + 16*a^5*b^8*c - 37*a^6*b^6*c^2 -
184*a^7*b^4*c^3 + 40*a^8*b^2*c^4 + 32*a^9*c^5)*d^3*e^15 + (6*a^5*b^9 + 17*a
^6*b^7*c - 172*a^7*b^5*c^2 + 9*a^8*b^3*c^3 + 92*a^9*b*c^4)*d^2*e^16 - 4*(a^
6*b^8 - 3*a^7*b^6*c - 7*a^8*b^4*c^2 + 13*a^9*b^2*c^3 - 4*a^10*c^4)*d*e^17 +
 (a^7*b^7 - 6*a^8*b^5*c + 9*a^9*b^3*c^2 - 4*a^10*b*c^3)*e^18 + ((a*b^6*c^11
 - 12*a^2*b^4*c^12 + 48*a^3*b^2*c^13 - 64*a^4*c^14)*d^14 - 4*(a*b^7*c^10 -
12*a^2*b^5*c^11 + 48*a^3*b^3*c^12 - 64*a^4*b*c^13)*d^13*e + 3*(2*a*b^8*c^9
- 25*a^2*b^6*c^10 + 108*a^3*b^4*c^11 - 176*a^4*b^2*c^12 + 64*a^5*c^13)*d^12
*e^2 - 4*(a*b^9*c^8 - 17*a^2*b^7*c^9 + 108*a^3*b^5*c^10 - 304*a^4*b^3*c^11
+ 320*a^5*b*c^12)*d^11*e^3 + (a*b^10*c^7 - 59*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.57, size = 2500, normalized size = 6.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] 2*atan(((x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*
c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 1
```

2*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e + 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*
c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^
2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d
^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4
- a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4
+ a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^
3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4
*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*
e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a
*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*
c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4
)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^
3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*
b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3
*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*
a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*
c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d
*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*c -
b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*
c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*c^7*d^2 - 32768*a^5*c^6*e^2 - 102
4*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 - 32768*a^3*b^2*c^6*d^2 - 2048*a^3*
b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 32768*a^4*b*c^6*d*e + 2048*a^2*b^5*c^
4*d*e - 16384*a^3*b^3*c^5*d*e)*1i - 4096*a^5*c^5*e^3 - 256*a*b^5*c^4*d^3 -
4096*a^3*b*c^6*d^3 + 12288*a^4*c^6*d^2*e + 2048*a^2*b^3*c^5*d^3 - 256*a^3*b
^4*c^3*e^3 + 2048*a^4*b^2*c^4*e^3 + 768*a^2*b^4*c^4*d^2*e - 6144*a^3*b^2*c^
5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2)
- 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) -
128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e
^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*
a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b
^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(
4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*
a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4) + (x*(4*a^3*b^3*c*e^6 - 12*a^4*b*c^2
*e^6 + 16*a^2*c^5*d^5*e + 16*a^4*c^3*d*e^5 + 32*a^3*c^4*d^3*e^3 + 4*a*b*c^5
*d^6 + 16*a^2*b^2*c^3*d^3*e^3 + 12*a^2*b^3*c^2*d^2*e^4 - 16*a*b^2*c^4*d^5*e
+ 4*a*b^5*c*d^2*e^4 - 8*a^2*b^4*c*d*e^5 + 24*a*b^3*c^3*d^4*e^2 - 16*a*b^4*
c^2*d^3*e^3 - 36*a^2*b*c^4*d^4*e^2 - 52*a^3*b*c^3*d^2*e^4 + 16*a^3*b^2*c^2*
d*e^5) + (-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*
a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128
*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 -
48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e + 6*a*b^5*c^2*d^2*e^2 + 64*a^2*
b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*b*c^4*d^2*e^2 - 128*a^3*b^2*c
^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d*e^3*(-(4*a*

```
c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c^3 - 16*a^2*b^6*c^4 + 96*a^3*
b^4*c^5 - 256*a^4*b^2*c^6)))^(3/4)*(x*(-(a*b^7*e^4 + b^5*c^3*d^4 + c^3*d^4*
(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b^2*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2*c*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 40*a^3*b^
3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^3*c^3*d^2*e^2 - 8*a*b^4*c^3*d^3*e +
6*a*b^5*c^2*d^2*e^2 + 64*a^2*b^2*c^4*d^3*e + 40*a^2*b^4*c^2*d*e^3 + 96*a^3*
b*c^4*d^2*e^2 - 128*a^3*b^2*c^3*d*e^3 - 6*a*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^
(1/2) + 4*a*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^5*c^7 + a*b^8*c
^3 - 16*a^2*b^6*c^4 + 96*a^3*b^4*c^5 - 256*a^4*b^2*c^6)))^(1/4)*(32768*a^4*
c^7*d^2 - 32768*a^5*c^6*e^2 - 1024*a*b^6*c^4*d^2 + 10240*a^2*b^4*c^5*d^2 -
32768*a^3*b^2*c^6*d^2 - 2048*a^3*b^4*c^4*e^2 + 16384*a^4*b^2*c^5*e^2 + 3276
8*a^4*b*c^6*d*e + 2048*a^2*b^5*c^4*d*e - 16384*a^3*b^3*c^5*d*e)*1i + 4096*a
^5*c^5*e^3 + 256*a*b^5*c^4*d^3 + 4096*a^3*b*c^6*d^3 - 12288*a^4*c^6*d^2*e -
 2048*a^2*b^3*c^5*d^3 + 256*a^3*b^4*c^3*e^3 - 2048*a^4*b^2*c^4*e^3 - 768*a^
2*b^4*c^4*d^2*e + 6144*a^3*b^2*c^5*d^2*e)*1i)*(-(a*b^7*e^4 + b^5*c^3*d^4 +
c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a*b^3*c^4*d^4 + 16*a^2*b*c^5*d^4 - a*b
^2*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a^2*b^5*c*e^4 - 48*a^4*b*c^3*e^4 + a^2
*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 128*a^3*c^5*d^3*e + 128*a^4*c^4*d*e^3 + 4
0*a^3*b^3*c^2*e^4 - 4*a*b^6*c*d*e^3 - 48*a^2*b^...
```

3.46 $\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=184

$$\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d
)/(-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arct
an(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*c*d)/(-4*a*c
+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1504, 1180, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
 - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((
e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1504

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x(d+ex^4)}{a+bx^4+cx^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex^2}{a+bx^2+cx^4}\,dx, x, x^2\right)$$
$$= \frac{1}{4}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, x^2\right)+\frac{1}{4}\left(e+\frac{2cd-}{\sqrt{b^2-}}\right.$$
$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{}{\sqrt{b}}\right.}{2\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-}}}$$

Mathematica [A]
time = 0.11, size = 179, normalized size = 0.97

$$\frac{\frac{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(-2cd+\left(b+\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{2\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^4))/(a + b*x^4 + c*x^8),x]
```

```
[Out] (((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((-2*c*d + (b + Sqrt[b
^2 - 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
Sqrt[b + Sqrt[b^2 - 4*a*c]])/(2*Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.06, size = 168, normalized size = 0.91

method	result

default	$2c\left(-\frac{\left(e\sqrt{-4ac+b^{2}}-eb+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{c\,x^{2}\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{8c\sqrt{-4ac+b^{2}}\,\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}+\frac{\left(e\sqrt{-4ac+b^{2}}+eb-2cd\right)\sqrt{}}{8c\sqrt{-4ac+b^{2}}}\right.$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^{3}c^{3}-8a^{2}b^{2}c^{2}+a\,b^{4}c\right)\_Z^{4}+\left(-4a^{2}bc\,e^{2}+16a^{2}c^{2}de+b^{3}e^{2}a-4a\,b^{2}cde-4ab\,c^{2}d^{2}+b^{3}c\,d^{2}\right)\_Z^{2}+a^{2}e^{4}-2abd\,e^{3}+2ac\,d^{2}e^{2}+b^{2}d^{2}e^{2}\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c*(-1/8*(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))+1/8*(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/c/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((x^4*e + d)*x/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1531 vs. $2(149) = 298$.
time = 0.54, size = 1531, normalized size = 8.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*
sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-c^2*d^4*x^2 + b*c*d^3*x^2*e - a*b*d*x^2*e^3 + a^2*x^2
*e^4 + 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a
*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*
```

```
c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e
 + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)
/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/4*sqrt(1/2)*sqrt(-
(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*
c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log
(-c^2*d^4*x^2 + b*c*d^3*x^2*e - a*b*d*x^2*e^3 + a^2*x^2*e^4 - 1/2*sqrt(1/2)
*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2*c)*d*e^2 - ((a*b^3*c - 4*a^2*b*c^2
)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)
/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + (a*b^2*
c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))) + 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e
 + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)
/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-c^2*d^4*x^2 + b*c*
d^3*x^2*e - a*b*d*x^2*e^3 + a^2*x^2*e^4 + 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*
d^3 - (a*b^2 - 4*a^2*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c -
 4*a^3*c^2)*e)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
4*a^2*c^2))) - 1/4*sqrt(1/2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*
c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-c^2*d^4*x^2 + b*c*d^3*x^2*e - a*b*d*x^
2*e^3 + a^2*x^2*e^4 - 1/2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d^3 - (a*b^2 - 4*a^2
*c)*d*e^2 + ((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e)*sqrt(
(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(b*c*
d^2 - 4*a*c*d*e + a*b*e^2 - (a*b^2*c - 4*a^2*c^2)*sqrt((c^2*d^4 - 2*a*c*d^2
*e^2 + a^2*e^4)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**4+d)/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. $2(149) = 298$.
time = 7.24, size = 1404, normalized size = 7.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] 1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)
*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sqrt((b + sqrt(b^
2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^
2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/8*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^
3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*
b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*
(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*d + 2*(2*a*b^2*c^2 - 8*a^2*c^3
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*e)*arc
tan(2*sqrt(1/2)*x^2/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c
- 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [**B**]
time = 7.05, size = 2500, normalized size = 13.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^4))/(a + b*x^4 + c*x^8),x)
```

```
[Out] atan((b^4*c*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i - a^2*e^3*
x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e^3
*x^2*4i - a*b^4*d*e^2*x^2*1i - b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c
```

^2 - 12*a*b^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i + a
*b*d*e^2*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a*
c*d^2*e*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2
*b*c^2*d^2*e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)/(8*a^2
*b^4*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)
/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*b^3*c^2*(-(
a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4
*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^
3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-(a*b^3*e^2 + b^
3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*
d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2
- 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^
2*c^2 + 32*a*b^4*c))^(1/2) + 64*a^4*c^2*e^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b
^4*c))^(1/2) + 128*a^2*b^5*c*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3
*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*
d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) +
2048*a^4*b*c^3*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*
c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(3/2) - 48*a^3*b^2*c*e
^2*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1
/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*
a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) + 16*a^2*b^2*c^2*d^2*(-(a*b^
3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^
(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*
b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 -
256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 16*a^2*b^3*c*d*e*(-(a*b^3*e^2 + b^3*
c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 -
4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*
c^2 + 32*a*b^4*c))^(1/2) + 64*a^3*b*c^2*d*e*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*
a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^
2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b
^4*c))^(1/2)))*(-(a*b^3*e^2 + b^3*c*d^2 + a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*
b^2*c^2 - 12*a*b^4*c)^(1/2) - c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12
*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*

```
c*d*e)/(512*a^3*c^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2)*2i + atan((b^4*c
*d^3*x^2*1i + a^2*b^3*e^3*x^2*1i + a^2*c^3*d^3*x^2*8i + a^2*e^3*x^2*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a^3*b*c*e^3*x^2*4i - a
*b^4*d*e^2*x^2*1i + b*c*d^3*x^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)^(1/2)*1i - a*b^2*c^2*d^3*x^2*6i - a^3*c^2*d*e^2*x^2*8i - a*b*d*e^2*x^
2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i - a*c*d^2*e*x^2
*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2)*1i + a^2*b*c^2*d^2*
e*x^2*4i + a^2*b^2*c*d*e^2*x^2*6i - a*b^3*c*d^2*e*x^2*1i)/(8*a^2*b^4*e^2*(-
(a*b^3*e^2 + b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^
4*c)^(1/2) + c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) -
 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c
^3 - 256*a^2*b^2*c^2 + 32*a*b^4*c))^(1/2) - 1024*a^3*b^3*c^2*(-(a*b^3*e^2 +
 b^3*c*d^2 - a*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) +
 c*d^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d
^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3*c^3 - 256*a^2
*b^2*c^2 + 32*a*b^4*c))^(3/2) - 64*a^3*c^3*d^2*(-(a*b^3*e^2 + b^3*c*d^2 - a
*e^2*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) + c*d^2*(b^6 -
64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c
*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(512*a^3...
```

3.47 $\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx$

Optimal. Leaf size=375

$$-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}-$$

```
[Out] -1/4*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)
/(-4*a*c+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arct
anh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(e+(b*e-2*c*d)/(-4*a*c
+b^2)^(1/2))*2^(3/4)/c^(1/4)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctan(2^(1/
4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1
/2))*2^(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)-1/4*arctanh(2^(1/4)*c^(1
/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))*2^
(3/4)/c^(1/4)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A]
time = 0.22, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1436, 218, 214, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(\sqrt{b^2-4ac}-b\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] -1/2*((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
- Sqrt[b^2 - 4*a*c])^(1/4)])/(2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])^(3/4
)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)*c^(1/4)*x)/(-b
+ Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*c])^(3
/4)) - ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x)/(
-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b - Sqrt[b^2 - 4*a*c])
^(3/4)) - ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(2^(1/4)*c^(1/4)*x
)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*c^(1/4)*(-b + Sqrt[b^2 - 4*a*
c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rubi steps

$$\int \frac{d+ex^4}{a+bx^4+cx^8}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx+\frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}-\frac{1}{2}\sqrt{}}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2\sqrt{-b-\sqrt{b^2-4ac}}}-\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int}{2\sqrt{-}}$$

$$= -\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}-\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{2\sqrt[4]{2}\sqrt[4]{c}\left(-b+\sqrt{b}\right.}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 61, normalized size = 0.16

$$\frac{1}{4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{d\log(x-\#1)+e\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(a + b*x^4 + c*x^8),x]
```

```
[Out] RootSum[a + b*#1^4 + c*#1^8 & , (d*Log[x - #1] + e*Log[x - #1]*#1^4)/(b*#1^
3 + 2*c*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 47, normalized size = 0.13

method	result	size
default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4 b+a\right)}\frac{\left(\_R^4 e+d\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}\right)}{4}$	47
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4 b+a\right)}\frac{\left(\_R^4 e+d\right)\ln\left(x-\_R\right)}{2\_R^7 c+\_R^3 b}\right)}{4}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((_R^4*e+d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((x^4*e + d)/(c*x^8 + b*x^4 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 16274 vs. 2(305) = 610.
time = 60.85, size = 16274, normalized size = 43.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] sqrt(sqrt(1/2)*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + (b^3*c - 3*a*b*c^
2)*d^4 + a^3*b*e^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*a^4*b^2
*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 - (b^4*c
^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 12*a^5*c*d^2*e^6 + 8*(a*b^3*c^2 - a^2*b*c
^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 - a^6*e^8 + 2*(a^3*b^2*c
```

- 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*
a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*arctan(1/2*(sqrt(1/2)
*sqrt((b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*d^12*x^2 - 2*(b^5*c^3 + 2*a*b^3*c^4
 - 3*a^2*b*c^5)*d^11*x^2*e + 30*a^6*b*c*d^3*x^2*e^9 + (b^6*c^2 + 16*a*b^4*c
^3 + 9*a^2*b^2*c^4 - 10*a^3*c^5)*d^10*x^2*e^2 - 10*(a*b^5*c^2 + 6*a^2*b^3*c
^3 + a^3*b*c^4)*d^9*x^2*e^3 - 2*a^7*b*d*x^2*e^11 + 15*(3*a^2*b^4*c^2 + 8*a^
3*b^2*c^3 + a^4*c^4)*d^8*x^2*e^4 + a^8*x^2*e^12 - 12*(9*a^3*b^3*c^2 + 11*a^
4*b*c^3)*d^7*x^2*e^5 - 2*(a^3*b^4*c - 71*a^4*b^2*c^2 - 26*a^5*c^3)*d^6*x^2*
e^6 + 12*(a^4*b^3*c - 7*a^5*b*c^2)*d^5*x^2*e^7 - 15*(2*a^5*b^2*c - a^6*c^2)
*d^4*x^2*e^8 + (a^6*b^2 - 10*a^7*c)*d^2*x^2*e^10 + 1/2*sqrt(1/2)*((b^8*c^2
- 8*a*b^6*c^3 + 21*a^2*b^4*c^4 - 22*a^3*b^2*c^5 + 8*a^4*c^6)*d^10 - 2*(5*a*
b^7*c^2 - 34*a^2*b^5*c^3 + 65*a^3*b^3*c^4 - 36*a^4*b*c^5)*d^9*e + 2*(23*a^2
*b^6*c^2 - 136*a^3*b^4*c^3 + 189*a^4*b^2*c^4 - 52*a^5*c^5)*d^8*e^2 - 8*(15*
a^3*b^5*c^2 - 77*a^4*b^3*c^3 + 68*a^5*b*c^4)*d^7*e^3 - 2*(a^3*b^6*c - 101*a
^4*b^4*c^2 + 438*a^5*b^2*c^3 - 200*a^6*c^4)*d^6*e^4 + 4*(3*a^4*b^5*c - 59*a
^5*b^3*c^2 + 188*a^6*b*c^3)*d^5*e^5 - 4*(8*a^5*b^4*c - 57*a^6*b^2*c^2 + 100
*a^7*c^3)*d^4*e^6 + 40*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e^7 + (a^6*b^4 - 30*a^
7*b^2*c + 104*a^8*c^2)*d^2*e^8 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^9 + 2*(a^8*b^2
 - 4*a^9*c)*e^10 + ((a^3*b^9*c^2 - 13*a^4*b^7*c^3 + 60*a^5*b^5*c^4 - 112*a^
6*b^3*c^5 + 64*a^7*b*c^6)*d^6 - 2*(3*a^4*b^8*c^2 - 37*a^5*b^6*c^3 + 156*a^6
*b^4*c^4 - 240*a^7*b^2*c^5 + 64*a^8*c^6)*d^5*e + 14*(a^5*b^7*c^2 - 12*a^6*b
^5*c^3 + 48*a^7*b^3*c^4 - 64*a^8*b*c^5)*d^4*e^2 - 12*(a^6*b^6*c^2 - 12*a^7*
b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)*d^3*e^3 - (a^6*b^7*c - 12*a^7*b^5*c^
2 + 48*a^8*b^3*c^3 - 64*a^9*b*c^4)*d^2*e^4 + 2*(a^7*b^6*c - 12*a^8*b^4*c^2
+ 48*a^9*b^2*c^3 - 64*a^10*c^4)*d*e^5)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*
b*c*d^3*e^5 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 12*a^5*c*d^2*e^6 + 8*
(a*b^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 - a^6
e^8 + 2(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 +
48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + (
b^3*c - 3*a*b*c^2)*d^4 + a^3*b*e^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b
^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*
d^3*e^5 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 12*a^5*c*d^2*e^6 + 8*(a*b
^3*c^2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 - a^6*e^8
 + 2*(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a
^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)))*((b^7
*c^2 - 9*a*b^5*c^3 + 24*a^2*b^3*c^4 - 16*a^3*b*c^5)*d^7 - (7*a*b^6*c^2 - 59
*a^2*b^4*c^3 + 136*a^3*b^2*c^4 - 48*a^4*c^5)*d^6*e + 18*(a^2*b^5*c^2 - 8*a^
3*b^3*c^3 + 16*a^4*b*c^4)*d^5*e^2 + (a^2*b^6*c - 27*a^3*b^4*c^2 + 168*a^4*b
^2*c^3 - 304*a^5*c^4)*d^4*e^3 - 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3
)*d^3*e^4 + 9*(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*e^5 - (a^5*b^4 -
 8*a^6*b^2*c + 16*a^7*c^2)*e^7 + ((a^3*b^8*c^2 - 14*a^4*b^6*c^3 + 72*a^5*b^
4*c^4 - 160*a^6*b^2*c^5 + 128*a^7*c^6)*d^3 - 3*(a^4*b^7*c^2 - 12*a^5*b^5*c^
3 + 48*a^6*b^3*c^4 - 64*a^7*b*c^5)*d^2*e + 6*(a^5*b^6*c^2 - 12*a^6*b^4*c^3
+ 48*a^7*b^2*c^4 - 64*a^8*c^5)*d*e^2 - (a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7
*b^3*c^3 - 64*a^8*b*c^4)*e^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e

```
^5 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 12*a^5*c*d^2*e^6 + 8*(a*b^3*c^
2 - a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 - a^6*e^8 + 2*
(a^3*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^
2*c^4 - 64*a^9*c^5)))*sqrt(-(6*a^2*b*c*d^2*e^2 - 8*a^3*c*d*e^3 + (b^3*c - 3
*a*b*c^2)*d^4 + a^3*b*e^4 - 4*(a*b^2*c - 2*a^2*c^2)*d^3*e - (a^3*b^4*c - 8*
a^4*b^2*c^2 + 16*a^5*c^3)*sqrt(-(48*a^3*b*c^2*d^5*e^3 - 8*a^4*b*c*d^3*e^5 -
 (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^8 + 12*a^5*c*d^2*e^6 + 8*(a*b^3*c^2 -
a^2*b*c^3)*d^7*e - 4*(7*a^2*b^2*c^2 - 3*a^3*c^3)*d^6*e^2 - a^6*e^8 + 2*(a^3
*b^2*c - 19*a^4*c^2)*d^4*e^4)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^
4 - 64*a^9*c^5)))/(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)) + sqrt(1/2)*((b
^9*c^4 - 10*a*b^7*c^5 + 33*a^2*b^5*c^6 - 40*a^3*b^3*c^7 + 16*a^4*b*c^8)*d^1
3*x - (b^10*c^3 + a*b^8*c^4 - 69*a^2*b^6*c^5 + 251*a^3*b^4*c^6 - 232*a^4*b^
2*c^7 + 48*a^5*c^8)*d^12*x*e + 4*(3*a*b^9*c^3 - 15*a^2*b^7*c^4 - 32*a^3*b^5
*c^5 + 208*a^4*b^3*c^6 - 128*a^5*b*c^7)*d^11*x*e^2 - 2*(31*a^2*b^8*c^3 - 20
6*a^3*b^6*c^4 + 143*a^4*b^4*c^5 + 808*a^5*b^2*c^6 - 272*a^6*c^7)*d^10*x*e^3
 - (a^2*b^9*c^2 - 184*a^3*b^7*c^3 + 1307*a^4*b^5*c^4 - 1880*a^5*b^3*c^5 - 1
872*a^6*b*c^6)*d^9*x*e^4 + (11*a^3*b^8*c^2 - 37...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [**B**]
time = 8.75, size = 2500, normalized size = 6.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(a + b*x^4 + c*x^8),x)
```

```
[Out] - atan((((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11
*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*
a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128
*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e -
 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*
b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c
^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*
c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*
b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(
-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(
-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-
(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3
*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 4
0*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*
b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*
c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^5*
c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e + 4
9152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4*b
*c^7*d) + x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2 +
 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 + 1024*a^2*b^5*c^4*e^2 - 8192*
a^3*b^3*c^5*e^2 + 65536*a^4*c^7*d*e - 2048*a*b^6*c^4*d*e + 20480*a^2*b^4*c^
5*d*e - 65536*a^3*b^2*c^6*d*e))*(-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a
*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*
c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*
d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2
*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3
*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2)
+ 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 1
6*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(3/4) + 64*a*c^7*d^5 -
16*b^2*c^6*d^5 + 64*a^3*b*c^4*e^5 - 192*a^3*c^5*d*e^4 + 16*b^3*c^5*d^4*e -
16*a^2*b^3*c^3*e^5 - 128*a^2*c^6*d^3*e^2 - 64*a*b*c^6*d^4*e + 16*a*b^4*c^3*
d*e^4 + 32*a*b^2*c^5*d^3*e^2 - 64*a*b^3*c^4*d^2*e^3 + 256*a^2*b*c^5*d^2*e^3
 - 16*a^2*b^2*c^4*d*e^4) + x*(8*c^7*d^6 - 8*a^3*c^4*e^6 + 8*a*c^6*d^4*e^2 +
 4*a^2*b^2*c^3*e^6 - 8*a^2*c^5*d^2*e^4 + 28*b^2*c^5*d^4*e^2 - 16*b^3*c^4*d^
3*e^3 + 4*b^4*c^3*d^2*e^4 - 24*b*c^6*d^5*e - 16*a*b*c^5*d^3*e^3 - 8*a*b^3*c
^3*d*e^5 + 8*a^2*b*c^4*d*e^5 + 16*a*b^2*c^4*d^2*e^4))*(-(b^7*c*d^4 + a^3*b^
5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*
d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e
^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d
*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^
3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^
3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(
4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a
```

```
^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(
1/4)*1i - ((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4*(-(4*a*c - b^2)^5)^(1/2) -
11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) -
8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) + 1
28*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b^3*c^3*d^4 - 4*a*b^6*c*d^3*e
 - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 + 40*a^2*b^4*c^2*d^3*e + 6*a^
2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^4*b*c^3*d^2*e^2 + 64*a^4*b^2
*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 4*a*b*c*d^3*e*(-(4*
a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^8*c - 16*a^4*b^6*c^2 + 96*a^
5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(((-(b^7*c*d^4 + a^3*b^5*e^4 + a^3*e^4
*(-(4*a*c - b^2)^5)^(1/2) - 11*a*b^5*c^2*d^4 - 48*a^3*b*c^4*d^4 + a*c^2*d^4
*(-(4*a*c - b^2)^5)^(1/2) - 8*a^4*b^3*c*e^4 + 16*a^5*b*c^2*e^4 - b^2*c*d^4*
(-(4*a*c - b^2)^5)^(1/2) + 128*a^4*c^4*d^3*e - 128*a^5*c^3*d*e^3 + 40*a^2*b
^3*c^3*d^4 - 4*a*b^6*c*d^3*e - 48*a^3*b^3*c^2*d^2*e^2 - 8*a^3*b^4*c*d*e^3 +
 40*a^2*b^4*c^2*d^3*e + 6*a^2*b^5*c*d^2*e^2 - 128*a^3*b^2*c^3*d^3*e + 96*a^
4*b*c^3*d^2*e^2 + 64*a^4*b^2*c^2*d*e^3 - 6*a^2*c*d^2*e^2*(-(4*a*c - b^2)^5)
^(1/2) + 4*a*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(256*a^7*c^5 + a^3*b^
8*c - 16*a^4*b^6*c^2 + 96*a^5*b^4*c^3 - 256*a^6*b^2*c^4)))^(1/4)*(262144*a^
5*c^7*e - 49152*a^2*b^5*c^5*d + 196608*a^3*b^3*c^6*d - 4096*a^2*b^6*c^4*e +
 49152*a^3*b^4*c^5*e - 196608*a^4*b^2*c^6*e + 4096*a*b^7*c^4*d - 262144*a^4
*b*c^7*d) - x*(1024*b^7*c^4*d^2 - 11264*a*b^5*c^5*d^2 - 49152*a^3*b*c^7*d^2
 + 16384*a^4*b*c^6*e^2 + 40960*a^2*b^3*c^6*d^2 ...
```

3.48 $\int \frac{d+ex^4}{x(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=78

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}$$

```
[Out] d*ln(x)/a-1/8*d*ln(c*x^8+b*x^4+a)/a+1/4*(-2*a*e+b*d)*arctanh((2*c*x^4+b)/(-
4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1488, 814, 648, 632, 212, 642}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^4)/Sqrt[b^2 - 4*a*c]])/(4*a*Sqrt[b^2 - 4*
a*c]) + (d*Log[x])/a - (d*Log[a + b*x^4 + c*x^8])/(8*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^4}{x\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{d+ex}{x\left(a+bx+cx^2\right)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-cdx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^4\right)\\
&= \frac{d\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{a+bx+cx^2}\,dx,x,x^4\right)}{4a}\\
&= \frac{d\log(x)}{a}-\frac{d\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^4\right)}{8a}+\frac{(-bd+2ae)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x,x\right.}{8a}\\
&= \frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}-\frac{(-bd+2ae)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2cx^4\right.}{4a}\\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^4}{\sqrt{b^2-4ac}}\right)}{4a\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{d\log\left(a+bx^4+cx^8\right)}{8a}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.02, size = 80, normalized size = 1.03

$$\frac{d\log(x)}{a}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b+2c\#1^4}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x]
```

```
[Out] (d*Log[x])/a - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x
 - #1] + c*d*Log[x - #1]*#1^4)/(b + 2*c*#1^4) & ]/(4*a)
```

Maple [A]
time = 0.06, size = 74, normalized size = 0.95

method	result
default	$\frac{-\frac{d\ln\left(c\,x^{8}+b\,x^{4}+a\right)}{4}+\frac{\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^{4}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a}+\frac{d\ln(x)}{a}$
risch	$\frac{d\ln(x)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c-a\,b^{2}\right)\_Z^{2}+\left(4acd-b^{2}d\right)\_Z+a\,e^{2}-deb+c\,d^{2}\right)}\_R\ln\left(\left(\left(18ac-5b^{2}\right)\_R^{2}+\left(-eb+9cd\right)\_R+4e^{2}\right)x^{4}-b\_\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*(-1/4*d*ln(c*x^8+b*x^4+a)+(a*e-1/2*b*d)/(4*a*c-b^2)^(1/2)*arctan((2*c
*x^4+b)/(4*a*c-b^2)^(1/2)))+d*ln(x)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.80, size = 242, normalized size = 3.10

$$\left[-\frac{(b^2-4\,ac)d\log\left(cx^8+bx^4+a\right)-8\,(b^2-4\,ac)d\log\left(x\right)+\sqrt{b^2-4\,ac}\,(bd-2\,ae)\log\left(\frac{2\,c^2x^8+2\,bcx^4+b^2-2\,ac-(2\,cx^4+b)\sqrt{b^2-4\,ac}}{cx^8+bx^4+a}\right)}{8\,(ab^2-4\,a^2c)},-\frac{(b^2-4\,ac)d\log\left(cx^8+bx^4+a\right)-8\,(b^2-4\,ac)d\log\left(x\right)-2\,\sqrt{-b^2+4\,ac}\,(bd-2\,ae)\arctan\left(-\frac{(2\,cx^4+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{8\,(ab^2-4\,a^2c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) +
sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^8 + 2*b*c*x^4 + b^2 - 2*a*c -
(2*c*x^4 + b)*sqrt(b^2 - 4*a*c))/(c*x^8 + b*x^4 + a)))/(a*b^2 - 4*a^2*c), -
1/8*((b^2 - 4*a*c)*d*log(c*x^8 + b*x^4 + a) - 8*(b^2 - 4*a*c)*d*log(x) - 2*
sqrt(-b^2 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^4 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)))/(a*b^2 - 4*a^2*c)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.55, size = 78, normalized size = 1.00

$$-\frac{d\log\left(cx^8+bx^4+a\right)}{8\,a}+\frac{d\log\left(x^4\right)}{4\,a}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^4+b}{\sqrt{-b^2+4\,ac}}\right)}{4\,\sqrt{-b^2+4\,ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*d*log(c*x^8 + b*x^4 + a)/a + 1/4*d*log(x^4)/a - 1/4*(b*d - 2*a*e)*arct
an((2*c*x^4 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

Mupad [**B**]
time = 5.27, size = 2500, normalized size = 32.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x*(a + b*x^4 + c*x^8)),x)
```

```
[Out] (d*log(x))/a - (log(a + b*x^4 + c*x^8)*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 -
 64*a^2*c)) + (atan((128*a^5*x^4*(((c^4*e^5 - ((4*b^2*d - 16*a*c*d)*(11*b*c
^4*e^4 + 9*c^5*d*e^3 - ((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^
2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*
(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e)
)/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c
^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^2*c^4*e^3 + 108*b*
c^5*d*e^2))/(2*(16*a*b^2 - 64*a^2*c))))/(2*(16*a*b^2 - 64*a^2*c)) - ((4*b^2
```

*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c
d)(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^
5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b
^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1
/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^
2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2))))/(2*(16*a*b^2 - 64*a^2*c)
) + ((2*a*e - b*d)*(((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 -
 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*
e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1
280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4
*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a
*e - b*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 460
8*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e +
3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^
5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*a*c - b^2)^(1/2))))/(8*a*(4*a*c - b^2)^(1
/2))))/(2*(16*a*b^2 - 64*a^2*c)) + (((((((2*a*e - b*d)*(((4*b^2*d - 16*a*c
d)(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^
5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b
^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a
*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1
/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^
2)/(128*a^2*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)))*(2*a*e - b*d))/(8*a*(4*a*
c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*
a*e - b*d)^3)/(1024*a^3*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2)))*(2*a*e
- b*d))/(8*a*(4*a*c - b^2)^(1/2)) - ((((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*
d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 6
4*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c
 - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a
*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*
a*c*d))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(
((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^
2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(2*(16*a*b^2 -
64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c^5*e^2 - 432*b^2*c^5*d*e))/(8*a*(4*
a*c - b^2)^(1/2))))/(2*(16*a*b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d -
 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 46
08*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e +
 3456*a*b^2*c^5*e))/(2*(16*a*b^2 - 64*a^2*c)) + 224*b^3*c^4*e^2 - 864*a*b*c
^5*e^2 - 432*b^2*c^5*d*e))/(2*(16*a*b^2 - 64*a^2*c)) + 72*a*c^5*e^3 + 16*b^
2*c^4*e^3 + 108*b*c^5*d*e^2))/(8*a*(4*a*c - b^2)^(1/2)))*(2*a*e - b*d))/(8*
a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c
^5)*(2*a*e - b*d)^4)/(8192*a^4*(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^2))*(5*b
^5*d - a^3*c^2*e - a*b^4*e - 24*a*b^3*c*d + 23*a^2*b*c^2*d + 3*a^2*b^2*c*e)
)/(32*a^5*c^4*(a^2*e^2 - 20*b^2*d^2 + 81*a*c*d^2 - a*b*d*e)) - ((((4*b^2*d
- 16*a*c*d)*(((((((2*a*e - b*d)*(((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608

```
*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b^3*c^5*d - 1024*b^4*c^4*e + 3
456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*c*d)*(1280*b
^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*(16*a*b^2 - 64*a^2*c)*(4*a*c
- b^2)^(1/2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^2*d - 16*a*
c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^2)/(128*a^2*(16*a*b^2 -
64*a^2*c)*(4*a*c - b^2)))*(2*a*e - b*d))/(8*a*(4*a*c - b^2)^(1/2)) + ((4*b^
2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d)^3)/(1024*a^3*
(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(3/2))))/(2*(16*a*b^2 - 64*a^2*c)) - ((
4*b^2*d - 16*a*c*d)*(((4*b^2*d - 16*a*c*d)*(((((2*a*e - b*d)*(((4*b^2*d - 1
6*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5))/(2*(16*a*b^2 - 64*a^2*c)) + 576*b
^3*c^5*d - 1024*b^4*c^4*e + 3456*a*b^2*c^5*e))/(8*a*(4*a*c - b^2)^(1/2)) +
((4*b^2*d - 16*a*c*d)*(1280*b^5*c^4 - 4608*a*b^3*c^5)*(2*a*e - b*d))/(16*a*
(16*a*b^2 - 64*a^2*c)*(4*a*c - b^2)^(1/2)))*(4*b^2*d - 16*a*c*d))/(2*(16*a*
b^2 - 64*a^2*c)) + ((2*a*e - b*d)*(((4*b^2*d - ...
```

3.49 $\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=392

$$-\frac{d}{ax}-\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}}-\frac{\sqrt[4]{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\ldots}{\sqrt[4]{-b+\sqrt{b^2\ldots}}}\right.}{2\,2^{3/4}a\sqrt[4]{-b+\sqrt{b^2-4ac}}}$$

```
[Out] -d/a/x-1/4*c^(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)+
1/4*c^(1/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*(d+(2*
a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(1/4)-1/4*c^
(1/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+b*
d)/(-4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)+1/4*c^(1/4)*
arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+b*d)/(-
4*a*c+b^2)^(1/2))*2^(1/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(1/4)
```

Rubi [A]

time = 0.44, antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1518, 1524, 304, 211, 214}

$$-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{-\sqrt{b^2-4ac}-b}}+\frac{\sqrt[4]{c}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\,2^{3/4}a\sqrt[4]{\sqrt{b^2-4ac}-b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -(d/(a*x)) - (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(1/4)
*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sqrt[b^2 -
4*a*c])^(1/4)) - (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b + Sqrt[b^
2 - 4*a*c])^(1/4)) + (c^(1/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh
[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b - Sq
rt[b^2 - 4*a*c])^(1/4)) + (c^(1/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Ar
cTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(3/4)*a*(-b
 + Sqrt[b^2 - 4*a*c])^(1/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rule 1524

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
 (c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 +
(2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (
2*c*d - b*e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b
, c, d, e, f, m}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{d+ex^4}{x^2(a+bx^4+cx^8)}\,dx = -\frac{d}{ax} - \frac{\int \frac{x^2(bd-ae+cdx^4)}{a+bx^4+cx^8}\,dx}{a}$$

$$= -\frac{d}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{x^2}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{d}{ax} + \frac{\left(\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2\sqrt{2}\,a} - \frac{\left(\sqrt{c}\right.}{}$$

$$= -\frac{d}{ax} - \frac{\sqrt[4]{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\,2^{3/4}a\sqrt[4]{-b-\sqrt{b^2-4ac}}} - \frac{\sqrt[4]{c}\left(d+\frac{bd}{\sqrt{b^2}}\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 85, normalized size = 0.22

$$-\frac{d}{ax} - \frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1+2c\#1^5}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x]
[Out] -(d/(a*x)) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log[x -
 #1] + c*d*Log[x - #1]*#1^4)/(b*#1 + 2*c*#1^5) & ]/(4*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.06, size = 73, normalized size = 0.19

method	result
default	$-\frac{d}{ax}+\frac{\sum_{\_R=\text{RootOf}\left(c\_Z^8+\_Z^4b+a\right)}\frac{\left(-cd\_R^6+(ae-bd)\_R^2\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4a}$
risch	$-\frac{d}{ax}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(256a^9c^4-256b^2c^3a^8+96b^4c^2a^7-16b^6ca^6+b^8a^5\right)\_Z^8+\left(16a^6bc^2e^4+128a^6c^3de^3-8a^5b^3ce^4-128a^5b^2c^2de^3-288a^5b\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d/a/x+1/4/a*sum((-c*d*_R^6+(a*e-b*d)*_R^2)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=R
ootOf(_Z^8*c+_Z^4*b+a))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^6 + (b*d - a*e)*x^2)/(c*x^8 + b*x^4 + a), x)/a - d/(a*x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**2/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^2/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**B**]
time = 9.46, size = 2500, normalized size = 6.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x^4)/(x^2*(a + b*x^4 + c*x^8)),x)

[Out] atan((((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(1/4)*(32768*a^16*c^8*d^2 - 32768*a^17*c^7*e^2 + 1024*a^12*b^8*c^4*d^2 - 12288*a^13*b^6*c^5*d^2 + 51200*a^14*b^4*c^6*d^2 - 81920*a^15*b^2*c^7*d^2 + 1024*a^14*b^6*c^4*e^2 - 10240*a^15*b^4*c^5*e^2 + 32768*a^16*b^2*c^6*e^2 + 98304*a^16*b*c^7*d*e - 2048*a^13*b^7*c^4*d*e + 22528*a^14*b^5*c^5*d*e - 81920*a^15*b^3*c^6*d*e) - 4096*a^15*c^8*d^3 + 4096*a^16*b*c^6*e^3 + 12288*a^16*c^7*d*e^2 - 256*a^11*b^8*c^4*d^3 + 2816*a^12*b^6*c^5*d^3 - 10496*a^13*b^4*c^6*d^3 + 14336*a^14*b^2*c^7*d^3 + 256*a^14*b^5*c^4*e^3 - 2048*a^15*b^3*c^5*e^3 - 24576*a^15*b*c^7*d^2*e + 768*a^12*b^7*c^4*d^2*e - 7680*a^13*b^5*c^5*d^2*e - 768*a^13*b^6*c^4*d*e^2 + 24576*a^14*b^3*c^6*d^2*e + 6912*a^14*b^4*c^5*d*e^2 - 18432*a^15*b^2*c^6*d*e^2) + x*(4*a^11*b*c^8*d^6 + 4*a^14*b*c^5*e^6 - 16*a^12*c^8*d^5*e - 16*a^14*c^6*d*e^5 - 32*a^13*c^7*d^3*e^3 + 4*a^11*b^3*c^6*d^4*e^2 - 32*a^12*b^2*c^6*d^3*e^3 + 4*a^12*b^3*c^5*d^2*e^4 - 8*a^11*b^2*c^7*d^5*e + 44*a^12*b*c^7*d^4*e^2 + 44*a^13*b*c^6*d^2*e^4 - 8*a^13*b^2*c^5*d*e^5))*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3

```
*e + 6*a^2*b^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 -
 3*a*b^2*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^
(1/2) - 4*a^3*b*d*e^3*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^
4*b^4*c*d*e^3 - 200*a^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*
c^3*d^3*e - 288*a^5*b*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*
(a^5*b^8 + 256*a^9*c^4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))
^(1/4)*1i + ((-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) +
b^4*d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*
a^6*b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 6
1*a^2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1
/2) + 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^
2*(-(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a
*c - b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b*d*e^3
*(-(4*a*c - b^2)^5)^(1/2) + 48*a^2*b^6*c*d^3*e + 40*a^4*b^4*c*d*e^3 - 200*a
^3*b^4*c^2*d^3*e - 66*a^3*b^5*c*d^2*e^2 + 320*a^4*b^2*c^3*d^3*e - 288*a^5*b
*c^3*d^2*e^2 - 128*a^5*b^2*c^2*d*e^3 - 6*a^3*c*d^2*e^2*(-(4*a*c - b^2)^5)^(
1/2) + 8*a^2*b*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^5*b^8 + 256*a^9*c^
4 - 16*a^6*b^6*c + 96*a^7*b^4*c^2 - 256*a^8*b^2*c^3)))^(3/4)*(4096*a^15*c^8
*d^3 + x*(-(b^9*d^4 + a^4*b^5*e^4 + a^4*e^4*(-(4*a*c - b^2)^5)^(1/2) + b^4*
d^4*(-(4*a*c - b^2)^5)^(1/2) + 80*a^4*b*c^4*d^4 - 8*a^5*b^3*c*e^4 + 16*a^6*
b*c^2*e^4 - 4*a^3*b^6*d*e^3 - 128*a^5*c^4*d^3*e + 128*a^6*c^3*d*e^3 + 61*a^
2*b^5*c^2*d^4 - 120*a^3*b^3*c^3*d^4 + a^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2)
+ 6*a^2*b^7*d^2*e^2 - 13*a*b^7*c*d^4 - 4*a*b^8*d^3*e + 6*a^2*b^2*d^2*e^2*(-
(4*a*c - b^2)^5)^(1/2) + 240*a^4*b^3*c^2*d^2*e^2 - 3*a*b^2*c*d^4*(-(4*a*c -
 b^2)^5)^(1/2) - 4*a*b^3*d^3*e*(-(4*a*c - b^2)^...
```

3.50 $\int \frac{d+ex^4}{x^3(a+bx^4+cx^8)}\,dx$

Optimal. Leaf size=199

$$-\frac{d}{2ax^2}-\frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*d/a/x^2-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c
^(1/2)*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))
^(1/2)-1/4*arctan(x^2*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)
*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1504, 1295, 1180, 211}

$$-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*d/(a*x^2) - (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x^2)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b - Sqrt[
b^2 - 4*a*c]]) - (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x^2)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[b + Sqrt[
b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1504

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^4}{x^3\left(a+bx^4+cx^8\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{d+ex^2}{x^2\left(a+bx^2+cx^4\right)}\,dx,x,x^2\right)\\ &= -\frac{d}{2ax^2}-\frac{\text{Subst}\left(\int \frac{bd-ae+cdx^2}{a+bx^2+cx^4}\,dx,x,x^2\right)}{2a}\\ &= -\frac{d}{2ax^2}-\frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx,x,x^2\right)}{4a}-\frac{\left(c\right.}{}\\ &= -\frac{d}{2ax^2}-\frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x^2}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(d-\frac{}{\sqrt{b}}\right.}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.03, size = 89, normalized size = 0.45

$$-\frac{d}{2ax^2}-\frac{\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1^2+2c\#1^6}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/2*d/(a*x^2) - RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e*Log
[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^2 + 2*c*#1^6) & ]/(4*a)
```

Maple [A]
time = 0.08, size = 177, normalized size = 0.89

method	result
default	$-\frac{d}{2ax^2}+\frac{2c\left(-\frac{\left(-bd+2ae-d\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx^2\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(bd-2ae-d\sqrt{-4ac+b^2}\right.}{8\sqrt{-4ac+}}\right.}{a}$
risch	$-\frac{d}{2ax^2}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^5c^2-8a^4b^2c+a^3b^4\right)\_Z^4+\left(-4a^3be^2c-16a^3dec^2+a^2b^3e^2+12a^2b^2dec+12a^2bc^2d^2-2ab^4de-7ab^3cd^2+b^5d^2\right)\_\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d/a/x^2+2/a*c*(-1/8*(-b*d+2*a*e-d*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/
2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x^2*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x^2*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="maxima")
```

```
[Out] -integrate((c*d*x^4 + b*d - a*e)*x/(c*x^8 + b*x^4 + a), x)/a - 1/2*d/(a*x^2
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2780 vs. $2(159) = 318$.
time = 0.88, size = 2780, normalized size = 13.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="fricas")

[Out] 1/4*(sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*
a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c +
a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*
d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(3*a*b^2*c*d^2*x^2*e
^2 - 3*a^2*b*c*d*x^2*e^3 + (b^2*c^2 - a*c^3)*d^4*x^2 + a^3*c*x^2*e^4 - (b^3
*c + a*b*c^2)*d^3*x^2*e + 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^
3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*
e^2 - (a^3*b^2 - 4*a^4*c)*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a
^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d
^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(
a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*
a^2*c)*d*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c +
a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*
d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x^2*sqrt
(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4
*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 +
4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))*log(3*a*b^2*c*d^2*x^2*e^2 - 3*a^2*b*c*d*x^2*e^3
+ (b^2*c^2 - a*c^3)*d^4*x^2 + a^3*c*x^2*e^4 - (b^3*c + a*b*c^2)*d^3*x^2*e -
 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2
*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)
*e^3 - ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sq
rt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 -
 a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt
(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + (a^3*b^2 - 4
*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 +
4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a
*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*
e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e
 - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)
)*log(3*a*b^2*c*d^2*x^2*e^2 - 3*a^2*b*c*d*x^2*e^3 + (b^2*c^2 - a*c^3)*d^4*x
^2 + a^3*c*x^2*e^4 - (b^3*c + a*b*c^2)*d^3*x^2*e + 1/2*sqrt(1/2)*((b^5 - 5*
a*b^3*c + 4*a^2*b*c^2)*d^3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3
*(a^2*b^3 - 4*a^3*b*c)*d*e^2 - (a^3*b^2 - 4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*
b^2*c + 8*a^5*c^2)*d - (a^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - (b^4
 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^
2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a
*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*
e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e
 - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)
)) - sqrt(1/2)*a*x^2*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*
a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c +
```

```
a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*
d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(3*a*b^2*c*d^2*x^2*e
^2 - 3*a^2*b*c*d*x^2*e^3 + (b^2*c^2 - a*c^3)*d^4*x^2 + a^3*c*x^2*e^4 - (b^3
*c + a*b*c^2)*d^3*x^2*e - 1/2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^
3 - (3*a*b^4 - 13*a^2*b^2*c + 4*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 4*a^3*b*c)*d*
e^2 - (a^3*b^2 - 4*a^4*c)*e^3 + ((a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*d - (a
^4*b^3 - 4*a^5*b*c)*e)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d
^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/(
a^6*b^2 - 4*a^7*c)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*
a^2*c)*d*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c +
a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*
d^2*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - 2*d)/(a*x^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**3/(c*x**8+b*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3007 vs. 2(159) = 318.
time = 11.20, size = 3007, normalized size = 15.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^3/(c*x^8+b*x^4+a),x, algorithm="giac")
```

```
[Out] -1/8*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^3*c^2 - 2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 +
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^2*c^3 + 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^4 - 32*a^2*c^4 + 2*(b^2 - 4*a*c)*b^2*c^2 - 8*(b^2 - 4*a*c)*a*
c^3)*d*x^4*abs(a) - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqr
```

t(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c
^2 + 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*
a^2*b*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
 - 2*a*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 + 16*a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^3 - 32*a^3*c^3 + 2*(b^2 - 4*a*c)*a*b^2*c - 8*(b^2 - 4*a*c)*a^2
*c^2)*abs(a)*e - (2*a*b^4*c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2)*d + (2*a^2*b^3*c^2 - 8
*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c +
 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2
 - 4*a*c)*a^2*b*c^2)*e)*arctan(2*sqrt(1/2)*x^2/sqrt((a*b + sqrt(a^2*b^2 - 4*
a^3*c))/(a*c)))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*
b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*abs(a)*abs(c)) - 1/8*((sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 2*b^4*c^2 + 16*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3
 - 16*a*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 + 32*a^2*c
^4 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*d*x^4*abs(a) - (2*a*b
^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3
 - 2*(b^2 - 4*a*c)*a*b*c^3)*d*x^4 + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c
)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2)*d*abs(a) - (sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*a*b^4*c + 16*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 16*
a^2*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 32*a^3*c^
3 - 2*(b^2 - 4*a*c)*a*b^2*c + 8*(b^2 - 4*a*c)*a^2*c^2)*abs(a)*e - (2*a*b^4*
c^2 - 8*a^2*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c

```
)*c)*a*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 -
2*(b^2 - 4*a*c)*a*b^2*c^2)*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^...
```

Mupad [B]
time = 7.62, size = 2500, normalized size = 12.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x^3*(a + b*x^4 + c*x^8)),x)
```

```
[Out] - atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*
c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b
^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 -
 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(3
2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 +
 a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^
12*b^6*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) + x^2*(9216*a^11*b^5*
c^5*d - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e -
8192*a^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^
2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2
*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)
))^(1/2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2
 - 3072*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e +
4096*a^12*b^2*c^6*d*e) + x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a
^12*c^7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7
*d^3 + 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e
- 960*a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2
))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a
```

*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b
^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2) + 64*a^10*c^8*d^4 + 64*a^12*c^6*e^4 + 16*a^8*b^4*c^6*d
^4 - 64*a^9*b^2*c^7*d^4 - 128*a^11*c^7*d^2*e^2 + 128*a^10*b^2*c^6*d^2*e^2 +
 128*a^10*b*c^7*d^3*e - 128*a^11*b*c^6*d*e^3 - 64*a^9*b^3*c^6*d^3*e) + x^2*
(8*a^11*c^6*e^5 - 8*a^9*c^8*d^4*e - 4*a^8*b^3*c^6*d^3*e^2 + 12*a^9*b^2*c^6*
d^2*e^3 - 16*a^10*b*c^6*d*e^4 + 4*a^8*b^2*c^7*d^4*e))*(-(b^5*d^2 + a^2*b^3*
e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*1i
 - ((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b
^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
 - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^
3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c
^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4
*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(4096*a^12*b^6
*c^4 - 32768*a^13*b^4*c^5 + 65536*a^14*b^2*c^6) - x^2*(9216*a^11*b^5*c^5*d
 - 1024*a^10*b^7*c^4*d - 24576*a^12*b^3*c^6*d + 1024*a^11*b^6*c^4*e - 8192*a
^12*b^4*c^5*e + 16384*a^13*b^2*c^6*e + 16384*a^13*b*c^7*d))*(-(b^5*d^2 + a^
2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(
1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c -
b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d
e(-(4*a*c - b^2)^3)^(1/2))/(32*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/
2) + 4096*a^12*b*c^7*d^2 - 4096*a^13*b*c^6*e^2 + 512*a^10*b^5*c^5*d^2 - 307
2*a^11*b^3*c^6*d^2 + 1024*a^12*b^3*c^5*e^2 - 1024*a^11*b^4*c^5*d*e + 4096*a
^12*b^2*c^6*d*e) - x^2*(512*a^11*c^8*d^3 - 768*a^12*b*c^6*e^3 - 512*a^12*c^
7*d*e^2 - 64*a^8*b^6*c^5*d^3 + 448*a^9*b^4*c^6*d^3 - 896*a^10*b^2*c^7*d^3 +
 192*a^11*b^3*c^5*e^3 + 768*a^11*b*c^7*d^2*e + 192*a^9*b^5*c^5*d^2*e - 960*
a^10*b^3*c^6*d^2*e - 320*a^10*b^4*c^5*d*e^2 + 1408*a^11*b^2*c^6*d*e^2))*(-(
b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d...

3.51 $\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx$

Optimal. Leaf size=394

$$-\frac{d}{3ax^3}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}}+\frac{c^{3/4}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-b+\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,a\left(-b+\sqrt{b^2-4ac}\right)^{3/4}}$$

```
[Out] -1/3*d/a/x^3+1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(
1/4))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^
(3/4)+1/4*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b-(-4*a*c+b^2)^(1/2))^(1/4))*
(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b-(-4*a*c+b^2)^(1/2))^(3/4)+
1/4*c^(3/4)*arctan(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*
a*e+b*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)+1/4*c^
(3/4)*arctanh(2^(1/4)*c^(1/4)*x/(-b+(-4*a*c+b^2)^(1/2))^(1/4))*(d+(-2*a*e+b
*d)/(-4*a*c+b^2)^(1/2))*2^(3/4)/a/(-b+(-4*a*c+b^2)^(1/2))^(3/4)
```

Rubi [A]
time = 0.40, antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1518, 1436, 218, 214, 211}

$$\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{-\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(-\sqrt{b^2-4ac}-b\right)^{3/4}}+\frac{c^{3/4}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{\sqrt{b^2-4ac}-b}}\right)}{2\sqrt[4]{2}\,a\left(\sqrt{b^2-4ac}-b\right)^{3/4}}-\frac{d}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/3*d/(a*x^3) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(2^(
1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b - Sqrt[b^
2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b + Sqr
t[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*Arc
Tanh[(2^(1/4)*c^(1/4)*x)/(-b - Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a*(-b
- Sqrt[b^2 - 4*a*c])^(3/4)) + (c^(3/4)*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c]
)*ArcTanh[(2^(1/4)*c^(1/4)*x)/(-b + Sqrt[b^2 - 4*a*c])^(1/4)])/(2*2^(1/4)*a
*(-b + Sqrt[b^2 - 4*a*c])^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{d+ex^4}{x^4\left(a+bx^4+cx^8\right)}\,dx = -\frac{d}{3ax^3} - \frac{\int \frac{3(bd-ae)+3cdx^4}{a+bx^4+cx^8}\,dx}{3a}$$

$$= -\frac{d}{3ax^3} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^4}\,dx}{2a} - \frac{\left(c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right.\right.}{}$$

$$= -\frac{d}{3ax^3} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{-b-\sqrt{b^2-4ac}}-\sqrt{2}\sqrt{c}\,x^2}\,dx}{2a\sqrt{-b-\sqrt{b^2-4ac}}} + \frac{\left(c\left(d\right.\right.}{}$$

$$= -\frac{d}{3ax^3} + \frac{c^{3/4}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\sqrt[4]{c}\,x}{\sqrt[4]{-b-\sqrt{b^2-4ac}}}\right)}{2\sqrt[4]{2}\,a\left(-b-\sqrt{b^2-4ac}\right)^{3/4}} + \frac{c^{3/4}\left(d+\frac{b}{\sqrt{b}}\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.05, size = 86, normalized size = 0.22

$$-\frac{\frac{4d}{x^3}+3\text{RootSum}\left[a+b\#1^4+c\#1^8\&,\frac{bd\log(x-\#1)-ae\log(x-\#1)+cd\log(x-\#1)\#1^4}{b\#1^3+2c\#1^7}\&\right]}{12a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x]
```

```
[Out] -1/12*((4*d)/x^3 + 3*RootSum[a + b*#1^4 + c*#1^8 & , (b*d*Log[x - #1] - a*e
*Log[x - #1] + c*d*Log[x - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) & ])/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.07, size = 68, normalized size = 0.17

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+\_Z^4 b+a\right)}\frac{\left(-cd\_R^4+ae-bd\right)\ln\left(x-\_R\right)}{2\_R^7c+\_R^3b}}{4a}-\frac{d}{3a\,x^3}$	68
risch	Expression too large to display	1633

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*sum((-_R^4*c*d+a*e-b*d)/(2*_R^7*c+_R^3*b)*ln(x-_R),_R=RootOf(_Z^8*c+_
Z^4*b+a))-1/3*d/a/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="maxima")
[Out] -integrate((c*d*x^4 + b*d - a*e)/(c*x^8 + b*x^4 + a), x)/a - 1/3*d/(a*x^3)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**4+d)/x**4/(c*x**8+b*x**4+a),x)
[Out] Timed out
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^4+d)/x^4/(c*x^8+b*x^4+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 10.22, size = 2500, normalized size = 6.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^4)/(x^4*(a + b*x^4 + c*x^8)),x)
```

```
[Out] atan((((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*a
^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*c - b
^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3 + 86
*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3*d^4*
(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a^6*b^
3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^2*b^2
*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^5)^(1
/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^2*e^2
*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a*b^
5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d*e^3
- 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 78*a^3
*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a^5*b^
4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b^3*c*
d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)^(1/2
) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-(4*a*
c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^9*b^4
*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(((-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4
*a*c - b^2)^5)^(1/2) - 112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*
e^4 - a^5*c*e^4*(-(4*a*c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^
3*e - 128*a^7*c^4*d*e^3 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^
4*b^3*c^4*d^4 - a^3*c^3*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c
 - b^2)^5)^(1/2) + 40*a^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4
- 4*a*b^10*d^3*e + 6*a^2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d
^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3
*d^2*e^2 + 6*a^4*c^2*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*
a*c - b^2)^5)^(1/2) - 4*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c
*d^3*e + 48*a^4*b^6*c*d*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 29
2*a^3*b^6*c^2*d^3*e - 78*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^
5*b^2*c^4*d^3*e - 200*a^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b
^2*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d
^3*e*(-(4*a*c - b^2)^5)^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/
2) + 8*a^4*b*c*d*e^3*(-(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4
 - 16*a^8*b^6*c + 96*a^9*b^4*c^2 - 256*a^10*b^2*c^3)))^(1/4)*(262144*a^17*c
^8*d + 4096*a^13*b^8*c^4*d - 53248*a^14*b^6*c^5*d + 245760*a^15*b^4*c^6*d -
 458752*a^16*b^2*c^7*d - 4096*a^14*b^7*c^4*e + 49152*a^15*b^5*c^5*e - 19660
8*a^16*b^3*c^6*e + 262144*a^17*b*c^7*e) + x*(81920*a^15*b*c^8*d^2 - 49152*a
^16*b*c^7*e^2 + 1024*a^11*b^9*c^4*d^2 - 13312*a^12*b^7*c^5*d^2 + 62464*a^13
*b^5*c^6*d^2 - 122880*a^14*b^3*c^7*d^2 + 1024*a^13*b^7*c^4*e^2 - 11264*a^14
*b^5*c^5*e^2 + 40960*a^15*b^3*c^6*e^2 - 65536*a^16*c^8*d*e - 2048*a^12*b^8*
c^4*d*e + 24576*a^13*b^6*c^5*d*e - 102400*a^14*b^4*c^6*d*e + 163840*a^15*b^
2*c^7*d*e))*(-(b^11*d^4 + a^4*b^7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) -
```

```
112*a^5*b*c^5*d^4 - 11*a^5*b^5*c*e^4 - 48*a^7*b*c^3*e^4 - a^5*c*e^4*(-(4*a*
c - b^2)^5)^(1/2) - 4*a^3*b^8*d*e^3 + 128*a^6*c^5*d^3*e - 128*a^7*c^4*d*e^3
 + 86*a^2*b^7*c^2*d^4 - 231*a^3*b^5*c^3*d^4 + 280*a^4*b^3*c^4*d^4 - a^3*c^3
*d^4*(-(4*a*c - b^2)^5)^(1/2) + a^4*b^2*e^4*(-(4*a*c - b^2)^5)^(1/2) + 40*a
^6*b^3*c^2*e^4 + 6*a^2*b^9*d^2*e^2 - 15*a*b^9*c*d^4 - 4*a*b^10*d^3*e + 6*a^
2*b^2*c^2*d^4*(-(4*a*c - b^2)^5)^(1/2) + 6*a^2*b^4*d^2*e^2*(-(4*a*c - b^2)^
5)^(1/2) + 366*a^4*b^5*c^2*d^2*e^2 - 720*a^5*b^3*c^3*d^2*e^2 + 6*a^4*c^2*d^
2*e^2*(-(4*a*c - b^2)^5)^(1/2) - 5*a*b^4*c*d^4*(-(4*a*c - b^2)^5)^(1/2) - 4
*a*b^5*d^3*e*(-(4*a*c - b^2)^5)^(1/2) + 56*a^2*b^8*c*d^3*e + 48*a^4*b^6*c*d
*e^3 - 4*a^3*b^3*d*e^3*(-(4*a*c - b^2)^5)^(1/2) - 292*a^3*b^6*c^2*d^3*e - 7
8*a^3*b^7*c*d^2*e^2 + 680*a^4*b^4*c^3*d^3*e - 640*a^5*b^2*c^4*d^3*e - 200*a
^5*b^4*c^2*d*e^3 + 480*a^6*b*c^4*d^2*e^2 + 320*a^6*b^2*c^3*d*e^3 + 16*a^2*b
^3*c*d^3*e*(-(4*a*c - b^2)^5)^(1/2) - 12*a^3*b*c^2*d^3*e*(-(4*a*c - b^2)^5)
^(1/2) - 18*a^3*b^2*c*d^2*e^2*(-(4*a*c - b^2)^5)^(1/2) + 8*a^4*b*c*d*e^3*(-
(4*a*c - b^2)^5)^(1/2))/(512*(a^7*b^8 + 256*a^11*c^4 - 16*a^8*b^6*c + 96*a^
9*b^4*c^2 - 256*a^10*b^2*c^3)))^(3/4) - 64*a^14*c^7*e^5 - 128*a^11*b*c^9*d^
5 + 192*a^12*c^9*d^4*e - 16*a^9*b^5*c^7*d^5 + 96*a^10*b^3*c^8*d^5 + 16*a^13
*b^2*c^6*e^5 + 128*a^13*c^8*d^2*e^3 - 64*a^10*b^5*c^6*d^3*e^2 + 288*a^11*b^
3*c^7*d^3*e^2 + 96*a^11*b^4*c^6*d^2*e^3 - 416*a^12*b^2*c^7*d^2*e^3 + 256*a^
13*b*c^7*d*e^4 + 16*a^9*b^6*c^6*d^4*e - 48*a^10*b^4*c^7*d^4*e - 112*a^11*b^
2*c^8*d^4*e - 128*a^12*b*c^8*d^3*e^2 - 64*a^12*b^3*c^6*d*e^4) + x*(8*a^13*c
^7*e^6 - 8*a^10*c^10*d^6 + 4*a^9*b^2*c^9*d^6 - 8*a^11*c^9*d^4*e^2 + 8*a^12*
c^8*d^2*e^4 + 4*a^9*b^4*c^7*d^4*e^2 + 16*a^10*b^2*c^8*d^4*e^2 - 16*a^10*b^3
*c^7*d^3*e^3 + 28*a^11*b^2*c^7*d^2*e^4 + 8*a^10*b*c^9*d^5*e - 24*a^12*b*c^7
*d*e^5 - 8*a^9*b^3*c^8*d^5*e - 16*a^11*b*c^8*d^3*e^3))*(-(b^11*d^4 + a^4*b^
7*e^4 + b^6*d^4*(-(4*a*c - b^2)^5)^(1/2) - 112*...
```

3.52 $\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=278

$$-x-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right.}{2\sqrt{6}}$$

```
[Out] -x-1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*6^
(1/2)+1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)
))*6^(1/2)+1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1+x
^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^
(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1516, 1360, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-x$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x - ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcT
an[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[
2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[
3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 -
Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]
/(4*Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1360

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1516

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[e*f^(n - 1)*(f*x)^(m - n + 1)*((a
+ b*x^n + c*x^(2*n))^(p + 1)/(c*(m + n*(2*p + 1) + 1))), x] - Dist[f^n/(c*(
m + n*(2*p + 1) + 1)), Int[(f*x)^(m - n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e
*(m - n + 1) + (b*e*(m + n*p + 1) - c*d*(m + n*(2*p + 1) + 1))*x^n, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*(2*p + 1) + 1, 0] && Integer
Q[p]
```

Rubi steps

$$\int \frac{x^4(1-x^4)}{1-x^4+x^8}\,dx = -x+\int \frac{1}{1-x^4+x^8}\,dx$$

$$= -x + \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -x + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-1+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-1+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \cdots$$

$$= -x - \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \cdots$$

$$= -x - \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\cdots}\right)}{4\sqrt{6}}\cdots$$

$$= -x - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}\cdots$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 46, normalized size = 0.17

$$-x+\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -x + RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 34, normalized size = 0.12

method	result	size
default	$-x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(3\_R^{2}+3\_Rx+x^{2}\right)\right)}{4}$	34
risch	$-x+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(3\_R^{2}+3\_Rx+x^{2}\right)\right)}{4}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -x+1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -x + integrate(1/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.41, size = 223, normalized size = 0.80

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}(x^3-x)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}x+2\right)}{3x^2-2}\right)+\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(36x^4+36\sqrt{3}\sqrt{2}(x^3+x)+108x^2+36\right)-\frac{1}{24}\sqrt{3}\sqrt{2}\log\left(36x^4-36\sqrt{3}\sqrt{2}(x^3+x)+108x^2+36\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(36*x^4 + 36*sqrt(3)*sqrt(2)*(x^3 + x) + 108
*x^2 + 36) - 1/24*sqrt(3)*sqrt(2)*log(36*x^4 - 36*sqrt(3)*sqrt(2)*(x^3 + x)
 + 108*x^2 + 36) - x
```

Sympy [A]
time = 0.12, size = 170, normalized size = 0.61

$$-x-\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}-\frac{\sqrt{6}\left(-2\operatorname{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)-2\operatorname{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right)\right)}{24}-\frac{\sqrt{6}\log\left(x^4-\sqrt{6}x^3+3x^2-\sqrt{6}x+1\right)}{24}+\frac{\sqrt{6}\log\left(x^4+\sqrt{6}x^3+3x^2+\sqrt{6}x+1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -x - sqrt(6)*(-2*atan(sqrt(6)*x/3 - 1/3) - 2*atan(sqrt(6)*x**3 - 4*x**2 + 2
*sqrt(6)*x - 3))/24 - sqrt(6)*(-2*atan(sqrt(6)*x/3 + 1/3) - 2*atan(sqrt(6)*
x**3 + 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*
x**2 - sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(
6)*x + 1)/24
```

Giac [A]
time = 3.78, size = 208, normalized size = 0.75

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - x
```

Mupad [B]
time = 1.92, size = 56, normalized size = 0.20

$$-x+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(-\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] - x - 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 + 1i/
12) - 6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 - 1i/
12)
```

3.53 $\int \frac{x^3(1-x^4)}{1-x^4+x^8} dx$

Optimal. Leaf size=39

$$-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] -1/8*ln(x^8-x^4+1)-1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1482, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(1 - 2*x^4)/Sqrt[3]]/Sqrt[3] - Log[1 - x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(1-x^4)}{1-x^4+x^8}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1-x}{1-x+x^2}\,dx, x, x^4\right)\\
&= \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right) - \frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{8}\log\left(1-x^4+x^8\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 39, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{-1+2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{8}\log\left(1-x^4+x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(-1 + 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 - x^4 + x^8]/8
```

Maple [A]
time = 0.02, size = 33, normalized size = 0.85

method	result	size
default	$-\frac{\ln(x^8-x^4+1)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	33

risch	$-\frac{\ln(4x^8-4x^4+4)}{8}+\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/8*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A]
time = 0.51, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Fricas [A]
time = 0.36, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Sympy [A]
time = 0.08, size = 37, normalized size = 0.95

$$-\frac{\log\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**4+1)/(x**8-x**4+1),x)
[Out] -log(x**8 - x**4 + 1)/8 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12
```

Giac [A]
time = 3.57, size = 32, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1)
```

Mupad [B]
time = 0.05, size = 34, normalized size = 0.87

$$-\frac{\ln\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\,\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^3*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] - log(x^8 - x^4 + 1)/8 - (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3))/12
```

3.54 $\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2
*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))-1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(
1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/
6*6^(1/2))-1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))/(3/2*2^(1/2)+1/2*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A]

time = 0.19, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1520, 1293, 1183, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2 - Sqrt[3])
]) - ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sqr
t[3])]) - ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
- Sqrt[3])]) + ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[
3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x^
2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sqr
t[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt[
3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1520

```
Int[(((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^(n_)))/((a_) + (b_.)*(x_)^(n_) +
(c_.)*(x_)^(n2_)), x_Symbol] :> With[{q = Rt[a*c, 2]}, With[{r = Rt[2*c*q -
 b*c, 2]}, Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r - (c*d - e*q)*x^(n/2), x]/
(q - r*x^(n/2) + c*x^n)), x], x] + Dist[c/(2*q*r), Int[(f*x)^m*(Simp[d*r +
(c*d - e*q)*x^(n/2), x]/(q + r*x^(n/2) + c*x^n)), x], x]] /;  !LtQ[2*c*q -
b*c, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[n2, 2*n] && LtQ[b^2 - 4*a*c
, 0] && IntegersQ[m, n/2] && LtQ[0, m, n] && PosQ[a*c]
```

Rubi steps

$$\int \frac{x^2(1-x^4)}{1-x^4+x^8}\,dx = \frac{\int \frac{x^2\left(\sqrt{3}-2x^2\right)}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{x^2\left(\sqrt{3}+2x^2\right)}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{-2+\sqrt{3}\,x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{2+\sqrt{3}\,x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{\int \frac{2\sqrt{2-\sqrt{3}}-\left(2-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{2\sqrt{2-\sqrt{3}}+\left(2-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{-2\sqrt{2+\sqrt{3}}-(}{1-\sqrt{2+\sqrt{3}}}}{4\sqrt{3\left(2+\right.}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1+\sqrt{2+}}$$

$$= \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\,\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\log\left(1+\sqrt{2-\sqrt{3}}\right.$$

$$= \frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) - \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\,\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 55, normalized size = 0.15

$$-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1+2\#1^5}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1 + 2
*#1^5) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 46, normalized size = 0.13

method	result	size
default	$-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(\_R^{6}-\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	46
risch	$\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^{8}-\_Z^{4}+1\right)}\frac{\left(-\_R^{6}+\_R^{2}\right)\ln\left(x-\_R\right)}{2\_R^{7}-\_R^{3}}\right)}{4}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*sum((_R^6-_R^2)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)*x^2/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(263) = 526$.
time = 0.41, size = 719, normalized size = 2.03

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(576*x^2 +
96*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 576) - 1
/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(576*x^2 - 9
6*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 576) + 1/
96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(576*x^2 +
 48*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 576)
 - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(576*
x^2 - 48*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) +
 576) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(
3)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3
) + 8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqr
```

```
t(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2
) - 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(3)*s
qrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)
*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) +
1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(
2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 1/3*sqrt(6*x^2 + sqrt(6)*(2*sqrt(3)
*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqr
t(2))*sqrt(sqrt(3) + 2) + sqrt(3) - 2) + 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
+ 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) +
 2) + 1/3*sqrt(6*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqr
t(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(3) + 2) - sqrt(3)
+ 2)
```

Sympy [A]
time = 1.39, size = 27, normalized size = 0.08

$$-\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t \log\left(442368t^7 - 384t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(442368*_t**7 - 3
84*_t**3 + x)))
```

Giac [A]
time = 3.80, size = 253, normalized size = 0.71

$$-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x
*(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1)
```

Mupad [B]
time = 1.99, size = 248, normalized size = 0.70

$$\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}-\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}-\frac{2^{3/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}+\frac{2^{3/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{3/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(x^4 - 1))/(x^8 - x^4 + 1),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)) + (3^(1/2)*x*
(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/4)*1i)
/12 - (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4)*1i)/(2*(3^(1/2)*1i - 1)) - (3
^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*(3^(1/2)*1i - 1)))*(8 - 3^(1/2)*8i)^(1/
4))/12 + (2^(3/4)*3^(1/2)*atan((2^(3/4)*x)/(2*(3^(1/2)*1i + 1)^(3/4)) - (2^
(3/4)*3^(1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/
12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(3/4)) + (2^(
3/4)*3^(1/2)*x)/(2*(3^(1/2)*1i + 1)^(3/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.55 $\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx$

Optimal. Leaf size=50

$$-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}$$

```
[Out] -1/12*ln(1+x^4-3^(1/2)*x^2)*3^(1/2)+1/12*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1504, 1178, 642}

$$\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^4))/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*Log[1 - Sqrt[3]*x^2 + x^4]/Sqrt[3] + Log[1 + Sqrt[3]*x^2 + x^4]/(4*Sqr
t[3])
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1504

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x(1-x^4)}{1-x^4+x^8}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{3}+2x}{-1-\sqrt{3}\,x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}} - \frac{\text{Subst}\left(\int \frac{\sqrt{3}-2x}{-1+\sqrt{3}\,x-x^2}\,dx, x, x^2\right)}{4\sqrt{3}}$$
$$= -\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}} + \frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}$$

Mathematica [A]
time = 0.01, size = 44, normalized size = 0.88

$$\frac{-\log\left(-1+\sqrt{3}\,x^2-x^4\right)+\log\left(1+\sqrt{3}\,x^2+x^4\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^4))/(1 - x^4 + x^8),x]
[Out] (-Log[-1 + Sqrt[3]*x^2 - x^4] + Log[1 + Sqrt[3]*x^2 + x^4])/(4*Sqrt[3])
```

Maple [A]
time = 0.03, size = 39, normalized size = 0.78

method	result	size
default	$-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	39
risch	$-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)\sqrt{3}}{12}+\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)\sqrt{3}}{12}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/12*ln(1+x^4-x^2*3^(1/2))*3^(1/2)+1/12*ln(1+x^4+x^2*3^(1/2))*3^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
[Out] -integrate((x^4 - 1)*x/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.37, size = 41, normalized size = 0.82

$$\frac{1}{12}\sqrt{3}\log\left(\frac{x^8+5x^4+2\sqrt{3}\left(x^6+x^2\right)+1}{x^8-x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
[Out] 1/12*sqrt(3)*log((x^8 + 5*x^4 + 2*sqrt(3)*(x^6 + x^2) + 1)/(x^8 - x^4 + 1))
```

Sympy [A]
time = 0.04, size = 42, normalized size = 0.84

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}x^2+1\right)}{12}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}x^2+1\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**4+1)/(x**8-x**4+1),x)
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/12 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/12
```

Giac [A]
time = 3.75, size = 31, normalized size = 0.62

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{x^2-\sqrt{3}+\frac{1}{x^2}}{x^2+\sqrt{3}+\frac{1}{x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
[Out] -1/12*sqrt(3)*log((x^2 - sqrt(3) + 1/x^2)/(x^2 + sqrt(3) + 1/x^2))
```

Mupad [B]
time = 1.89, size = 20, normalized size = 0.40

$$\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}x^2}{x^4+1}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x*(x^4 - 1))/(x^8 - x^4 + 1),x)
[Out] (3^(1/2)*atanh((3^(1/2)*x^2)/(x^4 + 1)))/6
```

3.56 $\int \frac{1-x^4}{1-x^4+x^8}\,dx$

Optimal. Leaf size=355

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] 1/8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(
1+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/4*arctan((-2
*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(3/2*2^(1/2)-1/2*6^(
1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))/(
3/2*2^(1/2)-1/2*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(
1/2)+1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)+1/
6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))/(3/2*2^(1/2)+1/2*6^(1/2))-1/4*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/
2*6^(1/2)-1/2*2^(1/2)))/(3/2*2^(1/2)+1/2*6^(1/2))
```

Rubi [A]
time = 0.14, antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1435, 1183, 648, 632, 210, 642}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/Sqrt[3*(2 - Sqrt[3
])] + ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt[3*(2 + Sq
rt[3])]) + ArcTan[(Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(4*Sqrt[3*(2
 - Sqrt[3])]) - ArcTan[(Sqrt[2 + Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(4*Sqrt
[3*(2 + Sqrt[3])]) + (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2])/8 - (Sq
rt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sqrt[(2 + Sqrt
[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1435

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x^
(n/2))/Simp[d/e + q*x^(n/2) - x^n, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*
x^(n/2))/Simp[d/e - q*x^(n/2) - x^n, x], x], x]] /; FreeQ[{a, b, c, d, e},
x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && IGtQ[
n/2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1-x^4}{1-x^4+x^8}\,dx = -\frac{\int \frac{\sqrt{3}+2x^2}{-1-\sqrt{3}\,x^2-x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{\sqrt{3}-2x^2}{-1+\sqrt{3}\,x^2-x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}-\left(-2+\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2-\sqrt{3}\right)}+\left(-2+\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3\left(2+\sqrt{\right.}}}{1-\sqrt{2}}}{4\sqrt{3}}$$

$$= -\left(\frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx\right) - \frac{1}{8}\sqrt{\frac{1}{3}\left(7-4\sqrt{3}\right)}\int \frac{}{1+\sqrt{2}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right) - \frac{1}{8}\sqrt{\frac{2}{3}-\frac{1}{\sqrt{3}}}\log\left(1+\sqrt{2-\sqrt{3}}\,x\right.$$

$$= -\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right) + \frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{}}}{\sqrt{2-}}\right.$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 57, normalized size = 0.16

$$-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{-\log(x-\#1)+\log(x-\#1)\#1^4}{-\#1^3+2\#1^7}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(1 - x^4 + x^8),x]
```

```
[Out] -1/4*RootSum[1 - #1^4 + #1^8 & , (-Log[x - #1] + Log[x - #1]*#1^4)/(-#1^3 +
 2*#1^7) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.02, size = 44, normalized size = 0.12

method	result	size

default	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	44
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\left(-\_R^4+1\right)\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*sum((-_R^4+1)/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -integrate((x^4 - 1)/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 719 vs. $2(263) = 526$.
time = 0.39, size = 719, normalized size = 2.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(144*x^2 +
24*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 144) - 1
/48*sqrt(6)*(sqrt(3)*sqrt(2) - 2*sqrt(2))*sqrt(sqrt(3) + 2)*log(144*x^2 - 2
4*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 144) + 1/
96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(144*x^2 +
 12*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + 144)
 - 1/96*sqrt(6)*(sqrt(3)*sqrt(2) + 2*sqrt(2))*sqrt(-4*sqrt(3) + 8)*log(144*
x^2 - 12*sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) +
 144) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(
3)*sqrt(12*x^2 + sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3
) + 8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqr
t(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) - sqrt(3) - 2
```

```
) + 1/24*sqrt(6)*sqrt(2)*sqrt(-4*sqrt(3) + 8)*arctan(1/36*sqrt(6)*sqrt(3)*s
qrt(12*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) +
8) + 12)*(2*sqrt(3)*sqrt(2) + 3*sqrt(2))*sqrt(-4*sqrt(3) + 8) - 1/6*sqrt(6)
*(2*sqrt(3)*sqrt(2)*x + 3*sqrt(2)*x)*sqrt(-4*sqrt(3) + 8) + sqrt(3) + 2) -
1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(
2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 1/3*sqrt(6*x^2 + sqrt(6)*(2*sqrt(3)
*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqr
t(2))*sqrt(sqrt(3) + 2) + sqrt(3) - 2) - 1/12*sqrt(6)*sqrt(2)*sqrt(sqrt(3)
+ 2)*arctan(-1/3*sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqrt(3) +
 2) + 1/3*sqrt(6*x^2 - sqrt(6)*(2*sqrt(3)*sqrt(2)*x - 3*sqrt(2)*x)*sqrt(sqr
t(3) + 2) + 6)*(2*sqrt(3)*sqrt(2) - 3*sqrt(2))*sqrt(sqrt(3) + 2) - sqrt(3)
+ 2)
```

Sympy [A]
time = 1.50, size = 26, normalized size = 0.07

$$-\operatorname{RootSum}\left(5308416t^8 - 2304t^4 + 1, \left(t \mapsto t\log\left(9216t^5 - 8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(9216*_t**5 - 8*_
t + x)))
```

Giac [A]
time = 3.72, size = 253, normalized size = 0.71

$$\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt
(2))) + 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6
) + sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2))
/(sqrt(6) - sqrt(2))) + 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) -
sqrt(2))/(sqrt(6) - sqrt(2))) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*
(sqrt(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt
(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6) -
 sqrt(2)) + 1) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqrt
(2)) + 1)
```

Mupad [B]
time = 0.00, size = 208, normalized size = 0.59

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}+\frac{\sqrt{3}\,x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,1{}\mathrm{i}}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}-\frac{\sqrt{3}\,x}{{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}-\frac{2^{1/4}\,\sqrt{3}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{1/4}\,x\,1{}\mathrm{i}}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}+\frac{2^{1/4}\,\sqrt{3}\,x}{2\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^8 - x^4 + 1),x)
```

```
[Out] (2^(3/4)*3^(1/2)*atan((2^(1/4)*x)/(2*(3^(1/2)*1i + 1)^(1/4)) - (2^(1/4)*3^(
1/2)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4)*1i)/12 - (3^(
1/2)*atan((x*1i)/(8 - 3^(1/2)*8i)^(1/4) - (3^(1/2)*x)/(8 - 3^(1/2)*8i)^(1/4
))*(8 - 3^(1/2)*8i)^(1/4))/12 - (3^(1/2)*atan(x/(8 - 3^(1/2)*8i)^(1/4) + (3
^(1/2)*x*1i)/(8 - 3^(1/2)*8i)^(1/4))*(8 - 3^(1/2)*8i)^(1/4)*1i)/12 + (2^(3/
4)*3^(1/2)*atan((2^(1/4)*x*1i)/(2*(3^(1/2)*1i + 1)^(1/4)) + (2^(1/4)*3^(1/2
)*x)/(2*(3^(1/2)*1i + 1)^(1/4)))*(3^(1/2)*1i + 1)^(1/4))/12
```

3.57 $\int \frac{1-x^4}{x(1-x^4+x^8)}\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)$$

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)+1/12*arctan(1/3*(-2*x^4+1)*3^(1/2))*3^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1488, 814, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

```
[Out] ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) + Log[x] - Log[1 - x^4 + x^8]/8
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1-x^4}{x\left(1-x^4+x^8\right)}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1-x}{x\left(1-x+x^2\right)}\,dx,x,x^4\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{x}-\frac{x}{1-x+x^2}\right)dx,x,x^4\right)\\
&= \log(x)-\frac{1}{4}\text{Subst}\left(\int \frac{x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)-\frac{1}{8}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx,x,x^4\right)-\frac{1}{8}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx,x,x^4\right)\\
&= \log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x^4\right)\\
&= \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}+\log(x)-\frac{1}{8}\log\left(1-x^4+x^8\right)
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 44, normalized size = 1.07

$$\log(x)-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1^4}{-1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x*(1 - x^4 + x^8)),x]
```

```
[Out] Log[x] - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^4)/(-1 + 2*#1^4) & ]/4
```

Maple [A]
time = 0.03, size = 35, normalized size = 0.85

method	result	size
risch	$\ln(x)-\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{2\left(x^4-\frac{1}{2}\right)\sqrt{3}}{3}\right)}{12}$	33
default	$\ln(x)-\frac{\ln(x^8-x^4+1)}{8}-\frac{\sqrt{3}\,\arctan\left(\frac{(2x^4-1)\sqrt{3}}{3}\right)}{12}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)-1/8*ln(x^8-x^4+1)-1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2))
```

Maxima [A]
time = 0.58, size = 38, normalized size = 0.93

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

Fricas [A]
time = 0.34, size = 34, normalized size = 0.83

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + lo
g(x)
```

Sympy [A]
time = 0.07, size = 41, normalized size = 1.00

$$\log\left(x\right)-\frac{\log\left(x^8-x^4+1\right)}{8}-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}\,x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x/(x**8-x**4+1),x)
```

```
[Out] log(x) - log(x**8 - x**4 + 1)/8 - sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3
)/12
```

Giac [A]
time = 4.15, size = 38, normalized size = 0.93

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)-\frac{1}{8}\log\left(x^8-x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) - 1/8*log(x^8 - x^4 + 1) + 1/
4*log(x^4)
```

Mupad [B]
time = 1.89, size = 36, normalized size = 0.88

$$\ln(x)-\frac{\ln\left(x^8-x^4+1\right)}{8}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}}{3}-\frac{2\sqrt{3}\,x^4}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x*(x^8 - x^4 + 1)),x)
```

```
[Out] log(x) - log(x^8 - x^4 + 1)/8 + (3^(1/2)*atan(3^(1/2)/3 - (2*3^(1/2)*x^4)/3
))/12
```

3.58 $\int \frac{1-x^4}{x^2(1-x^4+x^8)}\,dx$

Optimal. Leaf size=280

$$-\frac{1}{x}+\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right.}{2\sqrt{6}}$$

```
[Out] -1/x+1/12*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)))*
6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2))
)*6^(1/2)+1/12*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/
2)))*6^(1/2)-1/12*arctan((2*x+1/2*6^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(
1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)+1/24*ln(1
+x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*6^(1/2)-1/24*ln(1+x^2-x*(1/2*6^(1/2)+1/2*
2^(1/2)))*6^(1/2)+1/24*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^(1/2)))*6^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1518, 1386, 1183, 648, 632, 210, 642}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}+\frac{\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}\,x+1\right)}{4\sqrt{6}}-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) + ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) +
 ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 - Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(Sqrt[2 +
Sqrt[3]] + 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*
x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log
[1 - Sqrt[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x +
 x^2]/(4*Sqrt[6])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1183

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1386

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, -Dist[1/(2*c*r), Int[x^(
m - 3*(n/2))*((q - r*x^(n/2))/(q - r*x^(n/2) + x^n)), x], x] + Dist[1/(2*c*
r), Int[x^(m - 3*(n/2))*((q + r*x^(n/2))/(q + r*x^(n/2) + x^n)), x], x]]] /
; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n/2, 0
] && IGtQ[m, 0] && GeQ[m, 3*(n/2)] && LtQ[m, 2*n] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^4}{x^2\left(1-x^4+x^8\right)}\,dx = -\frac{1}{x} - \int \frac{x^6}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{x} + \frac{\int \frac{1-\sqrt{3}\,x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}} - \frac{\int \frac{1+\sqrt{3}\,x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{x} - \frac{\int \frac{\sqrt{2-\sqrt{3}}-\left(1-\sqrt{3}\right)x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\int \frac{\sqrt{2-\sqrt{3}}+\left(1-\sqrt{3}\right)x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}\cdots}{1-\sqrt{2+\cdots}}}{4\sqrt{3\left(2\cdots\right.}}$$

$$= -\frac{1}{x} - \frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}\,x+x^2}\,dx\cdots}{4\sqrt{6}}$$

$$= -\frac{1}{x} - \frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\cdots\right.}{\cdots}$$

$$= -\frac{1}{x} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}\cdots}}{\sqrt{2+\cdots}}\right.}{2\sqrt{6}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 47, normalized size = 0.17

$$-\frac{1}{x} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1^3}{-1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^2*(1 - x^4 + x^8)),x]
```

```
[Out] -x^(-1) - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^3)/(-1 + 2*#1^4) & ]/
4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.03, size = 38, normalized size = 0.14

method	result	size
default	$-\frac{1}{x}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(9x\_R^{3}-3\_R^{2}+x^{2}\right)\right)}{4}$	38
risch	$-\frac{1}{x}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(9\_Z^{4}+1\right)}\_R\ln\left(-9x\_R^{3}-3\_R^{2}+x^{2}\right)\right)}{4}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^2/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/x-1/4*sum(_R*ln(9*_R^3*x-3*_R^2+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/x - integrate(x^6/(x^8 - x^4 + 1), x)
```

Fricas [A]
time = 0.36, size = 229, normalized size = 0.82

$$\frac{4\sqrt{3}\sqrt{2}\,x\arctan\left(-\frac{\sqrt{3}\sqrt{2}\,(x^3-x)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\,(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}\,x-2\right)}{3x^2-2}\right)+4\sqrt{3}\sqrt{2}\,x\arctan\left(-\frac{\sqrt{3}\sqrt{2}\,(x^3-x)-x^2-\sqrt{x^4-\sqrt{3}\sqrt{2}\,(x^3+x)+3x^2+1}\left(\sqrt{3}\sqrt{2}\,x+2\right)}{3x^2-2}\right)+\sqrt{3}\sqrt{2}\,x\log\left(36x^4+36\sqrt{3}\sqrt{2}\,(x^3+x)+108x^2+36\right)-\sqrt{3}\sqrt{2}\,x\log\left(36x^4-36\sqrt{3}\sqrt{2}\,(x^3+x)+108x^2+36\right)-24}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/24*(4*sqrt(3)*sqrt(2)*x*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x
^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2
 - 2)) + 4*sqrt(3)*sqrt(2)*x*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqr
t(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*
x^2 - 2)) + sqrt(3)*sqrt(2)*x*log(36*x^4 + 36*sqrt(3)*sqrt(2)*(x^3 + x) + 1
08*x^2 + 36) - sqrt(3)*sqrt(2)*x*log(36*x^4 - 36*sqrt(3)*sqrt(2)*(x^3 + x)
+ 108*x^2 + 36) - 24)/x
```

Sympy [A]
time = 0.09, size = 168, normalized size = 0.60

$$-\frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^3-4x^2+2\sqrt{6}\,x-3\right)\right)}{24}-\frac{\sqrt{6}\cdot\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}\,x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}\,x^3+4x^2+2\sqrt{6}\,x+3\right)\right)}{24}-\frac{\sqrt{6}\,\log\left(x^4-\sqrt{6}\,x^3+3x^2-\sqrt{6}\,x+1\right)}{24}+\frac{\sqrt{6}\,\log\left(x^4+\sqrt{6}\,x^3+3x^2+\sqrt{6}\,x+1\right)}{24}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**2/(x**8-x**4+1),x)
```

```
[Out] -sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt
(6)*x - 3))/24 - sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
 4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
 sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
 1)/24 - 1/x
```

Giac [A]
time = 4.35, size = 210, normalized size = 0.75

$$-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{24}\sqrt{6}\log\left(x^2+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{24}\sqrt{6}\log\left(x^2-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^2/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*
sqrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) - 1/12*sqrt(6
)*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) - 1/12*sqrt(6)*arct
an((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 +
1/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sq
rt(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*
sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/x
```

Mupad [B]
time = 1.86, size = 58, normalized size = 0.21

$$-\frac{1}{x}+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}+\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}-\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}-\frac{1}{12}\mathrm{i}\right)+\sqrt{6}\,\mathrm{atan}\left(\frac{\sqrt{6}\,x\,\left(\frac{1}{3}-\frac{1}{3}\mathrm{i}\right)}{\frac{2\,x^2}{3}+\frac{2}{3}\mathrm{i}}\right)\,\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^2*(x^8 - x^4 + 1)),x)
```

```
[Out] 6^(1/2)*atan((6^(1/2)*x*(1/3 + 1i/3))/((2*x^2)/3 - 2i/3))*(1/12 - 1i/12) +
6^(1/2)*atan((6^(1/2)*x*(1/3 - 1i/3))/((2*x^2)/3 + 2i/3))*(1/12 + 1i/12) -
1/x
```

3.59 $\int \frac{1-x^4}{x^3(1-x^4+x^8)}\,dx$

Optimal. Leaf size=89

$$-\frac{1}{2x^2}+\frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right)-\frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}+\frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}$$

```
[Out] -1/2/x^2-1/4*arctan(2*x^2-3^(1/2))-1/4*arctan(2*x^2+3^(1/2))-1/24*ln(1+x^4-
3^(1/2)*x^2)*3^(1/2)+1/24*ln(1+x^4+3^(1/2)*x^2)*3^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1504, 1295, 1141, 1175, 632, 210, 1178, 642}

$$\frac{1}{4}\text{ArcTan}\left(\sqrt{3}-2x^2\right)-\frac{1}{4}\text{ArcTan}\left(2x^2+\sqrt{3}\right)-\frac{1}{2x^2}-\frac{\log\left(x^4-\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}+\frac{\log\left(x^4+\sqrt{3}\,x^2+1\right)}{8\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
```

```
[Out] -1/2*1/x^2 + ArcTan[Sqrt[3] - 2*x^2]/4 - ArcTan[Sqrt[3] + 2*x^2]/4 - Log[1
- Sqrt[3]*x^2 + x^4]/(8*Sqrt[3]) + Log[1 + Sqrt[3]*x^2 + x^4]/(8*Sqrt[3])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
```

```
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1504

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_)*((d_) + (e
_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subs
t[Int[x^((m + 1)/k - 1)*(d + e*x^(n/k))^q*(a + b*x^(n/k) + c*x^(2*(n/k)))^p
, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2,
2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1-x^4}{x^3(1-x^4+x^8)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{x^2(1-x^2+x^4)}\,dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} - \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{1}{4}\text{Subst}\left(\int \frac{1-x^2}{1-x^2+x^4}\,dx, x, x^2\right) - \frac{1}{4}\text{Subst}\left(\int \frac{1+x^2}{1-x^2+x^4}\,dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1-\sqrt{3}\,x+x^2}\,dx, x, x^2\right) - \frac{1}{8}\text{Subst}\left(\int \frac{1}{1+\sqrt{3}\,x+x^2}\,dx, x\right.$$
$$= -\frac{1}{2x^2} - \frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}} + \frac{\log\left(1+\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{-1-x^2}\right.$$
$$= -\frac{1}{2x^2} + \frac{1}{4}\tan^{-1}\left(\sqrt{3}-2x^2\right) - \frac{1}{4}\tan^{-1}\left(\sqrt{3}+2x^2\right) - \frac{\log\left(1-\sqrt{3}\,x^2+x^4\right)}{8\sqrt{3}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.01, size = 49, normalized size = 0.55

$$-\frac{1}{2x^2} - \frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&, \frac{\log(x-\#1)\#1^2}{-1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^3*(1 - x^4 + x^8)),x]
[Out] -1/2*1/x^2 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1^2)/(-1 + 2*#1^4) &
 ]/4
```

Maple [A]
time = 0.04, size = 82, normalized size = 0.92

method	result
risch	$-\frac{1}{2x^2} + \frac{\left(\sum_{\_R=\text{RootOf}\left(9\_Z^4+3\_Z^2+1\right)} \_R\ln\left(-6\_R^3+x^2-\_R\right)\right)}{4}$
default	$-\frac{1}{2x^2} + \frac{\sqrt{3}\left(-\frac{\ln\left(1+x^4-x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2-\sqrt{3}\right)\right)}{12} + \frac{\sqrt{3}\left(\frac{\ln\left(1+x^4+x^2\sqrt{3}\right)}{2}-\sqrt{3}\arctan\left(2x^2+\sqrt{3}\right.\right.}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^3/(x^8-x^4+1),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2+1/12*3^(1/2)*(-1/2*ln(1+x^4-x^2*3^(1/2))-3^(1/2)*arctan(2*x^2-3^(1
/2)))+1/12*3^(1/2)*(1/2*ln(1+x^4+x^2*3^(1/2))-3^(1/2)*arctan(2*x^2+3^(1/2))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] -1/2/x^2 - integrate(x^5/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(69) = 138.
time = 0.37, size = 189, normalized size = 2.12

$$\frac{4\sqrt{6}\sqrt{3}\sqrt{2}\,x^2\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}\,x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2\,x^4+\sqrt{6}\sqrt{2}\,x^2+2}-\sqrt{3}\right)+4\sqrt{6}\sqrt{3}\sqrt{2}\,x^2\arctan\left(-\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2}\,x^2+\frac{1}{3}\sqrt{6}\sqrt{3}\sqrt{2\,x^4-\sqrt{6}\sqrt{2}\,x^2+2}+\sqrt{3}\right)+\sqrt{6}\sqrt{2}\,x^2\log\left(36\,x^4+18\sqrt{6}\sqrt{2}\,x^2+36\right)-\sqrt{6}\sqrt{2}\,x^2\log\left(36\,x^4-18\sqrt{6}\sqrt{2}\,x^2+36\right)-24}{48\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/48*(4*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2
 + 1/3*sqrt(6)*sqrt(3)*sqrt(2*x^4 + sqrt(6)*sqrt(2)*x^2 + 2) - sqrt(3)) + 4
*sqrt(6)*sqrt(3)*sqrt(2)*x^2*arctan(-1/3*sqrt(6)*sqrt(3)*sqrt(2)*x^2 + 1/3*
sqrt(6)*sqrt(3)*sqrt(2*x^4 - sqrt(6)*sqrt(2)*x^2 + 2) + sqrt(3)) + sqrt(6)*
sqrt(2)*x^2*log(36*x^4 + 18*sqrt(6)*sqrt(2)*x^2 + 36) - sqrt(6)*sqrt(2)*x^2
*log(36*x^4 - 18*sqrt(6)*sqrt(2)*x^2 + 36) - 24)/x^2
```

Sympy [A]
time = 0.09, size = 76, normalized size = 0.85

$$-\frac{\sqrt{3}\log\left(x^4-\sqrt{3}\,x^2+1\right)}{24}+\frac{\sqrt{3}\log\left(x^4+\sqrt{3}\,x^2+1\right)}{24}-\frac{\operatorname{atan}\left(2x^2-\sqrt{3}\right)}{4}-\frac{\operatorname{atan}\left(2x^2+\sqrt{3}\right)}{4}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**3/(x**8-x**4+1),x)
```

```
[Out] -sqrt(3)*log(x**4 - sqrt(3)*x**2 + 1)/24 + sqrt(3)*log(x**4 + sqrt(3)*x**2
+ 1)/24 - atan(2*x**2 - sqrt(3))/4 - atan(2*x**2 + sqrt(3))/4 - 1/(2*x**2)
```

Giac [A]
time = 3.56, size = 81, normalized size = 0.91

$$-\frac{1}{24}\sqrt{3}\,x^4\log\left(x^4+\sqrt{3}\,x^2+1\right)+\frac{1}{24}\sqrt{3}\,x^4\log\left(x^4-\sqrt{3}\,x^2+1\right)-\frac{1}{4}\,x^4\arctan\left(2\,x^2+\sqrt{3}\right)-\frac{1}{4}\,x^4\arctan\left(2\,x^2-\sqrt{3}\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^3/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(3)*x^4*log(x^4 + sqrt(3)*x^2 + 1) + 1/24*sqrt(3)*x^4*log(x^4 - s
qrt(3)*x^2 + 1) - 1/4*x^4*arctan(2*x^2 + sqrt(3)) - 1/4*x^4*arctan(2*x^2 -
sqrt(3)) - 1/2/x^2
```

Mupad [B]
time = 0.10, size = 56, normalized size = 0.63

$$\operatorname{atan}\left(\frac{2\,x^2}{-1+\sqrt{3}\,\,1\mathrm{i}}\right)\,\left(\frac{1}{4}+\frac{\sqrt{3}\,\,1\mathrm{i}}{12}\right)+\operatorname{atan}\left(\frac{2\,x^2}{1+\sqrt{3}\,\,1\mathrm{i}}\right)\,\left(-\frac{1}{4}+\frac{\sqrt{3}\,\,1\mathrm{i}}{12}\right)-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^3*(x^8 - x^4 + 1)),x)
```

```
[Out] atan((2*x^2)/(3^(1/2)*1i - 1))*((3^(1/2)*1i)/12 + 1/4) + atan((2*x^2)/(3^(1
/2)*1i + 1))*((3^(1/2)*1i)/12 - 1/4) - 1/(2*x^2)
```

3.60 $\int \frac{1-x^4}{x^4(1-x^4+x^8)} \, dx$

Optimal. Leaf size=370

$$-\frac{1}{3x^3}-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{}$$

```
[Out] -1/3/x^3-1/4*arctan((-2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*6^(1/2)+1/2*2^(1/2)
))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((2*x+1/2*6^(1/2)-1/2*2^(1/2))/(1/2*
6^(1/2)+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))-1/8*ln(1+x^2-x*(1/2*6^(1/2)
+1/2*2^(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/8*ln(1+x^2+x*(1/2*6^(1/2)+1/2*2^
(1/2)))*(1/2*2^(1/2)-1/6*6^(1/2))+1/4*arctan((-2*x+1/2*6^(1/2)+1/2*2^(1/2))
/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/4*arctan((2*x+1/2*6
^(1/2)+1/2*2^(1/2))/(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))+1/
8*ln(1+x^2-x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))-1/8*ln(1+
x^2+x*(1/2*6^(1/2)-1/2*2^(1/2)))*(1/2*2^(1/2)+1/6*6^(1/2))
```

Rubi [A]
time = 0.18, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1518, 12, 1387, 1141, 1175, 632, 210, 1178, 642}

$$-\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\text{ArcTan}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\text{ArcTan}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)+\frac{1}{4}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\right)-\frac{1}{4}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\text{ArcTan}\left(\frac{2x+\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\right)-\frac{1}{3x^3}+\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2+\sqrt{3}\right)}\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)-\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)+\frac{1}{8}\sqrt{\frac{1}{3}\left(2-\sqrt{3}\right)}\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
```

```
[Out] -1/3*1/x^3 - (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2
 + Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]] - 2*x)/S
qrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 - Sqrt[3])/3]*ArcTan[(Sqrt[2 - Sqrt[3]] + 2
*x)/Sqrt[2 + Sqrt[3]]])/4 - (Sqrt[(2 + Sqrt[3])/3]*ArcTan[(Sqrt[2 + Sqrt[3]
] + 2*x)/Sqrt[2 - Sqrt[3]]])/4 + (Sqrt[(2 + Sqrt[3])/3]*Log[1 - Sqrt[2 - Sq
rt[3]]*x + x^2])/8 - (Sqrt[(2 + Sqrt[3])/3]*Log[1 + Sqrt[2 - Sqrt[3]]*x + x
^2])/8 - (Sqrt[(2 - Sqrt[3])/3]*Log[1 - Sqrt[2 + Sqrt[3]]*x + x^2])/8 + (Sq
rt[(2 - Sqrt[3])/3]*Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2])/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1141

```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Dist[1/2, Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Dist[1/2, I
nt[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b^2
 - 4*a*c, 0] && PosQ[a*c]
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e
+ q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^
2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c
*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

Rule 1387

```
Int[(x_)^(m_.)/((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> W
ith[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*r), Int[x^(m
 - n/2)/(q - r*x^(n/2) + x^n), x], x] - Dist[1/(2*c*r), Int[x^(m - n/2)/(q
+ r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && IGtQ[m, 0] && GeQ[m, n/2] && LtQ[m, 3*(n
/2)] && NegQ[b^2 - 4*a*c]
```

Rule 1518

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^n + c*x^
(2*n))^(p + 1)/(a*f*(m + 1))), x] + Dist[1/(a*f^n*(m + 1)), Int[(f*x)^(m +
n)*(a + b*x^n + c*x^(2*n))^p*Simp[a*e*(m + 1) - b*d*(m + n*(p + 1) + 1) - c
*d*(m + 2*n*(p + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x]
 && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IGtQ[n, 0] && LtQ[m, -1] && Inte
gerQ[p]
```

Rubi steps

$$\int \frac{1-x^4}{x^4\left(1-x^4+x^8\right)}\,dx = -\frac{1}{3x^3}-\frac{1}{3}\int \frac{3x^4}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}-\int \frac{x^4}{1-x^4+x^8}\,dx$$

$$= -\frac{1}{3x^3}-\frac{\int \frac{x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}+\frac{\int \frac{x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\int \frac{1-x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1+x^2}{1-\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}}-\frac{\int \frac{1-x^2}{1+\sqrt{3}\,x^2+x^4}\,dx}{4\sqrt{3}}+\frac{\int \frac{1+x^2}{1+\sqrt{3}\,x^2+x^4}}{4\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}+\frac{\int \frac{1}{1+\sqrt{2-\sqrt{3}}\,x+x^2}\,dx}{8\sqrt{3}}-\frac{\int \frac{1}{1-\sqrt{2+\sqrt{3}}\,x+x^2}}{8\sqrt{3}}$$

$$= -\frac{1}{3x^3}+\frac{\log\left(1-\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1+\sqrt{2-\sqrt{3}}\,x+x^2\right)}{8\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\log\left(1-\right.}{8\sqrt{}}$$

$$= -\frac{1}{3x^3}-\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{4\sqrt{3\left(2+\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{4\sqrt{3\left(2-\sqrt{3}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{2-}}{\sqrt{2+}}\right.}{4\sqrt{3\left(2+\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

time = 0.01, size = 47, normalized size = 0.13

$$-\frac{1}{3x^3}-\frac{1}{4}\text{RootSum}\left[1-\#1^4+\#1^8\&,\frac{\log(x-\#1)\#1}{-1+2\#1^4}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4)/(x^4*(1 - x^4 + x^8)),x]
[Out] -1/3*1/x^3 - RootSum[1 - #1^4 + #1^8 & , (Log[x - #1]*#1)/(-1 + 2*#1^4) & ]
/4
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.04, size = 46, normalized size = 0.12

method	result	size
risch	$-\frac{1}{3x^3}+\frac{\left(\sum_{\_R=\text{RootOf}\left(81\_Z^8-9\_Z^4+1\right)}\_R\ln\left(18\_R^5-\_R+x\right)\right)}{4}$	38
default	$-\frac{1}{3x^3}-\frac{\left(\sum_{\_R=\text{RootOf}\left(\_Z^8-\_Z^4+1\right)}\frac{\_R^4\ln\left(x-\_R\right)}{2\_R^7-\_R^3}\right)}{4}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+1)/x^4/(x^8-x^4+1),x,method=_RETURNVERBOSE)
[Out] -1/3/x^3-1/4*sum(_R^4/(2*_R^7-_R^3)*ln(x-_R),_R=RootOf(_Z^8-_Z^4+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="maxima")
[Out] -1/3/x^3 - integrate(x^4/(x^8 - x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(260) = 520.
time = 0.40, size = 622, normalized size = 1.68

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] 1/96*(8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3)*s
qrt(2)*x*sqrt(sqrt(3) + 2) + 1/18*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(6*sqrt(6)*sq
rt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 36*x^2 + 36)*sqrt(sqrt(3) + 2) - sqrt(3
) - 2) + 8*sqrt(6)*sqrt(2)*x^3*sqrt(sqrt(3) + 2)*arctan(-1/3*sqrt(6)*sqrt(3
)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 1/18*sqrt(6)*sqrt(3)*sqrt(2)*sqrt(-6*sqrt(6
)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 36*x^2 + 36)*sqrt(sqrt(3) + 2) + sq
rt(3) + 2) - 4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arctan(-1/6*sqrt(6)
*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/36*sqrt(6)*sqrt(3)*sqrt(2)*sqrt
(3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36*x^2 + 36)*sqrt(-4*sq
rt(3) + 8) + sqrt(3) - 2) - 4*sqrt(6)*sqrt(2)*x^3*sqrt(-4*sqrt(3) + 8)*arct
an(-1/6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 1/36*sqrt(6)*sqrt(
3)*sqrt(2)*sqrt(-3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36*x^2
+ 36)*sqrt(-4*sqrt(3) + 8) - sqrt(3) + 2) - 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3
- 2*sqrt(2)*x^3)*sqrt(sqrt(3) + 2)*log(6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqr
t(3) + 2) + 36*x^2 + 36) + 2*sqrt(6)*(sqrt(3)*sqrt(2)*x^3 - 2*sqrt(2)*x^3)*
sqrt(sqrt(3) + 2)*log(-6*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(sqrt(3) + 2) + 36*x
^2 + 36) - sqrt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) +
8)*log(3*sqrt(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36*x^2 + 36) + sq
rt(6)*(sqrt(3)*sqrt(2)*x^3 + 2*sqrt(2)*x^3)*sqrt(-4*sqrt(3) + 8)*log(-3*sqr
t(6)*sqrt(3)*sqrt(2)*x*sqrt(-4*sqrt(3) + 8) + 36*x^2 + 36) - 32)/x^3
```

Sympy [**A**]
time = 1.42, size = 32, normalized size = 0.09

$$- \operatorname{RootSum} \left(5308416t^{8} - 2304t^{4} + 1, \left(t \mapsto t \log \left(-18432t^{5} + 4t + x \right) \right)\right) - \frac{1}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+1)/x**4/(x**8-x**4+1),x)
```

```
[Out] -RootSum(5308416*_t**8 - 2304*_t**4 + 1, Lambda(_t, _t*log(-18432*_t**5 + 4
*_t + x))) - 1/(3*x**3)
```

Giac [**A**]
time = 2.96, size = 258, normalized size = 0.70

$$-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{4x-\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)-\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^{2}+\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(x^{2}-\frac{1}{2}x\left(\sqrt{6}+\sqrt{2}\right)+1\right)-\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^{2}+\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)+\frac{1}{48}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(x^{2}-\frac{1}{2}x\left(\sqrt{6}-\sqrt{2}\right)+1\right)-\frac{1}{3x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+1)/x^4/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] -1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqr
t(2))) - 1/24*(sqrt(6) - 3*sqrt(2))*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(
6) + sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x + sqrt(6) + sqrt(2)
)/(sqrt(6) - sqrt(2))) - 1/24*(sqrt(6) + 3*sqrt(2))*arctan((4*x - sqrt(6) -
 sqrt(2))/(sqrt(6) - sqrt(2))) - 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 + 1/2*x
```

```
*(sqrt(6) + sqrt(2)) + 1) + 1/48*(sqrt(6) - 3*sqrt(2))*log(x^2 - 1/2*x*(sqr
t(6) + sqrt(2)) + 1) - 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 + 1/2*x*(sqrt(6)
- sqrt(2)) + 1) + 1/48*(sqrt(6) + 3*sqrt(2))*log(x^2 - 1/2*x*(sqrt(6) - sqr
t(2)) + 1) - 1/3/x^3
```

Mupad [B]
time = 0.07, size = 479, normalized size = 1.29

$$-\frac{1}{3\,x^3}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8{}\mathrm{i}}\,1{}\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8{}\mathrm{i}}}{4}\right)}+\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8{}\mathrm{i}}\,1{}\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8{}\mathrm{i}}}{4}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8{}\mathrm{i}}\,1{}\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8{}\mathrm{i}}}{4}\right)}-\frac{\sqrt{3}\,x\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{2\,\left(\frac{\sqrt{3}\,\sqrt{8-\sqrt{3}\,8{}\mathrm{i}}\,1{}\mathrm{i}}{4}+\frac{\sqrt{8-\sqrt{3}\,8{}\mathrm{i}}}{4}\right)}\right)\,{\left(8-\sqrt{3}\,8{}\mathrm{i}\right)}^{1/4}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{2\,\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}-\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{12}-\frac{2^{3/4}\,\sqrt{3}\,\mathrm{atan}\left(\frac{2^{3/4}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}\,1{}\mathrm{i}}{2\,\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}+\frac{2^{3/4}\,\sqrt{3}\,x\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{2\,\left(\frac{\sqrt{2}\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}}{2}-\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{1+\sqrt{3}\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}\right)\,{\left(1+\sqrt{3}\,1{}\mathrm{i}\right)}^{1/4}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^4 - 1)/(x^4*(x^8 - x^4 + 1)),x)
```

```
[Out] (3^(1/2)*atan((x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2
)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)) + (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4)*1i
)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(
8 - 3^(1/2)*8i)^(1/4)*1i)/12 - 1/(3*x^3) + (3^(1/2)*atan((x*(8 - 3^(1/2)*8i
)^(1/4)*1i)/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1
/2)/4)) - (3^(1/2)*x*(8 - 3^(1/2)*8i)^(1/4))/(2*((3^(1/2)*(8 - 3^(1/2)*8i)^
(1/2)*1i)/4 + (8 - 3^(1/2)*8i)^(1/2)/4)))*(8 - 3^(1/2)*8i)^(1/4))/12 - (2^(
3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/2)*
1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)) - (2^(3/
4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*((2^(1/2)*(3^(1/2)*1i + 1)^(1/2)
)/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(1/2)*1i + 1)^(1/
4)*1i)/12 - (2^(3/4)*3^(1/2)*atan((2^(3/4)*x*(3^(1/2)*1i + 1)^(1/4)*1i)/(2*
((2^(1/2)*(3^(1/2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/
2)*1i)/2)) + (2^(3/4)*3^(1/2)*x*(3^(1/2)*1i + 1)^(1/4))/(2*((2^(1/2)*(3^(1/
2)*1i + 1)^(1/2))/2 - (2^(1/2)*3^(1/2)*(3^(1/2)*1i + 1)^(1/2)*1i)/2)))*(3^(
1/2)*1i + 1)^(1/4))/12
```

3.61 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=280

$$\frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}+\frac{\left(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4ab^2c^2e-2a^2c^3e\right)\tanh}{a^4\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}$$

```
[Out] (a^2*d^2+b^2*e^2+a*e*(b*d-c*e))*x/a^3/e^3-1/2*(a*d+b*e)*x^2/a^2/e^2+1/3*x^3
/a/e-d^5*ln(e*x+d)/e^4/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c^2*d-3*a*b^2*c*d+2*a*b
*c^2*e+b^4*d-b^3*c*e)*ln(a*x^2+b*x+c)/a^4/(a*d^2-e*(b*d-c*e))+(5*a^2*b*c^2*
d-2*a^2*c^3*e-5*a*b^3*c*d+4*a*b^2*c^2*e+b^5*d-b^4*c*e)*arctanh((2*a*x+b)/(-
4*a*c+b^2)^(1/2))/a^4/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 1642, 648, 632, 212, 642}

$$-\frac{x^2(ad+be)}{2a^2e^2}+\frac{\left(a^2c^2d-3ab^2cd+2abc^2e+b^4d-b^3ce\right)\log\left(ax^2+bx+c\right)}{2a^4\left(ad^2-e(bd-ce)\right)}+\frac{\left(5a^2bc^2d-2a^2c^3e-5ab^3cd+4ab^2c^2e+b^5d-b^4ce\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^4\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}+\frac{x\left(a^2d^2+ae(bd-ce)+b^2e^2\right)}{a^3e^3}-\frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}+\frac{x^3}{3ae}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + b^2*e^2 + a*e*(b*d - c*e))*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(2*
a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e +
4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^4*S
qrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^5*Log[d + e*x])/(e^4*(a*d^2
- e*(b*d - c*e))) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2
*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^5}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{a^2d^2+b^2e^2+ae(bd-ce)}{a^3e^3} - \frac{(ad+be)x}{a^2e^2} + \frac{x^2}{ae} + \frac{d^5}{e^3\left(-ad^2+e(bd-ce)\right)}\right.$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} - \frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} - \frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} - \frac{d^5\log(d+ex)}{e^4\left(ad^2-e(bd-ce)\right)}$$

$$= \frac{\left(a^2d^2+b^2e^2+ae(bd-ce)\right)x}{a^3e^3} - \frac{(ad+be)x^2}{2a^2e^2} + \frac{x^3}{3ae} + \frac{\left(b^5d-5ab^3cd+5a^2b\right.}{}$$

Mathematica [A]
time = 0.16, size = 283, normalized size = 1.01

$$\frac{(a^2d^2+abde+b^2e^2-ace^2)\,x}{a^3e^3}-\frac{(ad+be)x^2}{2a^2e^2}+\frac{x^3}{3ae}+\frac{(b^5d-5ab^3cd+5a^2bc^2d-b^4ce+4ab^2c^2e-2a^2c^3e)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^4\sqrt{-b^2+4ac}\,(-ad^2+bde-ce^2)}-\frac{d^5\log(d+ex)}{e^4\,(ad^2-bde+ce^2)}+\frac{(b^4d-3ab^2cd+a^2c^2d-b^3ce+2abc^2e)\log(c+bx+ax^2)}{2a^4\,(ad^2-bde+ce^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] ((a^2*d^2 + a*b*d*e + b^2*e^2 - a*c*e^2)*x)/(a^3*e^3) - ((a*d + b*e)*x^2)/(
2*a^2*e^2) + x^3/(3*a*e) + ((b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e
+ 4*a*b^2*c^2*e - 2*a^2*c^3*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^4
*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^5*Log[d + e*x])/(e^4*(
a*d^2 - b*d*e + c*e^2)) + ((b^4*d - 3*a*b^2*c*d + a^2*c^2*d - b^3*c*e + 2*a
*b*c^2*e)*Log[c + b*x + a*x^2])/(2*a^4*(a*d^2 - b*d*e + c*e^2))
```

Maple [A]
time = 0.24, size = 286, normalized size = 1.02

method	result
default	$\frac{\frac{1}{3}a^2e^2x^3-\frac{1}{2}a^2de\,x^2-\frac{1}{2}ab\,e^2x^2+a^2d^2x+abdex-ac\,e^2x+e^2b^2x}{e^3a^3}-\frac{d^5\ln(ex+d)}{e^4(a\,d^2-deb+c\,e^2)}+\frac{\frac{\left(a^2c^2d-3a\,b^2cd+2ab\,c^2e+b^4d-b^3ce\right)\ln\left(a\,x^2+b\right.}{2a}}{}$
risch	$\frac{x^3}{3ae}-\frac{d\,x^2}{2e^2a}-\frac{b\,x^2}{2e\,a^2}+\frac{d^2x}{e^3a}+\frac{bdx}{e^2a^2}-\frac{cx}{e\,a^2}+\frac{b^2x}{e\,a^3}-\frac{d^5\ln(ex+d)}{e^4(a\,d^2-deb+c\,e^2)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c\,d^2-b^2d^2a^2-4a^2bcde+4e^2c^2a^2+\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^3/a^3*(1/3*a^2*e^2*x^3-1/2*a^2*d*e*x^2-1/2*a*b*e^2*x^2+a^2*d^2*x+a*b*d*
e*x-a*c*e^2*x+e^2*b^2*x)-1/e^4*d^5/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/(a*d^2-b
*d*e+c*e^2)/a^3*(1/2*(a^2*c^2*d-3*a*b^2*c*d+2*a*b*c^2*e+b^4*d-b^3*c*e)/a*ln
(a*x^2+b*x+c)+2*(-2*a*b*c^2*d+a*c^3*e+b^3*c*d-b^2*c^2*e-1/2*(a^2*c^2*d-3*a*
b^2*c*d+2*a*b*c^2*e+b^4*d-b^3*c*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/
(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 18.39, size = 1017, normalized size = 3.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a^4*b^2 - 4*a^5*c)*d^2*x^3*e^3 - 3*(a^4*b^2 - 4*a^5*c)*d^3*x^2*e^2
 + 6*(a^4*b^2 - 4*a^5*c)*d^4*x*e - 6*(a^4*b^2 - 4*a^5*c)*d^5*log(x*e + d) -
 3*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^
3)*e^5)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2
 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + (2*(a^3*b^2*c - 4*a^4*c^2)*x^3
- 3*(a^2*b^3*c - 4*a^3*b*c^2)*x^2 + 6*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)
*x)*e^5 - (2*(a^3*b^3 - 4*a^4*b*c)*d*x^3 - 3*(a^2*b^4 - 5*a^3*b^2*c + 4*a^4
*c^2)*d*x^2 + 6*(a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*x)*e^4 + 3*((b^6 - 7*
a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3*c^2 + 8*a^2*
b*c^3)*e^5)*log(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4 - (a^4*b^3 -
 4*a^5*b*c)*d*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6), 1/6*(2*(a^4*b^2 - 4*a^5*c
)*d^2*x^3*e^3 - 3*(a^4*b^2 - 4*a^5*c)*d^3*x^2*e^2 + 6*(a^4*b^2 - 4*a^5*c)*d
^4*x*e - 6*(a^4*b^2 - 4*a^5*c)*d^5*log(x*e + d) + 6*((b^5 - 5*a*b^3*c + 5*a
^2*b*c^2)*d*e^4 - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e^5)*sqrt(-b^2 + 4*a*c)
*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + (2*(a^3*b^2*c - 4*
a^4*c^2)*x^3 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*x^2 + 6*(a*b^4*c - 5*a^2*b^2*c^2
 + 4*a^3*c^3)*x)*e^5 - (2*(a^3*b^3 - 4*a^4*b*c)*d*x^3 - 3*(a^2*b^4 - 5*a^3*
b^2*c + 4*a^4*c^2)*d*x^2 + 6*(a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d*x)*e^4 +
 3*((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d*e^4 - (b^5*c - 6*a*b^3
*c^2 + 8*a^2*b*c^3)*e^5)*log(a*x^2 + b*x + c))/((a^5*b^2 - 4*a^6*c)*d^2*e^4
 - (a^4*b^3 - 4*a^5*b*c)*d*e^5 + (a^4*b^2*c - 4*a^5*c^2)*e^6)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.42, size = 295, normalized size = 1.05

$$-\frac{d^{5} \log\left(|xe + d|\right)}{ad^{2}e^{4} - bde^{5} + ce^{6}} + \frac{\left(b^{4}d - 3\,ab^{2}cd + a^{2}c^{2}d - b^{3}ce + 2\,abc^{2}e\right)\log\left(ax^{2} + bx + c\right)}{2\left(a^{5}d^{2} - a^{4}bde + a^{4}ce^{2}\right)} - \frac{\left(b^{5}d - 5\,ab^{3}cd + 5\,a^{2}bc^{2}d - b^{4}ce + 4\,ab^{2}c^{2}e - 2\,a^{2}c^{3}e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^{2} + 4\,ac}}\right)}{\left(a^{5}d^{2} - a^{4}bde + a^{4}ce^{2}\right)\sqrt{-b^{2} + 4\,ac}} + \frac{\left(2\,a^{2}x^{3}e^{2} - 3\,a^{2}dx^{2}e + 6\,a^{2}d^{2}x - 3\,abx^{2}e^{2} + 6\,abdxe + 6\,b^{2}xe^{2} - 6\,acxe^{2}\right)e^{(-3)}}{6\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] -d^5*log(abs(x*e + d))/(a*d^2*e^4 - b*d*e^5 + c*e^6) + 1/2*(b^4*d - 3*a*b^2
*c*d + a^2*c^2*d - b^3*c*e + 2*a*b*c^2*e)*log(a*x^2 + b*x + c)/(a^5*d^2 - a
^4*b*d*e + a^4*c*e^2) - (b^5*d - 5*a*b^3*c*d + 5*a^2*b*c^2*d - b^4*c*e + 4*
a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^5*d^2
 - a^4*b*d*e + a^4*c*e^2)*sqrt(-b^2 + 4*a*c)) + 1/6*(2*a^2*x^3*e^2 - 3*a^2*
d*x^2*e + 6*a^2*d^2*x - 3*a*b*x^2*e^2 + 6*a*b*d*x*e + 6*b^2*x*e^2 - 6*a*c*x
*e^2)*e^(-3)/a^3
```

Mupad [B]
time = 6.21, size = 2490, normalized size = 8.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(4*a^5*c*d^7 - a^4*b^2*d^7 + b^3*c^3*e^7 - b^6*d^3*e^4 - 6*a^2*c^4*d*e^
6 - 3*b^4*c^2*d*e^6 + 3*b^5*c*d^2*e^5 - 2*a^2*c^4*e^7*x - b^2*c^3*e^7*(b^2
- 4*a*c)^(1/2) + b^5*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 2*a^3*c^3*d^3*e^4 - 4*a^
4*c^2*d^5*e^2 - 3*a*b*c^4*e^7 + a^4*b*d^7*(b^2 - 4*a*c)^(1/2) + a*c^4*e^7*(
b^2 - 4*a*c)^(1/2) + 2*a^5*d^7*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d^2*e^5*(b
^2 - 4*a*c)^(1/2) + 8*a^5*c*d^6*e*x - 9*a^2*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e
*(b^2 - 4*a*c)^(1/2) + 12*a*b^2*c^3*d*e^6 + 6*a*b^4*c*d^3*e^4 + a*b^2*c^3*e
^7*x - a*b^5*d^3*e^4*x - 2*a^4*b^2*d^6*e*x + 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^
(1/2) - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 15*a*b^3*c^2*d^2*e^5 + 15*a^2
*b*c^3*d^2*e^5 + a^3*b^2*c*d^5*e^2 + a^3*b^3*d^5*e^2*x + 6*a^3*c^3*d^2*e^5*
x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^(
1/2) + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c
)^(1/2) - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 5*a^2*b^3*c*d^3*e^4*x - 5
*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c^
2*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + a^3
*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) - 12*a^2*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d
*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^3*e^7*x*(b^2 - 4*a*c)^(1/2) - 2*a^4*b*d^6*
e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c^2*d*e^6*x + 3*a*b^4*c*d^2*e^5*x + 9*a^2
*b*c^3*d*e^6*x - 4*a^4*b*c*d^5*e^2*x + 3*a*b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^(1
/2) - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c^2*d^2*e^5*x*(b^2
- 4*a*c)^(1/2) - 3*a^2*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^5*d*(b^2 - 4
```

```
*a*c)^(1/2) - b^6*d + 4*a^3*c^3*d + b^5*c*e - 13*a^2*b^2*c^2*d + 7*a*b^4*c*
d - b^4*c*e*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c^2*e + 8*a^2*b*c^3*e - 2*a^2*c^3
*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*d*(b^2 - 4*a*c)^(1/2) + 4*a*b^2*c^2*e*
(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^2 - a
^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2 + a^4*b^3*d*e - 4*a^5*b*c*d*e))
- (d^5*log(d + e*x))/(c*e^6 + a*d^2*e^4 - b*d*e^5) - x*((b*d + c*e)/(a^2*e^
2) - (a*d + b*e)^2/(a^3*e^3)) + (log(a^4*b^2*d^7 - 4*a^5*c*d^7 - b^3*c^3*e^
7 + b^6*d^3*e^4 + 6*a^2*c^4*d*e^6 + 3*b^4*c^2*d*e^6 - 3*b^5*c*d^2*e^5 + 2*a
^2*c^4*e^7*x - b^2*c^3*e^7*(b^2 - 4*a*c)^(1/2) + b^5*d^3*e^4*(b^2 - 4*a*c)^
(1/2) - 2*a^3*c^3*d^3*e^4 + 4*a^4*c^2*d^5*e^2 + 3*a*b*c^4*e^7 + a^4*b*d^7*(
b^2 - 4*a*c)^(1/2) + a*c^4*e^7*(b^2 - 4*a*c)^(1/2) + 2*a^5*d^7*x*(b^2 - 4*a
*c)^(1/2) - 3*a^2*c^3*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 8*a^5*c*d^6*e*x + 9*a^2
*b^2*c^2*d^3*e^4 - 4*a^4*c*d^6*e*(b^2 - 4*a*c)^(1/2) - 12*a*b^2*c^3*d*e^6 -
 6*a*b^4*c*d^3*e^4 - a*b^2*c^3*e^7*x + a*b^5*d^3*e^4*x + 2*a^4*b^2*d^6*e*x
+ 3*b^3*c^2*d*e^6*(b^2 - 4*a*c)^(1/2) - 3*b^4*c*d^2*e^5*(b^2 - 4*a*c)^(1/2)
 + 15*a*b^3*c^2*d^2*e^5 - 15*a^2*b*c^3*d^2*e^5 - a^3*b^2*c*d^5*e^2 - a^3*b^
3*d^5*e^2*x - 6*a^3*c^3*d^2*e^5*x - 4*a*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) +
 a^3*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + a*b^4*d^3*e^4*x*(b^2 - 4*a*c)^(1/2)
- 3*a^2*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 2*a^4*c*d^5*e^2*x*(b^2 - 4*a*c)^(
1/2) - 5*a^2*b^3*c*d^3*e^4*x + 5*a^3*b*c^2*d^3*e^4*x + 9*a*b^2*c^2*d^2*e^5*
(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) + a^3*b^2*d^5
*e^2*x*(b^2 - 4*a*c)^(1/2) + a^3*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 12*a^2
*b^2*c^2*d^2*e^5*x - 6*a*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^3*e^7*x*(b
^2 - 4*a*c)^(1/2) - 2*a^4*b*d^6*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^3*c^2*d*e^6
*x - 3*a*b^4*c*d^2*e^5*x - 9*a^2*b*c^3*d*e^6*x + 4*a^4*b*c*d^5*e^2*x + 3*a*
b^2*c^2*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^3*c*d^2*e^5*x*(b^2 - 4*a*c)^(1/
2) + 6*a^2*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a^2*b^2*c*d^3*e^4*x*(b^2
 - 4*a*c)^(1/2))*(4*a^3*c^3*d - b^5*d*(b^2 - 4*a*c)^(1/2) - b^6*d + b^5*c*e
 - 13*a^2*b^2*c^2*d + 7*a*b^4*c*d + b^4*c*e*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c
^2*e + 8*a^2*b*c^3*e + 2*a^2*c^3*e*(b^2 - 4*a*c)^(1/2) - 5*a^2*b*c^2*d*(b^2
 - 4*a*c)^(1/2) - 4*a*b^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 5*a*b^3*c*d*(b^2 - 4*
a*c)^(1/2)))/(2*(4*a^6*c*d^2 - a^5*b^2*d^2 + 4*a^5*c^2*e^2 - a^4*b^2*c*e^2
+ a^4*b^3*d*e - 4*a^5*b*c*d*e)) + x^3/(3*a*e) - (x^2*(a*d + b*e))/(2*a^2*e^
2)
```

3.62 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=218

$$-\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}-\frac{(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{d^4\log(d+ex)}{e^3\,(ad^2-e(bd-ce))}-$$

```
[Out] -(a*d+b*e)*x/a^2/e^2+1/2*x^2/a/e+d^4*ln(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))-1/2*
(-2*a*b*c*d+a*c^2*e+b^3*d-b^2*c*e)*ln(a*x^2+b*x+c)/a^3/(a*d^2-e*(b*d-c*e))-
(2*a^2*c^2*d-4*a*b^2*c*d+3*a*b*c^2*e+b^4*d-b^3*c*e)*arctanh((2*a*x+b)/(-4*a
*c+b^2)^(1/2))/a^3/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 1642, 648, 632, 212, 642}

$$-\frac{(-2abcd+ac^2e+b^3d-b^2ce)\log(ax^2+bx+c)}{2a^3(ad^2-e(bd-ce))}-\frac{x(ad+be)}{a^2e^2}-\frac{(2a^2c^2d-4ab^2cd+3abc^2e+b^4d-b^3ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{d^4\log(d+ex)}{e^3(ad^2-e(bd-ce))}+\frac{x^2}{2ae}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) - ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^3
*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + (d^4*Log[d + e*x])/(e^3*(a*d^
2 - e*(b*d - c*e))) - ((b^3*d - 2*a*b*c*d - b^2*c*e + a*c^2*e)*Log[c + b*x
+ a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx = \int \frac{x^4}{(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{-ad-be}{a^2e^2}+\frac{x}{ae}+\frac{d^4}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{-c(b^2d-acd-bce)}{a^2\left(ad^2-e\right.}\right.$$

$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{-c\left(b^2d-acd-bce\right)-\left(b^3d-2abcd-b^2\right.}{c+bx+ax^2}}{a^2\left(ad^2-e(bd-ce\right.}$$

$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-2abcd-b^2ce+ac^2e\right)}{2a^3\left(ad^2-e(bd-c\right.}$$

$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{d^4\log(d+ex)}{e^3\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-2abcd-b^2ce+ac^2e\right)\mathrm{l}}{2a^3\left(ad^2-e(bd-\right.}$$

$$= -\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}-\frac{\left(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e\right)\tanh^{-1}\left(\frac{}{\sqrt{b}}\right.}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}$$

Mathematica [A]

time = 0.12, size = 218, normalized size = 1.00

$$-\frac{(ad+be)x}{a^2e^2}+\frac{x^2}{2ae}+\frac{\left(b^4d-4ab^2cd+2a^2c^2d-b^3ce+3abc^2e\right)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^3\sqrt{-b^2+4ac}\left(ad^2+e(-bd+ce)\right)}+\frac{d^4\log(d+ex)}{e^3\left(ad^2+e(-bd+ce)\right)}+\frac{\left(-b^3d+2abcd+b^2ce-ac^2e\right)\log(c+x(b+ax))}{2a^3\left(ad^2+e(-bd+ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((a*d + b*e)*x)/(a^2*e^2)) + x^2/(2*a*e) + ((b^4*d - 4*a*b^2*c*d + 2*a^2*
c^2*d - b^3*c*e + 3*a*b*c^2*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3
*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))) + (d^4*Log[d + e*x])/(e^3*(
a*d^2 + e*(-(b*d) + c*e))) + ((-(b^3*d) + 2*a*b*c*d + b^2*c*e - a*c^2*e)*Lo
g[c + x*(b + a*x)])/(2*a^3*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [A]
time = 0.25, size = 208, normalized size = 0.95

method	result
default	$-\frac{-\frac{1}{2}ae\,x^2+adx+ebx}{e^2a^2}+\frac{d^4\ln(ex+d)}{e^3(a\,d^2-deb+c\,e^2)}+\frac{\frac{(2abcd-a\,c^2e-b^3d+b^2ce)\ln(a\,x^2+bx+c)}{2a}+\frac{2\left(a\,c^2d-b^2cd+b\,c^2e-\frac{(2abcd-a\,c^2e-b^3d+b^2c\ldots}{2a}\ldots\right.}{\sqrt{4ac-b^2}\ldots}}{(a\,d^2-deb+c\,e^2)a^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^2/a^2*(-1/2*a*e*x^2+a*d*x+e*b*x)+1/e^3*d^4/(a*d^2-b*d*e+c*e^2)*ln(e*x+
d)+1/(a*d^2-b*d*e+c*e^2)/a^2*(1/2*(2*a*b*c*d-a*c^2*e-b^3*d+b^2*c*e)/a*ln(a*
x^2+b*x+c)+2*(a*c^2*d-b^2*c*d+b*c^2*e-1/2*(2*a*b*c*d-a*c^2*e-b^3*d+b^2*c*e)
*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 10.06, size = 788, normalized size = 3.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((a^3*b^2 - 4*a^4*c)*d^2*x^2*e^2 - 2*(a^3*b^2 - 4*a^4*c)*d^3*x*e + 2*(
a^3*b^2 - 4*a^4*c)*d^4*log(x*e + d) + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d*e^3
- (b^3*c - 3*a*b*c^2)*e^4)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2
 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a^2*b^2*c
- 4*a^3*c^2)*x^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*x)*e^4 - ((a^2*b^3 - 4*a^3*b*c
)*d*x^2 - 2*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d*x)*e^3 - ((b^5 - 6*a*b^3*c
+ 8*a^2*b*c^2)*d*e^3 - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b
*x + c))/((a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*
b^2*c - 4*a^4*c^2)*e^5), 1/2*((a^3*b^2 - 4*a^4*c)*d^2*x^2*e^2 - 2*(a^3*b^2
- 4*a^4*c)*d^3*x*e + 2*(a^3*b^2 - 4*a^4*c)*d^4*log(x*e + d) - 2*((b^4 - 4*a
*b^2*c + 2*a^2*c^2)*d*e^3 - (b^3*c - 3*a*b*c^2)*e^4)*sqrt(-b^2 + 4*a*c)*arc
tan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((a^2*b^2*c - 4*a^3*c^
2)*x^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*x)*e^4 - ((a^2*b^3 - 4*a^3*b*c)*d*x^2 -
2*(a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d*x)*e^3 - ((b^5 - 6*a*b^3*c + 8*a^2*b*
c^2)*d*e^3 - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e^4)*log(a*x^2 + b*x + c))/(
(a^4*b^2 - 4*a^5*c)*d^2*e^3 - (a^3*b^3 - 4*a^4*b*c)*d*e^4 + (a^3*b^2*c - 4*
a^4*c^2)*e^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.64, size = 224, normalized size = 1.03

$$\frac{d^4 \log\left(|xe+d|\right)}{ad^2e^3-bde^4+ce^5}-\frac{\left(b^3d-2\,abcd-b^2ce+ac^2e\right)\log\left(ax^2+bx+c\right)}{2\left(a^4d^2-a^3bde+a^3ce^2\right)}+\frac{\left(b^4d-4\,ab^2cd+2\,a^2c^2d-b^3ce+3\,abc^2e\right)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(a^4d^2-a^3bde+a^3ce^2\right)\sqrt{-b^2+4\,ac}}+\frac{\left(ax^2e-2\,adx-2\,bxe\right)e^{(-2)}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^4*log(abs(x*e + d))/(a*d^2*e^3 - b*d*e^4 + c*e^5) - 1/2*(b^3*d - 2*a*b*c*
d - b^2*c*e + a*c^2*e)*log(a*x^2 + b*x + c)/(a^4*d^2 - a^3*b*d*e + a^3*c*e^
2) + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - b^3*c*e + 3*a*b*c^2*e)*arctan((2*
a*x + b)/sqrt(-b^2 + 4*a*c))/((a^4*d^2 - a^3*b*d*e + a^3*c*e^2)*sqrt(-b^2 +
 4*a*c)) + 1/2*(a*x^2*e - 2*a*d*x - 2*b*x*e)*e^(-2)/a^2
```

Mupad [**B**]
time = 5.24, size = 2051, normalized size = 9.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (d^4*log(d + e*x))/(c*e^5 + a*d^2*e^3 - b*d*e^4) - (log(4*a^4*c*d^6 - 2*a*c
^4*e^6 - a^3*b^2*d^6 + b^2*c^3*e^6 - b^5*d^3*e^3 - 3*b^3*c^2*d*e^5 + 3*b^4*
c*d^2*e^4 + b^4*d^3*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^3*d^2*e^4 - 4*a^3*c^2
*d^4*e^2 + a^3*b*d^6*(b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2) +
2*a^4*d^6*x*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b^2 -
4*a*c)^(1/2) + a*b*c^3*e^6*x + 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 - 4*a*c
)^(1/2) - 4*a^3*c*d^5*e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a*c)^(1/
2) + 5*a*b^3*c*d^3*e^3 - a*b^4*d^3*e^3*x - 2*a^3*b^2*d^5*e*x + 6*a^2*c^3*d*
e^5*x + 3*b^2*c^2*d*e^5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2 - 4*a*c)
^(1/2) - 12*a*b^2*c^2*d^2*e^4 - 5*a^2*b*c^2*d^3*e^3 + a^2*b^2*c*d^4*e^2 + a
^2*b^3*d^4*e^2*x - 2*a^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1
/2) - 3*a*b^2*c*d^3*e^3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(b^2 - 4*a*c)
^(1/2) + a*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2*x*(b^2 - 4*a
*c)^(1/2) - 9*a^2*b*c^2*d^2*e^4*x + 4*a^2*b^2*c*d^3*e^3*x + a^2*b^2*d^4*e^2
*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*b*
d^5*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c^2*d*e^5*x + 3*a*b^3*c*d^2*e^4*x - 4
*a^3*b*c*d^4*e^2*x + 3*a*b*c^2*d*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*
e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d
*(b^2 - 4*a*c)^(1/2) - b^5*d + 4*a^2*c^3*e + b^4*c*e + 6*a*b^3*c*d - b^3*c*
e*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c^2*d - 5*a*b^2*c^2*e + 2*a^2*c^2*d*(b^2 -
4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*e*(b^2 - 4*a*c)^
(1/2)))/(2*(4*a^5*c*d^2 - a^4*b^2*d^2 + 4*a^4*c^2*e^2 - a^3*b^2*c*e^2 + a^3
*b^3*d*e - 4*a^4*b*c*d*e)) + (log(2*a*c^4*e^6 - 4*a^4*c*d^6 + a^3*b^2*d^6 -
 b^2*c^3*e^6 + b^5*d^3*e^3 + 3*b^3*c^2*d*e^5 - 3*b^4*c*d^2*e^4 + b^4*d^3*e^
3*(b^2 - 4*a*c)^(1/2) - 6*a^2*c^3*d^2*e^4 + 4*a^3*c^2*d^4*e^2 + a^3*b*d^6*(
b^2 - 4*a*c)^(1/2) - b*c^3*e^6*(b^2 - 4*a*c)^(1/2) + 2*a^4*d^6*x*(b^2 - 4*a
*c)^(1/2) - 9*a*b*c^3*d*e^5 + a^2*c^2*d^3*e^3*(b^2 - 4*a*c)^(1/2) - a*b*c^3
*e^6*x - 8*a^4*c*d^5*e*x - 3*a*c^3*d*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*
e*(b^2 - 4*a*c)^(1/2) - a*c^3*e^6*x*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*d^3*e^3
 + a*b^4*d^3*e^3*x + 2*a^3*b^2*d^5*e*x - 6*a^2*c^3*d*e^5*x + 3*b^2*c^2*d*e^
5*(b^2 - 4*a*c)^(1/2) - 3*b^3*c*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 12*a*b^2*c^2*
d^2*e^4 + 5*a^2*b*c^2*d^3*e^3 - a^2*b^2*c*d^4*e^2 - a^2*b^3*d^4*e^2*x + 2*a
^3*c^2*d^3*e^3*x + 6*a*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^3*e^
3*(b^2 - 4*a*c)^(1/2) + a^2*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) + a*b^3*d^3*e^3
*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 9*a^2*b*c^
2*d^2*e^4*x - 4*a^2*b^2*c*d^3*e^3*x + a^2*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2)
 + 3*a^2*c^2*d^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 2*a^3*b*d^5*e*x*(b^2 - 4*a*c)^
(1/2) + 3*a*b^2*c^2*d*e^5*x - 3*a*b^3*c*d^2*e^4*x + 4*a^3*b*c*d^4*e^2*x + 3
*a*b*c^2*d*e^5*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*e^4*x*(b^2 - 4*a*c)^(1
/2) - 2*a^2*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^5*d + b^4*d*(b^2 - 4*a*c)
^(1/2) - 4*a^2*c^3*e - b^4*c*e - 6*a*b^3*c*d - b^3*c*e*(b^2 - 4*a*c)^(1/2)
```

```
+ 8*a^2*b*c^2*d + 5*a*b^2*c^2*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2
*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^5*c*d^
2 - a^4*b^2*d^2 + 4*a^4*c^2*e^2 - a^3*b^2*c*e^2 + a^3*b^3*d*e - 4*a^4*b*c*d
*e)) + x^2/(2*a*e) - (x*(a*d + b*e))/(a^2*e^2)
```

3.63 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=176

$$\frac{x}{ae}+\frac{(b^3d-3abcd-b^2ce+2ac^2e)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{d^3\log(d+ex)}{e^2\,(ad^2-e(bd-ce))}+\frac{(b^2d-acd-bce)\log\left(c+\right.}{2a^2\,(ad^2-e(bd-c}$$

```
[Out] x/a/e-d^3*ln(e*x+d)/e^2/(a*d^2-e*(b*d-c*e))+1/2*(-a*c*d+b^2*d-b*c*e)*ln(a*x
^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))+(-3*a*b*c*d+2*a*c^2*e+b^3*d-b^2*c*e)*arct
anh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a^2/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2
)
```

Rubi [A]
time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1583, 1642, 648, 632, 212, 642}

$$\frac{(-acd+b^2d-bce)\log(ax^2+bx+c)}{2a^2\,(ad^2-e(bd-ce))}+\frac{(-3abcd+2ac^2e+b^3d-b^2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{d^3\log(d+ex)}{e^2\,(ad^2-e(bd-ce))}+\frac{x}{ae}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTanh[(b + 2*a*x)/Sq
rt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - (d^3*Lo
g[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))) + ((b^2*d - a*c*d - b*c*e)*Log[c
+ b*x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx &= \int \frac{x^3}{(d+ex)\left(c+bx+ax^2\right)}\,dx \\
&= \int \left(\frac{1}{ae}+\frac{d^3}{e\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{c(bd-ce)+\left(b^2d-acd-bce\right)}{a\left(ad^2-e(bd-ce)\right)\left(c+bx+a\right.}\right. \\
&= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{c(bd-ce)+\left(b^2d-acd-bce\right)x}{c+bx+ax^2}\,dx}{a\left(ad^2-bde+ce^2\right)} \\
&= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a^2\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^3d-3abc\right.}{2} \\
&= \frac{x}{ae}-\frac{d^3\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-acd-bce\right)\log\left(c+bx+ax^2\right)}{2a^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^3d-\right.}{} \\
&= \frac{x}{ae}+\frac{\left(b^3d-3abcd-b^2ce+2ac^2e\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+e}{e^2\left(ad^2-e(bd\right.}
\end{aligned}$$

Mathematica [A]

time = 0.13, size = 178, normalized size = 1.01

$$\frac{x}{ae}+\frac{(b^3d-3abcd-b^2ce+2ac^2e)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^2\sqrt{-b^2+4ac}\,(-ad^2+bde-ce^2)}-\frac{d^3\log(d+ex)}{e^2\,(ad^2-bde+ce^2)}+\frac{(b^2d-acd-bce)\log\left(c+bx+ax^2\right)}{2a^2\,(ad^2-bde+ce^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] x/(a*e) + ((b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*ArcTan[(b + 2*a*x)/Sqr
t[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + b*d*e - c*e^2)) - (d^
3*Log[d + e*x])/(e^2*(a*d^2 - b*d*e + c*e^2)) + ((b^2*d - a*c*d - b*c*e)*Lo
g[c + b*x + a*x^2])/(2*a^2*(a*d^2 - b*d*e + c*e^2))
```

Maple [A]
time = 0.25, size = 164, normalized size = 0.93

method	result
default	$\frac{x}{ae}-\frac{d^3\ln(ex+d)}{e^2(a\,d^2-deb+c\,e^2)}+\frac{\frac{\left(-acd+b^2d-bce\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(bcd-c^2e-\frac{\left(-acd+b^2d-bce\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{(a\,d^2-deb+c\,e^2)a}$
risch	$\frac{x}{ae}-\frac{d^3\ln(ex+d)}{e^2(a\,d^2-deb+c\,e^2)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c\,d^2-b^2d^2a^2-4a^2bcde+4e^2c^2a^2+a\,b^3de-a\,b^2c\,e^2\right)\_Z^2+\left(4a^2c^2de-5a\,b^2cde+4c^2e^2ab+de\,b^4-\right.\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] x/a/e-1/e^2*d^3/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/(a*d^2-b*d*e+c*e^2)/a*(1/2*
(-a*c*d+b^2*d-b*c*e)/a*ln(a*x^2+b*x+c)+2*(b*c*d-c^2*e-1/2*(-a*c*d+b^2*d-b*c
*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 3.31, size = 581, normalized size = 3.30

$$\left[\frac{2\,(a^2b^2-4\,a^3c)d^2xe-2\,(a^2b^2-4\,a^3c)d^3\log\left(xe+d\right)-2\,(ab^3-4\,a^2bc)dxe^2+2\,(ab^2c-4\,a^2c^2)xe^3+((b^3-3\,abc)de^2-(b^2c-2\,ac^2)e^3)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,ax+b)}{ax^2+bx+c}\right)+((b^4-5\,ab^2c+4\,a^2c^2)de^2-(b^3c-4\,abc^2)e^3)\log\left(ax^2+bx+c\right)}{2\,((a^3b^2-4\,a^4c)d^2e^2-(a^2b^3-4\,a^3bc)de^3+(a^2b^2c-4\,a^3c^2)e^4)}, \frac{2\,(a^2b^2-4\,a^3c)d^2xe-2\,(a^2b^2-4\,a^3c)d^3\log\left(xe+d\right)-2\,(ab^3-4\,a^2bc)dxe^2+2\,(ab^2c-4\,a^2c^2)xe^3+2\,((b^3-3\,abc)de^2-(b^2c-2\,ac^2)e^3)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,ax+b)}{b^2-4\,ac}\right)+((b^4-5\,ab^2c+4\,a^2c^2)de^2-(b^3c-4\,abc^2)e^3)\log\left(ax^2+bx+c\right)}{2\,((a^3b^2-4\,a^4c)d^2e^2-(a^2b^3-4\,a^3bc)de^3+(a^2b^2c-4\,a^3c^2)e^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*b^2 - 4*a^3*c)*d^2*x*e - 2*(a^2*b^2 - 4*a^3*c)*d^3*log(x*e + d
) - 2*(a*b^3 - 4*a^2*b*c)*d*x*e^2 + 2*(a*b^2*c - 4*a^2*c^2)*x*e^3 + ((b^3 -
 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 +
 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))
+ ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2
 + b*x + c))/((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (
a^2*b^2*c - 4*a^3*c^2)*e^4), 1/2*(2*(a^2*b^2 - 4*a^3*c)*d^2*x*e - 2*(a^2*b^
2 - 4*a^3*c)*d^3*log(x*e + d) - 2*(a*b^3 - 4*a^2*b*c)*d*x*e^2 + 2*(a*b^2*c
- 4*a^2*c^2)*x*e^3 + 2*((b^3 - 3*a*b*c)*d*e^2 - (b^2*c - 2*a*c^2)*e^3)*sqrt
(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (b^3*c - 4*a*b*c^2)*e^3)*log(a*x^2 + b*x
 + c))/((a^3*b^2 - 4*a^4*c)*d^2*e^2 - (a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^2*b^
2*c - 4*a^3*c^2)*e^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.26, size = 185, normalized size = 1.05

$$-\frac{d^3\log\left(|xe+d|\right)}{ad^2e^2-bde^3+ce^4}+\frac{xe^{(-1)}}{a}+\frac{(b^2d-acd-bce)\log\left(ax^2+bx+c\right)}{2\,(a^3d^2-a^2bde+a^2ce^2)}-\frac{(b^3d-3\,abcd-b^2ce+2\,ac^2e)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^3d^2-a^2bde+a^2ce^2)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] -d^3*log(abs(x*e + d))/(a*d^2*e^2 - b*d*e^3 + c*e^4) + x*e^(-1)/a + 1/2*(b^
2*d - a*c*d - b*c*e)*log(a*x^2 + b*x + c)/(a^3*d^2 - a^2*b*d*e + a^2*c*e^2)
 - (b^3*d - 3*a*b*c*d - b^2*c*e + 2*a*c^2*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a^3*d^2 - a^2*b*d*e + a^2*c*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B]
time = 4.34, size = 1367, normalized size = 7.77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] x/(a*e) - (log(c^3*e^5*(b^2 - 4*a*c)^(1/2) - b*c^3*e^5 - 4*a^3*c*d^5 + a^2*
b^2*d^5 + b^4*d^3*e^2 + 3*b^2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 - b^3*d^3*e^2*(b^
2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x - a^2*
b*d^5*(b^2 - 4*a*c)^(1/2) - 2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e
*x + 4*a^2*c*d^4*e*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2)
+ 9*a*b*c^2*d^2*e^3 - 5*a*b^2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x - 3*a*c^2*d^2*e
^3*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^
2*e^3*x - 2*a*b^2*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a^2*c*d^3*e^2*x*(b^2 -
4*a*c)^(1/2) + 3*a*b*c^2*d*e^4*x + a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 2*a^
2*b*d^4*e*x*(b^2 - 4*a*c)^(1/2) - 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a
*b^2*c*d^2*e^3*x + a^2*b*c*d^3*e^2*x + 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2
))*(b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d
 + 4*a*b*c^2*e - 2*a*c^2*e*(b^2 - 4*a*c)^(1/2) + b^2*c*e*(b^2 - 4*a*c)^(1/2
) + 3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c
^2*e^2 - a^2*b^2*c*e^2 + a^2*b^3*d*e - 4*a^3*b*c*d*e)) - (log(a^2*b^2*d^5 -
 b*c^3*e^5 - c^3*e^5*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5 + b^4*d^3*e^2 + 3*b^
2*c^2*d*e^4 - 3*b^3*c*d^2*e^3 + b^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2
*d^3*e^2 - 6*a*c^3*d*e^4 - 2*a*c^3*e^5*x + a^2*b*d^5*(b^2 - 4*a*c)^(1/2) +
2*a^3*d^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^3*c*d^4*e*x - 4*a^2*c*d^4*e*(b^2 - 4*
a*c)^(1/2) + 3*b*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) + 9*a*b*c^2*d^2*e^3 - 5*a*b^
2*c*d^3*e^2 + 2*a^2*b^2*d^4*e*x + 3*a*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 3*b
^2*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 6*a^2*c^2*d^2*e^3*x + 2*a*b^2*d^3*e^2*x*
(b^2 - 4*a*c)^(1/2) - 3*a^2*c*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*e
^4*x - a*b*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 2*a^2*b*d^4*e*x*(b^2 - 4*a*c)^(1
/2) + 3*a*c^2*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 3*a*b^2*c*d^2*e^3*x + a^2*b*c*d
^3*e^2*x - 3*a*b*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d + b^3*d*(b^2 - 4*a
*c)^(1/2) + 4*a^2*c^2*d - b^3*c*e - 5*a*b^2*c*d + 4*a*b*c^2*e + 2*a*c^2*e*(
b^2 - 4*a*c)^(1/2) - b^2*c*e*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d*(b^2 - 4*a*c)^
(1/2)))/(2*(4*a^4*c*d^2 - a^3*b^2*d^2 + 4*a^3*c^2*e^2 - a^2*b^2*c*e^2 + a^2
*b^3*d*e - 4*a^3*b*c*d*e)) - (d^3*log(d + e*x))/(c*e^4 + a*d^2*e^2 - b*d*e^
3)
```

3.64 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx$

Optimal. Leaf size=149

$$-\frac{(b^2d-2acd-bce)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{d^2\log(d+ex)}{e\,(ad^2-bde+ce^2)}-\frac{(bd-ce)\log\left(c+bx+ax^2\right)}{2a\,(ad^2-e(bd-ce))}$$

```
[Out] d^2*ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2)-1/2*(b*d-c*e)*ln(a*x^2+b*x+c)/a/(a*d^2-
e*(b*d-c*e))-(-2*a*c*d+b^2*d-b*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a
/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1459, 1642, 648, 632, 212, 642}

$$-\frac{(-2acd+b^2d-bce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{d^2\log(d+ex)}{e\,(ad^2-bde+ce^2)}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2a\,(ad^2-e(bd-ce))}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -(((b^2*d - 2*a*c*d - b*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a*Sqr
t[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e)))) + (d^2*Log[d + e*x])/(e*(a*d^2 - b
*d*e + c*e^2)) - ((b*d - c*e)*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d -
c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1459

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)}\,dx &= \int \frac{x^2}{(d+ex)\left(c+bx+ax^2\right)}\,dx \\
&= \int \left(\frac{d^2}{\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{-cd-(bd-ce)x}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right)dx \\
&= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}+\frac{\int \frac{-cd-(bd-ce)x}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)} \\
&= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2a\left(ad^2-e(bd-ce)\right)}+\frac{\left(b^2d-2acd-bce\right)\int \frac{1}{c+bx+ax^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce)\log\left(c+bx+ax^2\right)}{2a\left(ad^2-e(bd-ce)\right)}-\frac{\left(b^2d-2acd-bce\right)\text{Subs}}{a\left(ad^2-\right.} \\
&= -\frac{\left(b^2d-2acd-bce\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{d^2\log(d+ex)}{e\left(ad^2-bde+ce^2\right)}-\frac{(bd-ce}{2a(a}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 132, normalized size = 0.89

$$-\frac{2e(-b^2d+2acd+bce)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,(-2ad^2\log(d+ex)+e(bd-ce)\log(c+x(b+ax)))}{2a\sqrt{-b^2+4ac}\,e\,(ad^2+e(-bd+ce))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)),x]
```

```
[Out] -1/2*(2*e*(-(b^2*d) + 2*a*c*d + b*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c
]] + Sqrt[-b^2 + 4*a*c]*(-2*a*d^2*Log[d + e*x] + e*(b*d - c*e)*Log[c + x*(b
 + a*x)]))/(a*Sqrt[-b^2 + 4*a*c]*e*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [A]
time = 0.25, size = 130, normalized size = 0.87

method	result
default	$\frac{d^2\ln(ex+d)}{e(a\,d^2-deb+c\,e^2)}+\frac{\frac{(-bd+ce)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-cd-\frac{(-bd+ce)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a\,d^2-deb+c\,e^2}$
risch	$\frac{d^2\ln(ex+d)}{e(a\,d^2-deb+c\,e^2)}+\left(\sum_{\_R=\mathrm{RootOf}\left((4a^3c\,d^2-b^2d^2a^2-4a^2bcde+4e^2c^2a^2+a\,b^3de-a\,b^2c\,e^2)\_Z^2+(4abcd-4a\,c^2e-b^3d+b^2ce)\_Z+\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] d^2*ln(e*x+d)/e/(a*d^2-b*d*e+c*e^2)+1/(a*d^2-b*d*e+c*e^2)*(1/2*(-b*d+c*e)/a
*ln(a*x^2+b*x+c)+2*(-c*d-1/2*(-b*d+c*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*
x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 1.17, size = 405, normalized size = 2.72

$$\left[\frac{2\,(ab^2-4\,a^2c)d^2\log\left(xe+d\right)+(bce^2-(b^2-2\,ac)de)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,ax+b)}{ax^2+bx+c}\right)-((b^3-4\,abc)de-(b^2c-4\,ac^2)e^2)\log\left(ax^2+bx+c\right)}{2\,((a^2b^2-4\,a^3c)d^2e-(ab^3-4\,a^2bc)de^2+(ab^2c-4\,a^2c^2)e^3)},\ \frac{2\,(ab^2-4\,a^2c)d^2\log\left(xe+d\right)+2\,(bce^2-(b^2-2\,ac)de)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,ax+b)}{b^2-4\,ac}\right)-((b^3-4\,abc)de-(b^2c-4\,ac^2)e^2)\log\left(ax^2+bx+c\right)}{2\,((a^2b^2-4\,a^3c)d^2e-(ab^3-4\,a^2bc)de^2+(ab^2c-4\,a^2c^2)e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(x*e + d) + (b*c*e^2 - (b^2 - 2*a*c)*d*e)*
sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c
)*(2*a*x + b))/(a*x^2 + b*x + c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a*c^2
)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^2*b*
c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/2*(2*(a*b^2 - 4*a^2*c)*d^2*log(x*e
 + d) + 2*(b*c*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^
2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e - (b^2*c - 4*a
*c^2)*e^2)*log(a*x^2 + b*x + c))/((a^2*b^2 - 4*a^3*c)*d^2*e - (a*b^3 - 4*a^
2*b*c)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.32, size = 149, normalized size = 1.00

$$\frac{d^2\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}-\frac{(bd-ce)\log\left(ax^2+bx+c\right)}{2\,(a^2d^2-abde+ace^2)}+\frac{(b^2d-2\,acd-bce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(a^2d^2-abde+ace^2)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d),x, algorithm="giac")
```

```
[Out] d^2*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) - 1/2*(b*d - c*e)*log(a*x
^2 + b*x + c)/(a^2*d^2 - a*b*d*e + a*c*e^2) + (b^2*d - 2*a*c*d - b*c*e)*arc
tan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a^2*d^2 - a*b*d*e + a*c*e^2)*sqrt(-b^
2 + 4*a*c))
```

Mupad [**B**]
time = 3.67, size = 966, normalized size = 6.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (d^2*log(d + e*x))/(c*e^3 + a*d^2*e - b*d*e^2) - (log(a*b^2*d^4 - 2*c^3*e^4
 - 4*a^2*c*d^4 + b^3*d^3*e + c^2*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a*c^2*d^2*e
^2 - 4*b^2*c*d^2*e^2 - b^3*d^2*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) + 3*b*c^
2*d*e^3 - b*c^2*e^4*x + b^2*d^3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^2 -
4*a*c)^(1/2) + 2*a^2*d^4*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*d^3*e*x + 6*a*c^2*
d*e^3*x - 10*a^2*c*d^3*e*x - 2*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d^
3*e + b^2*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) -
 a*b*d^3*e*x*(b^2 - 4*a*c)^(1/2) + a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*x*(b^2 -
 4*a*c)^(1/2))*(e*((b^2*c)/2 - 2*a*c^2 + (b*c*(b^2 - 4*a*c)^(1/2))/2) - (b^
3*d)/2 - (b^2*d*(b^2 - 4*a*c)^(1/2))/2 + a*c*d*(b^2 - 4*a*c)^(1/2) + 2*a*b*
c*d))/(4*a^3*c*d^2 - a^2*b^2*d^2 + 4*a^2*c^2*e^2 + a*b^3*d*e - a*b^2*c*e^2
- 4*a^2*b*c*d*e) + (log(2*c^3*e^4 - a*b^2*d^4 + 4*a^2*c*d^4 - b^3*d^3*e + c
^2*e^4*x*(b^2 - 4*a*c)^(1/2) - 10*a*c^2*d^2*e^2 + 4*b^2*c*d^2*e^2 + b^3*d^2
*e^2*x + a*b*d^4*(b^2 - 4*a*c)^(1/2) - 3*b*c^2*d*e^3 + b*c^2*e^4*x + b^2*d^
3*e*(b^2 - 4*a*c)^(1/2) + 3*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^2*d^4*x*(b^
2 - 4*a*c)^(1/2) - 3*a*b^2*d^3*e*x - 6*a*c^2*d*e^3*x + 10*a^2*c*d^3*e*x - 2
*b*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*d^3*e + b^2*d^2*e^2*x*(b^2 - 4*a
*c)^(1/2) - 5*a*c*d^3*e*(b^2 - 4*a*c)^(1/2) - a*b*d^3*e*x*(b^2 - 4*a*c)^(1/
2) - a*b*c*d^2*e^2*x - 5*a*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2))*((b^3*d)/2 + e*
(2*a*c^2 - (b^2*c)/2 + (b*c*(b^2 - 4*a*c)^(1/2))/2) - (b^2*d*(b^2 - 4*a*c)^
(1/2))/2 + a*c*d*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d))/(4*a^3*c*d^2 - a^2*b^2*d
^2 + 4*a^2*c^2*e^2 + a*b^3*d*e - a*b^2*c*e^2 - 4*a^2*b*c*d*e)
```

3.65 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)}\,dx$

Optimal. Leaf size=124

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}+\frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-e(bd-ce)\right)}$$

```
[Out] -d*ln(e*x+d)/(a*d^2-e*(b*d-c*e))+1/2*d*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))+
(b*d-2*c*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))/(-4*a
*c+b^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 814, 648, 632, 212, 642}

$$\frac{(bd-2ce)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-e(bd-ce)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] ((b*d - 2*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(
a*d^2 - e*(b*d - c*e))) - (d*Log[d + e*x])/(a*d^2 - e*(b*d - c*e)) + (d*Log
[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right) x(d+ex)}\,dx &= \int \frac{x}{(d+ex)\left(c+bx+ax^2\right)}\,dx\\
&= \int \left(\frac{de}{\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{ce+adx}{\left(ad^2-e(bd-ce)\right)\left(c+bx+ax^2\right)}\right) d\\
&= -\frac{d\log(d+ex)}{ad^2-bde+ce^2}+\frac{\int \frac{ce+adx}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}\\
&= -\frac{d\log(d+ex)}{ad^2-bde+ce^2}+\frac{d\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)}+\frac{(-bd+2ce)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)}\\
&= -\frac{d\log(d+ex)}{ad^2-bde+ce^2}+\frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{(bd-2ce)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx\right.}{ad^2-e(bd-ce)}\\
&= \frac{(bd-2ce)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}-\frac{d\log(d+ex)}{ad^2-bde+ce^2}+\frac{d\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 107, normalized size = 0.86

$$
\frac{2(bd-2ce)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,d(2\log(d+ex)-\log(c+x(b+ax)))}{2\sqrt{-b^2+4ac}\,\left(-ad^2+e(bd-ce)\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)),x]
```

```
[Out] (2*(b*d - 2*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c
]*d*(2*Log[d + e*x] - Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2
) + e*(b*d - c*e)))
```

Maple [A]
time = 0.21, size = 105, normalized size = 0.85

method	result	size
default	$-\frac{d \ln(ex+d)}{a\,d^2-deb+c\,e^2}+\frac{\frac{d\ln\left(a\,x^2+bx+c\right)}{2}+\frac{2\left(-\frac{bd}{2}+ce\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a\,d^2-deb+c\,e^2}$	105
risch	Expression too large to display	3391

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -d/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/(a*d^2-b*d*e+c*e^2)*(1/2*d*ln(a*x^2+b*x+
c)+2*(-1/2*b*d+c*e)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.54, size = 309, normalized size = 2.49

$$\left[\frac{(b^2-4\,ac)d\log\left(ax^2+bx+c\right)-2\,(b^2-4\,ac)d\log\left(xe+d\right)-\sqrt{b^2-4\,ac}\,(bd-2\,ce)\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac-\sqrt{b^2-4\,ac}\,(2\,ax+b)}{ax^2+bx+c}\right)}{2\left((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2\right)},\ \frac{(b^2-4\,ac)d\log\left(ax^2+bx+c\right)-2\,(b^2-4\,ac)d\log\left(xe+d\right)+2\,\sqrt{-b^2+4\,ac}\,(bd-2\,ce)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,ax+b)}{b^2-4\,ac}\right)}{2\left((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2 - 4*a*c)*d*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(x*e + d)
 - sqrt(b^2 - 4*a*c)*(b*d - 2*c*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c -
```

```
sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
- (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), 1/2*((b^2 - 4*a*c)*d*log(a*
x^2 + b*x + c) - 2*(b^2 - 4*a*c)*d*log(x*e + d) + 2*sqrt(-b^2 + 4*a*c)*(b*d
 - 2*c*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2 -
4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.37, size = 127, normalized size = 1.02

$$-\frac{de\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{d\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(bd-2\,ce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ad^2-bde+ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d),x, algorithm="giac")
```

```
[Out] -d*e*log(abs(x*e + d))/(a*d^2*e - b*d*e^2 + c*e^3) + 1/2*d*log(a*x^2 + b*x
+ c)/(a*d^2 - b*d*e + c*e^2) - (b*d - 2*c*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [**B**]
time = 3.41, size = 801, normalized size = 6.46

$$\frac{\ln\left(a\,e\,x-\frac{\left(d\,\left(\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\,\left(x\,\left(d\,a^2\,e+b\,a\,e^2\right)+\frac{\left(d\,\left(\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\,\left(x\,\left(2\,a^3\,d^2\,e-2\,a^2\,b\,d\,e^2-6\,c\,a^2\,e^3+2\,a\,b^2\,e^3\right)+a\,b\,c\,e^3+a\,b^2\,d\,e^2+a^2\,b\,d^2\,e-8\,a^2\,c\,d\,e^2\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}+a\,c\,e^2+a\,b\,d\,e\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}\right)\,\left(d\,\left(\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-2\,a\,c+\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}-\frac{\ln\left(\frac{\left(d\,\left(2\,a\,c+\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\,\left(x\,\left(d\,a^2\,e+b\,a\,e^2\right)-\frac{\left(d\,\left(2\,a\,c+\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)\,\left(x\,\left(2\,a^3\,d^2\,e-2\,a^2\,b\,d\,e^2-6\,c\,a^2\,e^3+2\,a\,b^2\,e^3\right)+a\,b\,c\,e^3+a\,b^2\,d\,e^2+a^2\,b\,d^2\,e-8\,a^2\,c\,d\,e^2\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}+a\,c\,e^2+a\,b\,d\,e\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}+a\,e\,x\right)\,\left(d\,\left(2\,a\,c+\frac{b\,\sqrt{b^2-4\,a\,c}}{2}-\frac{b^2}{2}\right)-c\,e\,\sqrt{b^2-4\,a\,c}\right)}{-4\,a^2\,c\,d^2+a\,b^2\,d^2+4\,a\,b\,c\,d\,e-4\,a\,c^2\,e^2-b^3\,d\,e+b^2\,c\,e^2}-\frac{d\,\ln\left(d+e\,x\right)}{a\,d^2-b\,d\,e+c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(a*e*x - ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - c*e*(b^2 - 4
*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) + ((d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*
c + b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(2*a*b^2*e^3 - 6*a^2*c*e^3 + 2*a^3
*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3 + a*b^2*d*e^2 + a^2*b*d^2*e - 8*a^2*c*d
*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b
*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*
c*e^2 - b^3*d*e + 4*a*b*c*d*e))*(d*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2
```

```
/2) - c*e*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^
2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(((d*(2*a*c + (b*(b^2 - 4*a*c)^(1/2)
)/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(a*b*e^2 + a^2*d*e) - ((d*(2*a*c
 + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2))*(x*(2*a*b^
2*e^3 - 6*a^2*c*e^3 + 2*a^3*d^2*e - 2*a^2*b*d*e^2) + a*b*c*e^3 + a*b^2*d*e^
2 + a^2*b*d^2*e - 8*a^2*c*d*e^2))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 +
b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) + a*c*e^2 + a*b*d*e))/(a*b^2*d^2 - 4*a^2
*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) + a*e*x)*(d*(2*a*
c + (b*(b^2 - 4*a*c)^(1/2))/2 - b^2/2) - c*e*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d
^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (d*lo
g(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

3.66 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx$

Optimal. Leaf size=123

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}$$

```
[Out] e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)-1/2*e*ln(a*x^2+b*x+c)/(a*d^2-b*d*e+c*e^2)-(
2*a*d-b*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))/(-4*a*
c+b^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1583, 719, 31, 648, 632, 212, 642}

$$-\frac{(2ad-be)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] -(((2*a*d - b*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e)))) + (e*Log[d + e*x])/(a*d^2 - b*d*e + c*e^2) - (e*
Log[c + b*x + a*x^2])/(2*(a*d^2 - b*d*e + c*e^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)}\,dx &= \int \frac{1}{(d+ex)\left(c+bx+ax^2\right)}\,dx\\
&= \frac{e^2\int \frac{1}{d+ex}\,dx}{ad^2-bde+ce^2}+\frac{\int \frac{ad-be-aex}{c+bx+ax^2}\,dx}{ad^2-e(bd-ce)}\\
&= \frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\int \frac{b+2ax}{c+bx+ax^2}\,dx}{2\left(ad^2-bde+ce^2\right)}+\frac{(2ad-be)\int \frac{1}{c+bx+ax^2}\,dx}{2\left(ad^2-e(bd-ce)\right)}\\
&= \frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax^2\right)}{2\left(ad^2-bde+ce^2\right)}-\frac{(2ad-be)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x
\end{aligned}$$

$$= -\frac{(2ad-be)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}+\frac{e\log(d+ex)}{ad^2-bde+ce^2}-\frac{e\log\left(c+bx+ax\right.}{2\left(ad^2-bde+ce\right.}$$

(denominator of the Subst term: $ad^2-e(bd-ce)$)

Mathematica [A]
time = 0.05, size = 105, normalized size = 0.85

$$\frac{(-4ad+2be)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,e(-2\log(d+ex)+\log(c+x(b+ax)))}{2\sqrt{-b^2+4ac}\,(-ad^2+e(bd-ce))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)),x]
```

```
[Out] ((-4*a*d + 2*b*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*
c]*e*(-2*Log[d + e*x] + Log[c + x*(b + a*x)]))/(2*Sqrt[-b^2 + 4*a*c]*(-(a*d
^2) + e*(b*d - c*e)))
```

Maple [A]
time = 0.21, size = 104, normalized size = 0.85

method	result	size
default	$\frac{e\ln(ex+d)}{a\,d^2-deb+c\,e^2}+\frac{-\frac{e\ln\left(a\,x^2+bx+c\right)}{2}+\frac{2\left(ad-\frac{eb}{2}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{a\,d^2-deb+c\,e^2}$	104
risch	Expression too large to display	2885

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e*ln(e*x+d)/(a*d^2-b*d*e+c*e^2)+1/(a*d^2-b*d*e+c*e^2)*(-1/2*e*ln(a*x^2+b*x+
c)+2*(a*d-1/2*e*b)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.55, size = 313, normalized size = 2.54

$$\left[-\frac{(b^2-4\,ac)e\log\left(ax^2+bx+c\right)-2\,(b^2-4\,ac)e\log\left(xe+d\right)+\sqrt{b^2-4\,ac}\,(2\,ad-be)\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,ax+b)}{ax^2+bx+c}\right)}{2\,((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2)},\ -\frac{(b^2-4\,ac)e\log\left(ax^2+bx+c\right)-2\,(b^2-4\,ac)e\log\left(xe+d\right)+2\,\sqrt{-b^2+4\,ac}\,(2\,ad-be)\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,ax+b)}{b^2-4\,ac}\right)}{2\,((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2 - 4*a*c)*e*log(a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(x*e + d
) + sqrt(b^2 - 4*a*c)*(2*a*d - b*e)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c
+ sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)))/((a*b^2 - 4*a^2*c)*d^2
 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2), -1/2*((b^2 - 4*a*c)*e*log(
a*x^2 + b*x + c) - 2*(b^2 - 4*a*c)*e*log(x*e + d) + 2*sqrt(-b^2 + 4*a*c)*(2
*a*d - b*e)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)))/((a*b^2
- 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.77, size = 126, normalized size = 1.02

$$-\frac{e\log\left(ax^2+bx+c\right)}{2\left(ad^2-bde+ce^2\right)}+\frac{e^2\log\left(|xe+d|\right)}{ad^2e-bde^2+ce^3}+\frac{(2\,ad-be)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(ad^2-bde+ce^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*e*log(a*x^2 + b*x + c)/(a*d^2 - b*d*e + c*e^2) + e^2*log(abs(x*e + d))
/(a*d^2*e - b*d*e^2 + c*e^3) + (2*a*d - b*e)*arctan((2*a*x + b)/sqrt(-b^2 +
 4*a*c))/((a*d^2 - b*d*e + c*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B]
time = 3.82, size = 521, normalized size = 4.24

$$\frac{\ln\left(3a^2e^2x+abe^2+a^2de-\frac{ae\left(\frac{b^2e}{2}-2ace+ad\sqrt{b^2-4ac}-\frac{be\sqrt{b^2-4ac}}{2}\right)\left(2xa^2d^2+abd^2-2xabde-8cade-6cxae^2+b^2de+2xb^2e^2+cbe^2\right)}{(4ac-b^2)\left(ad^2-bde+ce^2\right)}\right)\left(e\left(2ac+\frac{b\sqrt{b^2-4ac}}{2}-\frac{b^2}{2}\right)-ad\sqrt{b^2-4ac}\right)}{-4a^2cd^2+ab^2d^2+4abcde-4ac^2e^2-b^3de+b^2ce^2}-\frac{\ln\left(3a^2e^2x+abe^2+a^2de-\frac{ae\left(\frac{b^2e}{2}-2ace-ad\sqrt{b^2-4ac}+\frac{be\sqrt{b^2-4ac}}{2}\right)\left(2xa^2d^2+abd^2-2xabde-8cade-6cxae^2+b^2de+2xb^2e^2+cbe^2\right)}{(4ac-b^2)\left(ad^2-bde+ce^2\right)}\right)\left(e\left(\frac{b\sqrt{b^2-4ac}}{2}-2ac+\frac{b^2}{2}\right)-ad\sqrt{b^2-4ac}\right)}{-4a^2cd^2+ab^2d^2+4abcde-4ac^2e^2-b^3de+b^2ce^2}+\frac{e\ln\left(d+ex\right)}{ad^2-bde+ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(3*a^2*e^2*x + a*b*e^2 + a^2*d*e - (a*e*((b^2*e)/2 - 2*a*c*e + a*d*(b^2
 - 4*a*c)^(1/2) - (b*e*(b^2 - 4*a*c)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x +
```

```
 a*b*d^2 + b*c*e^2 + b^2*d*e - 6*a*c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/((4*
a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(e*(2*a*c + (b*(b^2 - 4*a*c)^(1/2))/2
- b^2/2) - a*d*(b^2 - 4*a*c)^(1/2)))/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2
 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*e) - (log(3*a^2*e^2*x + a*b*e^2 + a^2*d*
e - (a*e*((b^2*e)/2 - 2*a*c*e - a*d*(b^2 - 4*a*c)^(1/2) + (b*e*(b^2 - 4*a*c
)^(1/2))/2)*(2*a^2*d^2*x + 2*b^2*e^2*x + a*b*d^2 + b*c*e^2 + b^2*d*e - 6*a*
c*e^2*x - 8*a*c*d*e - 2*a*b*d*e*x))/((4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e))
)*(e*((b*(b^2 - 4*a*c)^(1/2))/2 - 2*a*c + b^2/2) - a*d*(b^2 - 4*a*c)^(1/2))
)/(a*b^2*d^2 - 4*a^2*c*d^2 - 4*a*c^2*e^2 + b^2*c*e^2 - b^3*d*e + 4*a*b*c*d*
e) + (e*log(d + e*x))/(a*d^2 + c*e^2 - b*d*e)
```

3.67 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx$

Optimal. Leaf size=158

$$\frac{(abd-b^2e+2ace)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\,(ad^2-bde+ce^2)}-\frac{(ad-be)\log\left(c+bx+ax^2\right)}{2c\,(ad^2-e(bd-ce))}$$

```
[Out] ln(x)/c/d-e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)-1/2*(a*d-b*e)*ln(a*x^2+b*x+c)
/c/(a*d^2-e*(b*d-c*e))+(a*b*d+2*a*c*e-b^2*e)*arctanh((2*a*x+b)/(-4*a*c+b^2)
^(1/2))/c/(a*d^2-e*(b*d-c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 907, 648, 632, 212, 642}

$$\frac{(abd+2ace+b^2(-e))\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{e^2\log(d+ex)}{d\,(ad^2-e(bd-ce))}-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2c\,(ad^2-e(bd-ce))}+\frac{\log(x)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] ((a*b*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + Log[x]/(c*d) - (e^2*Log[d + e*x])/(
d*(a*d^2 - e*(b*d - c*e))) - ((a*d - b*e)*Log[c + b*x + a*x^2])/(2*c*(a*d^2
 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)}\,dx = \int \frac{1}{x(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cdx}+\frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)}+\frac{b^2e-a(bd+ce)-a(ad-}{c\left(ad^2-e(bd-ce)\right)(c+bx}\right.$$

$$= \frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{b^2e-a(bd+ce)-a(ad-be)x}{c+bx+ax^2}\,dx}{c\left(ad^2-bde+ce^2\right)}$$

$$= \frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}+\frac{\left(-abd+b^2e-2ace\right)\int \frac{1}{c+bx+ax^2}\,dx}{2c\left(ad^2-bde+ce^2\right)}-\frac{(ac}{2c}$$

$$= \frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce)\right)}-\frac{(ad-be)\log\left(c+bx+ax^2\right)}{2c\left(ad^2-e(bd-ce)\right)}-\frac{(-abd+}{}$$

$$= \frac{\left(abd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\left(ad^2-bde+ce^2\right)}+\frac{\log(x)}{cd}-\frac{e^2\log(d+ex)}{d\left(ad^2-e(bd-ce\right.}$$

Mathematica [A]
time = 0.13, size = 152, normalized size = 0.96

$$-\frac{2d(abd-b^2e+2ace)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,(-2(ad^2+e(-bd+ce))\log(x)+2ce^2\log(d+ex)+d(ad-be)\log(c+x(b+ax)))}{2c\sqrt{-b^2+4ac}\,d\,(ad^2+e(-bd+ce))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)),x]
```

```
[Out] -1/2*(2*d*(a*b*d - b^2*e + 2*a*c*e)*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]]
+ Sqrt[-b^2 + 4*a*c]*(-2*(a*d^2 + e*(-(b*d) + c*e))*Log[x] + 2*c*e^2*Log[d
+ e*x] + d*(a*d - b*e)*Log[c + x*(b + a*x)]))/(c*Sqrt[-b^2 + 4*a*c]*d*(a*d^
2 + e*(-(b*d) + c*e)))
```

Maple [A]
time = 0.20, size = 160, normalized size = 1.01

method	result
default	$-\frac{e^2\ln(ex+d)}{d(a\,d^2-deb+c\,e^2)}+\frac{\frac{\left(-d\,a^2+abe\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-abd-ace+b^2e-\frac{\left(-d\,a^2+abe\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{(a\,d^2-deb+c\,e^2)c}+\frac{\ln(x)}{cd}$
risch	$-\frac{e^2\ln(-ex-d)}{d(a\,d^2-deb+c\,e^2)}+\frac{\ln(-x)}{cd}+\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^2c^2d^2-a\,b^2c\,d^2-4ed\,c^2ba+4a\,c^3e^2+b^3cde-b^2c^2e^2\right)\_Z^2+\left(4a^2cd-a\,b^2d-4abc\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^2*ln(e*x+d)/d/(a*d^2-b*d*e+c*e^2)+1/(a*d^2-b*d*e+c*e^2)/c*(1/2*(-a^2*d+a
*b*e)/a*ln(a*x^2+b*x+c)+2*(-a*b*d-a*c*e+b^2*e-1/2*(-a^2*d+a*b*e)*b/a)/(4*a*
c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+ln(x)/c/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 177.70, size = 508, normalized size = 3.22

$$\left[-\frac{2\,(b^2c-4\,ac^2)e^2\log\left(xe+d\right)-(abd^2-(b^2-2\,ac)de)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac+\sqrt{b^2-4\,ac}\,(2\,ax+b)}{ax^2+bx+c}\right)+((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de)\log\left(ax^2+bx+c\right)-2\,((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2)\log\left(x\right)}{2\,((ab^2c-4\,a^2c^2)d^3-(b^3c-4\,abc^2)d^2e+(b^2c^2-4\,ac^3)de^2)},-\frac{2\,(b^2c-4\,ac^2)e^2\log\left(xe+d\right)-2\,(abd^2-(b^2-2\,ac)de)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{\sqrt{-b^2+4\,ac}\,(2\,ax+b)}{b^2-4\,ac}\right)+((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de)\log\left(ax^2+bx+c\right)-2\,((ab^2-4\,a^2c)d^2-(b^3-4\,abc)de+(b^2c-4\,ac^2)e^2)\log\left(x\right)}{2\,((ab^2c-4\,a^2c^2)d^3-(b^3c-4\,abc^2)d^2e+(b^2c^2-4\,ac^3)de^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(b^2*c - 4*a*c^2)*e^2*log(x*e + d) - (a*b*d^2 - (b^2 - 2*a*c)*d*e)
*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*
c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*
c)*d*e)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^2 - (b^3 - 4*a*b*c)*d
*e + (b^2*c - 4*a*c^2)*e^2)*log(x))/((a*b^2*c - 4*a^2*c^2)*d^3 - (b^3*c - 4
*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2), -1/2*(2*(b^2*c - 4*a*c^2)*e^2
*log(x*e + d) - 2*(a*b*d^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arctan(-
sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*d^2 - (b
^3 - 4*a*b*c)*d*e)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^2 - (b^3 -
 4*a*b*c)*d*e + (b^2*c - 4*a*c^2)*e^2)*log(x))/((a*b^2*c - 4*a^2*c^2)*d^3 -
 (b^3*c - 4*a*b*c^2)*d^2*e + (b^2*c^2 - 4*a*c^3)*d*e^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.42, size = 164, normalized size = 1.04

$$-\frac{(ad-be)\log\left(ax^2+bx+c\right)}{2\,(acd^2-bcde+c^2e^2)}-\frac{e^3\log\left(|xe+d|\right)}{ad^3e-bd^2e^2+cde^3}-\frac{(abd-b^2e+2\,ace)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(acd^2-bcde+c^2e^2)\sqrt{-b^2+4\,ac}}+\frac{\log\left(|x|\right)}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(a*d - b*e)*log(a*x^2 + b*x + c)/(a*c*d^2 - b*c*d*e + c^2*e^2) - e^3*l
og(abs(x*e + d))/(a*d^3*e - b*d^2*e^2 + c*d*e^3) - (a*b*d - b^2*e + 2*a*c*e
)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - b*c*d*e + c^2*e^2)*sqr
t(-b^2 + 4*a*c)) + log(abs(x))/(c*d)
```

Mupad [B]
time = 5.40, size = 2399, normalized size = 15.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log(b^3*c^3*e^5 - 6*a^4*c^2*d^5 + 2*a^3*b^2*c*d^5 + 8*a^2*c^4*d*e^4 - b^4*
c^2*d*e^4 - 2*b^5*c*d^2*e^3 + 2*a^3*b^3*d^5*x + 8*a^2*c^4*e^5*x + b^4*c^2*e
^5*x - 2*b^6*d^2*e^3*x + b^2*c^3*e^5*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*d^3*e
^2 - 4*a*b*c^4*e^5 - 4*a*c^4*e^5*(b^2 - 4*a*c)^(1/2) - 5*a^2*c^3*d^2*e^3*(b
^2 - 4*a*c)^(1/2) - 7*a^4*b*c*d^5*x - b^5*c*d*e^4*x - 27*a^2*b^2*c^2*d^3*e^
2 + 2*a^3*b*c*d^5*(b^2 - 4*a*c)^(1/2) - 3*a^4*c*d^5*x*(b^2 - 4*a*c)^(1/2) +
 2*a*b^2*c^3*d*e^4 + 6*a*b^4*c*d^3*e^2 - 6*a^2*b^3*c*d^4*e + 21*a^3*b*c^2*d
^4*e - 6*a*b^2*c^3*e^5*x + 6*a*b^5*d^3*e^2*x - 6*a^2*b^4*d^4*e*x - 14*a^4*c
^2*d^4*e*x + 7*a^3*c^2*d^4*e*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d*e^4*(b^2 - 4*a
*c)^(1/2) - 2*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^3*b^2*d^5*x*(b^2 - 4*
a*c)^(1/2) + b^3*c^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*x*(b^2 - 4*a
*c)^(1/2) + 13*a*b^3*c^2*d^2*e^3 - 21*a^2*b*c^3*d^2*e^3 + 10*a^3*c^3*d^2*e^
3*x + 6*a*b^3*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^2*c*d^4*e*(b^2 - 4*a*
c)^(1/2) + 6*a*b^4*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^3*d^4*e*x*(b^2 -
 4*a*c)^(1/2) + 4*a^2*c^3*d*e^4*x*(b^2 - 4*a*c)^(1/2) - 32*a^2*b^3*c*d^3*e^
2*x + 35*a^3*b*c^2*d^3*e^2*x + 7*a*b^2*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 13
*a^2*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 9*a^3*c^2*d^3*e^2*x*(b^2 - 4*a*c)^
(1/2) - 27*a^2*b^2*c^2*d^2*e^3*x + 4*a*b*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) - 4*
a*b*c^3*e^5*x*(b^2 - 4*a*c)^(1/2) - b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 5*a
*b^3*c^2*d*e^4*x + 14*a*b^4*c*d^2*e^3*x - 4*a^2*b*c^3*d*e^4*x + 26*a^3*b^2*
c*d^4*e*x + 14*a^3*b*c*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c^2*d*e^4*x*(b
^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2*b*c^2
*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 20*a^2*b^2*c*d^3*e^2*x*(b^2 - 4*a*c)^(1/2)
)*(d*((a*b^2)/2 - 2*a^2*c + (a*b*(b^2 - 4*a*c)^(1/2))/2) - (b^3*e)/2 - (b^2
*e*(b^2 - 4*a*c)^(1/2))/2 + a*c*e*(b^2 - 4*a*c)^(1/2) + 2*a*b*c*e))/(4*a*c^
3*e^2 + 4*a^2*c^2*d^2 - b^2*c^2*e^2 + b^3*c*d*e - a*b^2*c*d^2 - 4*a*b*c^2*d
*e) - (log(6*a^4*c^2*d^5 - b^3*c^3*e^5 - 2*a^3*b^2*c*d^5 - 8*a^2*c^4*d*e^4
+ b^4*c^2*d*e^4 + 2*b^5*c*d^2*e^3 - 2*a^3*b^3*d^5*x - 8*a^2*c^4*e^5*x - b^4
*c^2*e^5*x + 2*b^6*d^2*e^3*x + b^2*c^3*e^5*(b^2 - 4*a*c)^(1/2) - 18*a^3*c^3
*d^3*e^2 + 4*a*b*c^4*e^5 - 4*a*c^4*e^5*(b^2 - 4*a*c)^(1/2) - 5*a^2*c^3*d^2*
e^3*(b^2 - 4*a*c)^(1/2) + 7*a^4*b*c*d^5*x + b^5*c*d*e^4*x + 27*a^2*b^2*c^2*
d^3*e^2 + 2*a^3*b*c*d^5*(b^2 - 4*a*c)^(1/2) - 3*a^4*c*d^5*x*(b^2 - 4*a*c)^(
1/2) - 2*a*b^2*c^3*d*e^4 - 6*a*b^4*c*d^3*e^2 + 6*a^2*b^3*c*d^4*e - 21*a^3*b
*c^2*d^4*e + 6*a*b^2*c^3*e^5*x - 6*a*b^5*d^3*e^2*x + 6*a^2*b^4*d^4*e*x + 14
*a^4*c^2*d^4*e*x + 7*a^3*c^2*d^4*e*(b^2 - 4*a*c)^(1/2) - b^3*c^2*d*e^4*(b^2
 - 4*a*c)^(1/2) - 2*b^4*c*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 2*a^3*b^2*d^5*x*(b^
2 - 4*a*c)^(1/2) + b^3*c^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 2*b^5*d^2*e^3*x*(b^2
```

```
 - 4*a*c)^(1/2) - 13*a*b^3*c^2*d^2*e^3 + 21*a^2*b*c^3*d^2*e^3 - 10*a^3*c^3*
d^2*e^3*x + 6*a*b^3*c*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^2*c*d^4*e*(b^2
- 4*a*c)^(1/2) + 6*a*b^4*d^3*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*b^3*d^4*e*x*
(b^2 - 4*a*c)^(1/2) + 4*a^2*c^3*d*e^4*x*(b^2 - 4*a*c)^(1/2) + 32*a^2*b^3*c*
d^3*e^2*x - 35*a^3*b*c^2*d^3*e^2*x + 7*a*b^2*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2
) - 13*a^2*b*c^2*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 9*a^3*c^2*d^3*e^2*x*(b^2 - 4
*a*c)^(1/2) + 27*a^2*b^2*c^2*d^2*e^3*x + 4*a*b*c^3*d*e^4*(b^2 - 4*a*c)^(1/2
) - 4*a*b*c^3*e^5*x*(b^2 - 4*a*c)^(1/2) - b^4*c*d*e^4*x*(b^2 - 4*a*c)^(1/2)
 - 5*a*b^3*c^2*d*e^4*x - 14*a*b^4*c*d^2*e^3*x + 4*a^2*b*c^3*d*e^4*x - 26*a^
3*b^2*c*d^4*e*x + 14*a^3*b*c*d^4*e*x*(b^2 - 4*a*c)^(1/2) + 3*a*b^2*c^2*d*e^
4*x*(b^2 - 4*a*c)^(1/2) + 10*a*b^3*c*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 13*a^2
*b*c^2*d^2*e^3*x*(b^2 - 4*a*c)^(1/2) - 20*a^2*b^2*c*d^3*e^2*x*(b^2 - 4*a*c)
^(1/2))*((b^3*e)/2 + d*(2*a^2*c - (a*b^2)/2 + (a*b*(b^2 - 4*a*c)^(1/2))/2)
- (b^2*e*(b^2 - 4*a*c)^(1/2))/2 + a*c*e*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*e))/(
4*a*c^3*e^2 + 4*a^2*c^2*d^2 - b^2*c^2*e^2 + b^3*c*d*e - a*b^2*c*d^2 - 4*a*b
*c^2*d*e) - (e^2*log(d + e*x))/(a*d^3 - b*d^2*e + c*d*e^2) + log(x)/(c*d)
```

3.68 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx$

Optimal. Leaf size=193

$$-\frac{1}{cdx}+\frac{(2a^2cd+b^3e-ab(bd+3ce))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\,(ad^2-e(bd-ce))}+(abd$$

```
[Out] -1/c/d/x-(b*d+c*e)*ln(x)/c^2/d^2+e^3*ln(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))+1/2*
(a*b*d+a*c*e-b^2*e)*ln(a*x^2+b*x+c)/c^2/(a*d^2-e*(b*d-c*e))+(2*a^2*c*d+b^3*
e-a*b*(b*d+3*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*d^2-e*(b*d-
c*e))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 907, 648, 632, 212, 642}

$$\frac{(2a^2cd-ab(bd+3ce)+b^3e)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{(abd+ace+b^2(-e))\log\left(ax^2+bx+c\right)}{2c^2\,(ad^2-e(bd-ce))}+\frac{e^3\log(d+ex)}{d^2\,(ad^2-e(bd-ce))}-\frac{\log(x)(bd+ce)}{c^2d^2}-\frac{1}{cdx}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTanh[(b + 2*a*x)
/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) - ((b*
d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e)
)) + ((a*b*d - b^2*e + a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d
- c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)}\,dx=\int \frac{1}{x^2(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$=\int \left(\frac{1}{cdx^2}+\frac{-bd-ce}{c^2d^2x}+\frac{e^4}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{-a^2cd-b^3e+}{c^2\left(ad^2\right.}\right.$$

$$=-\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\int \frac{-a^2cd-b^3e+ab(bd+2ce)+}{c+bx+ax}}{c^2\left(ad^2-e(b\right.}$$

$$=-\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd-b^2e+ace\right)\int}{2c^2\left(ad^2-e(bd-\right.}$$

$$=-\frac{1}{cdx}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{d^2\left(ad^2-e(bd-ce)\right)}+\frac{\left(abd-b^2e+ace\right)\log}{2c^2\left(ad^2-e(b\right.}$$

$$=-\frac{1}{cdx}+\frac{\left(2a^2cd+b^3e-ab(bd+3ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)}-\frac{(bd+ce)}{c^2d^2}$$

Mathematica [A]
time = 0.12, size = 194, normalized size = 1.01

$$-\frac{1}{cdx}+\frac{(2a^2cd+b^3e-ab(bd+3ce))\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}\,(-ad^2+e(bd-ce))}-\frac{(bd+ce)\log(x)}{c^2d^2}+\frac{e^3\log(d+ex)}{ad^4+d^2e(-bd+ce)}+\frac{(abd-b^2e+ace)\log(c+x(b+ax))}{2c^2\,(ad^2+e(-bd+ce))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)),x]
```

```
[Out] -(1/(c*d*x)) + ((2*a^2*c*d + b^3*e - a*b*(b*d + 3*c*e))*ArcTan[(b + 2*a*x)/
Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) -
((b*d + c*e)*Log[x])/(c^2*d^2) + (e^3*Log[d + e*x])/(a*d^4 + d^2*e*(-(b*d)
+ c*e)) + ((a*b*d - b^2*e + a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2 + e*
(-(b*d) + c*e)))
```

Maple [A]
time = 0.28, size = 207, normalized size = 1.07

method	result
default	$\frac{e^3\ln(ex+d)}{d^2(a\,d^2-deb+c\,e^2)}+\frac{\frac{\left(a^2bd+a^2ce-a\,b^2e\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-a^2cd+a\,b^2d+2abce-b^3e-\frac{\left(a^2bd+a^2ce-a\,b^2e\right)b}{2a}\right)\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{(a\,d^2-deb+c\,e^2)c^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] e^3/d^2/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/(a*d^2-b*d*e+c*e^2)/c^2*(1/2*(a^2*b
*d+a^2*c*e-a*b^2*e)/a*ln(a*x^2+b*x+c)+2*(-a^2*c*d+a*b^2*d+2*a*b*c*e-b^3*e-1
/2*(a^2*b*d+a^2*c*e-a*b^2*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c
-b^2)^(1/2)))-1/c/d/x+1/c^2/d^2*(-b*d-c*e)*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.51, size = 210, normalized size = 1.09

$$\frac{(abd-b^2e+ace)\log(ax^2+bx+c)}{2\,(ac^2d^2-bc^2de+c^3e^2)}+\frac{e^4\log(|xe+d|)}{ad^4e-bd^3e^2+cd^2e^3}+\frac{(ab^2d-2\,a^2cd-b^3e+3\,abce)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ac^2d^2-bc^2de+c^3e^2)\sqrt{-b^2+4\,ac}}-\frac{(bd+ce)\log(|x|)}{c^2d^2}-\frac{1}{cdx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d),x, algorithm="giac")
```

```
[Out] 1/2*(a*b*d - b^2*e + a*c*e)*log(a*x^2 + b*x + c)/(a*c^2*d^2 - b*c^2*d*e + c
^3*e^2) + e^4*log(abs(x*e + d))/(a*d^4*e - b*d^3*e^2 + c*d^2*e^3) + (a*b^2*
d - 2*a^2*c*d - b^3*e + 3*a*b*c*e)*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/(
(a*c^2*d^2 - b*c^2*d*e + c^3*e^2)*sqrt(-b^2 + 4*a*c)) - (b*d + c*e)*log(abs
(x))/(c^2*d^2) - 1/(c*d*x)
```

Mupad [B]
time = 20.39, size = 2388, normalized size = 12.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (e^3*log(d + e*x))/(a*d^4 + c*d^2*e^2 - b*d^3*e) + (log((a^4*e^4*x)/(c^2*d^
2) - (((a*e*x*(a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 + 2*a^3*c*d^2*e^2 - 4*a*b^
2*c*e^4 + 2*a^2*b*c*d*e^3))/(c^2*d^2) - (((a*e*(a^2*b^2*d^4 - 4*a*c^3*e^4 -
```

a^3*c*d^4 + b^2*c^2*e^4 + b^4*d^2*e^2 + 4*a^2*c^2*d^2*e^2 - 2*a*b^3*d^3*e + b^3*c*d*e^3 - 4*a*b*c^2*d*e^3 + 5*a^2*b*c*d^3*e - 5*a*b^2*c*d^2*e^2))/(c*d) + (a*e*x*(2*a^3*b*d^4 + 2*b^3*c*e^4 + 2*b^4*d*e^3 - 2*a*b^3*d^2*e^2 - 2*a^2*b^2*d^3*e + 12*a^2*c^2*d*e^3 - 8*a*b*c^2*e^4 + a^3*c*d^3*e - 11*a*b^2*c*d*e^3 + 8*a^2*b*c*d^2*e^2))/(c*d) + (a*e*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(c^2*d^2))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e - a*b^2*d*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^2 + 4*a^2*c^3*d^2 - b^2*c^3*e^2 - a*b^2*c^2*d^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) + (log((a^4*e^4*x)/(c^2*d^2) - (((a*e*x*(a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 + 2*a^3*c*d^2*e^2 - 4*a*b^2*c*e^4 + 2*a^2*b*c*d*e^3))/(c^2*d^2) - (((a*e*(a^2*b^2*d^4 - 4*a*c^3*e^4 - a^3*c*d^4 + b^2*c^2*e^4 + b^4*d^2*e^2 + 4*a^2*c^2*d^2*e^2 - 2*a*b^3*d^3*e + b^3*c*d*e^3 - 4*a*b*c^2*d*e^3 + 5*a^2*b*c*d^3*e - 5*a*b^2*c*d^2*e^2))/(c*d) + (a*e*x*(2*a^3*b*d^4 + 2*b^3*c*e^4 + 2*b^4*d*e^3 - 2*a*b^3*d^2*e^2 - 2*a^2*b^2*d^3*e + 12*a^2*c^2*d*e^3 - 8*a*b*c^2*e^4 + a^3*c*d^3*e - 11*a*b^2*c*d*e^3 + 8*a^2*b*c*d^2*e^2))/(c*d) + (a*e*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(b*d + c*e)*(a^3*d^3 + b^3*e^3 - 3*a*b*c*e^3))/(c^2*d^2))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*e - b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - a*b^3*d + 4*a^2*b*c*d - 5*a*b

```
^2*c*e + a*b^2*d*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*d*(b^2 - 4*a*c)^(1/2) + 3*a*
b*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^4*e^2 + 4*a^2*c^3*d^2 - b^2*c^3*e^2 -
 a*b^2*c^2*d^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) - 1/(c*d*x) - (log(x)*(b*d +
 c*e))/(c^2*d^2)
```

3.69 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx$

Optimal. Leaf size=252

$$-\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}-\frac{(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{(b^2d^2+bcde-c(ad^2-c\ldots}{c^3d^3}$$

```
[Out] -1/2/c/d/x^2+(b*d+c*e)/c^2/d^2/x+(b^2*d^2+b*c*d*e-c*(a*d^2-c*e^2))*ln(x)/c^
3/d^3-e^4*ln(e*x+d)/d^3/(a*d^2-e*(b*d-c*e))+1/2*(a^2*c*d+b^3*e-a*b*(b*d+2*c
*e))*ln(a*x^2+b*x+c)/c^3/(a*d^2-e*(b*d-c*e))-(b^4*e+a^2*c*(3*b*d+2*c*e)-a*b
^2*(b*d+4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*d^2-e*(b*d-c*e
))/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 907, 648, 632, 212, 642}

$$\frac{(a^2cd-ab(bd+2ce)+b^3e)\log(ax^2+bx+c)}{2c^3(ad^2-e(bd-ce))}-\frac{(a^2c(3bd+2ce)-ab^2(bd+4ce)+b^4e)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}+\frac{\log(x)(-c(ad^2-ce^2)+b^2d^2+bcde)}{c^3d^3}-\frac{e^4\log(d+ex)}{d^3(ad^2-e(bd-ce))}+\frac{bd+ce}{c^2d^2x}-\frac{1}{2cdx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/2*1/(c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e
) - a*b^2*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^3*Sqrt[
b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e - c*(a*d^2 - c*
e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e)))
+ ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + b*x + a*x^2])/(2*c^3*(a*d^
2 - e*(b*d - c*e)))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)}\,dx = \int \frac{1}{x^3(d+ex)\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cdx^3} + \frac{-bd-ce}{c^2d^2x^2} + \frac{b^2d^2+bcde-c(ad^2-ce^2)}{c^3d^3x} + \frac{e^5}{d^3\left(-ad^2+e(bd-\right.}\right.$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} + \frac{\left(b^2d^2+bcde-c(ad^2-ce^2)\right)\log(x)}{c^3d^3} - \frac{e^4\log(d+}{d^3\left(ad^2-e(bc\right.}$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} + \frac{\left(b^2d^2+bcde-c(ad^2-ce^2)\right)\log(x)}{c^3d^3} - \frac{e^4\log(d+}{d^3\left(ad^2-e(bc\right.}$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} + \frac{\left(b^2d^2+bcde-c(ad^2-ce^2)\right)\log(x)}{c^3d^3} - \frac{e^4\log(d+}{d^3\left(ad^2-e(bc\right.}$$

$$= -\frac{1}{2cdx^2} + \frac{bd+ce}{c^2d^2x} - \frac{\left(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce)\right)\tanh^{-1}\left(\frac{}{\sqrt{b}}\right.}{c^3\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)}$$

Mathematica [A]

time = 0.16, size = 252, normalized size = 1.00

$$-\frac{1}{2cdx^2}+\frac{bd+ce}{c^2d^2x}-\frac{\left(b^4e+a^2c(3bd+2ce)-ab^2(bd+4ce)\right)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^3\sqrt{-b^2+4ac}\,\left(-ad^2+e(bd-ce)\right)}+\frac{\left(b^2d^2+bcde+c(-ad^2+ce^2)\right)\log(x)}{c^3d^3}-\frac{e^4\log(d+ex)}{ad^5+d^3e(-bd+ce)}+\frac{\left(a^2cd+b^3e-ab(bd+2ce)\right)\log(c+x(b+ax))}{2c^3\left(ad^2+e(-bd+ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)),x]
```

```
[Out] -1/2*1/(c*d*x^2) + (b*d + c*e)/(c^2*d^2*x) - ((b^4*e + a^2*c*(3*b*d + 2*c*e
) - a*b^2*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[
-b^2 + 4*a*c]*(-(a*d^2) + e*(b*d - c*e))) + ((b^2*d^2 + b*c*d*e + c*(-(a*d^
2) + c*e^2))*Log[x])/(c^3*d^3) - (e^4*Log[d + e*x])/(a*d^5 + d^3*e*(-(b*d)
+ c*e)) + ((a^2*c*d + b^3*e - a*b*(b*d + 2*c*e))*Log[c + x*(b + a*x)])/(2*c
^3*(a*d^2 + e*(-(b*d) + c*e)))
```

Maple [A]

time = 0.24, size = 277, normalized size = 1.10

method	result
default	$-\frac{e^4\ln(ex+d)}{d^3(a\,d^2-deb+c\,e^2)}+\frac{\frac{\left(a^3cd-a^2b^2d-2a^2bce+a\,b^3e\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(2a^2bcd+a^2c^2e-a\,b^3d-3a\,b^2ce+b^4e-\frac{\left(a^3cd-a^2b^2d-2a^2bce+a\,b^3e\right)}{2a}\right.}{\sqrt{4ac-b^2}}}{(a\,d^2-deb+c\,e^2)c^3}$
risch	$\frac{\frac{(bd+ce)x}{c^2d^2}-\frac{1}{2cd}}{x^2}-\frac{e^4\ln(-ex-d)}{d^3(a\,d^2-deb+c\,e^2)}-\frac{\ln(x)a}{c^2d}+\frac{\ln(x)b^2}{c^3d}+\frac{\ln(x)be}{c^2d^2}+\frac{\ln(x)e^2}{c\,d^3}+\left(\sum_{\_R=\mathrm{RootOf}\left((4a^2c^4d^2-a\,b^2c^3d^2-4abc^4c\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e^4/d^3/(a*d^2-b*d*e+c*e^2)*ln(e*x+d)+1/(a*d^2-b*d*e+c*e^2)/c^3*(1/2*(a^3*
c*d-a^2*b^2*d-2*a^2*b*c*e+a*b^3*e)/a*ln(a*x^2+b*x+c)+2*(2*a^2*b*c*d+a^2*c^2
*e-a*b^3*d-3*a*b^2*c*e+b^4*e-1/2*(a^3*c*d-a^2*b^2*d-2*a^2*b*c*e+a*b^3*e)*b/
a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-1/2/c/d/x^2-(-b*d
-c*e)/c^2/d^2/x+1/c^3/d^3*(-a*c*d^2+b^2*d^2+b*c*d*e+c^2*e^2)*ln(x)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.44, size = 279, normalized size = 1.11

$$-\frac{(ab^2d-a^2cd-b^3e+2\,abce)\log\left(ax^2+bx+c\right)}{2\left(ac^3d^2-bc^3de+c^4e^2\right)}-\frac{e^5\log\left(|xe+d|\right)}{ad^5e-bd^4e^2+cd^3e^3}-\frac{(ab^3d-3\,a^2bcd-b^4e+4\,ab^2ce-2\,a^2c^2e)\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{(ac^3d^2-bc^3de+c^4e^2)\sqrt{-b^2+4\,ac}}+\frac{(b^2d^2-acd^2+bcde+c^2e^2)\log\left(|x|\right)}{c^3d^3}-\frac{c^2d^2-2\left(bcd^2+c^2de\right)x}{2\,c^3d^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d),x, algorithm="giac")
```

```
[Out] -1/2*(a*b^2*d - a^2*c*d - b^3*e + 2*a*b*c*e)*log(a*x^2 + b*x + c)/(a*c^3*d^
2 - b*c^3*d*e + c^4*e^2) - e^5*log(abs(x*e + d))/(a*d^5*e - b*d^4*e^2 + c*d
^3*e^3) - (a*b^3*d - 3*a^2*b*c*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*arcta
n((2*a*x + b)/sqrt(-b^2 + 4*a*c))/((a*c^3*d^2 - b*c^3*d*e + c^4*e^2)*sqrt(-
b^2 + 4*a*c)) + (b^2*d^2 - a*c*d^2 + b*c*d*e + c^2*e^2)*log(abs(x))/(c^3*d^
3) - 1/2*(c^2*d^2 - 2*(b*c*d^2 + c^2*d*e)*x)/(c^3*d^3*x^2)
```

Mupad [B]
time = 26.16, size = 2500, normalized size = 9.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(d + e*x)*(a + b/x + c/x^2)),x)
```

```
[Out] (log((a^4*e^4*(b^2*d^2 + c^2*e^2 - a*c*d^2 + b*c*d*e))/(c^4*d^4) - (((((a*e
*(a^2*b^3*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e +
 b^3*c^2*d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a*b^
4*d^4*e - 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a*b^2
*c^2*d^2*e^3 + 8*a^2*b*c^2*d^3*e^2))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5 - 3*
a^4*c*d^5 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b^3*d^4
*e + 8*a^2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e^4 + 4*
a^3*b*c*d^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c^2*d^2*e
^3 + 10*a^2*b^2*c*d^3*e^2))/(c^2*d^2) - (a*e*(b^4*e*(b^2 - 4*a*c)^(1/2) - b
^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) -
5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c
*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e
+ b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4
*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x -
2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6
*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2
*e^2*x))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^4*e*(b^2 - 4*a*c
)^(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*
c)^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2)
+ 3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^
3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(a^3*b^3*d^6 + b^3*c^3*e^6
+ b^6*d^3*e^3 + 4*a^2*c^4*d*e^5 + a^4*c^2*d^5*e + 2*b^4*c^2*d*e^5 + 2*b^5*c
*d^2*e^4 - 4*a^3*c^3*d^3*e^3 - a^4*b*c*d^6 - 3*a*b*c^4*e^6 + 9*a^2*b^2*c^2*
d^3*e^3 - 8*a*b^2*c^3*d*e^5 - 6*a*b^4*c*d^3*e^3 - 9*a*b^3*c^2*d^2*e^4 + 7*a
^2*b*c^3*d^2*e^4))/(c^4*d^4) + (a*e*x*(a^4*b^2*d^6 + 2*a^2*c^4*e^6 + b^4*c^
2*e^6 + b^6*d^2*e^4 - 4*a*b^2*c^3*e^6 - 6*a^3*c^3*d^2*e^4 + 2*a^4*c^2*d^4*e
^2 + 2*b^5*c*d*e^5 + 11*a^2*b^2*c^2*d^2*e^4 - 10*a*b^3*c^2*d*e^5 - 6*a*b^4*
c*d^2*e^4 + 10*a^2*b*c^3*d*e^5))/(c^4*d^4))*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^
5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) - 5
*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*
d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b
^2)*(a*d^2 + c*e^2 - b*d*e)) - (a^5*e^5*x)/(c^3*d^3))*(b^4*e*(b^2 - 4*a*c)^
(1/2) - b^5*e + 4*a^3*c^2*d + a*b^4*d + 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)
^(1/2) - 5*a^2*b^2*c*d - 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) +
3*a^2*b*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a
*c^5*e^2 + 4*a^2*c^4*d^2 - b^2*c^4*e^2 - a*b^2*c^3*d^2 + b^3*c^3*d*e - 4*a*
b*c^4*d*e)) - (e^4*log(d + e*x))/(a*d^5 + c*d^3*e^2 - b*d^4*e) - (log((((a*
e*(a^3*b^3*d^6 + b^3*c^3*e^6 + b^6*d^3*e^3 + 4*a^2*c^4*d*e^5 + a^4*c^2*d^5*
e + 2*b^4*c^2*d*e^5 + 2*b^5*c*d^2*e^4 - 4*a^3*c^3*d^3*e^3 - a^4*b*c*d^6 - 3
*a*b*c^4*e^6 + 9*a^2*b^2*c^2*d^3*e^3 - 8*a*b^2*c^3*d*e^5 - 6*a*b^4*c*d^3*e^
3 - 9*a*b^3*c^2*d^2*e^4 + 7*a^2*b*c^3*d^2*e^4))/(c^4*d^4) - (((a*e*(a^2*b^3
*d^5 - 4*a*c^4*e^5 + b^2*c^3*e^5 + b^5*d^3*e^2 - 3*a^3*c^2*d^4*e + b^3*c^2*
d*e^4 + b^4*c*d^2*e^3 + 4*a^2*c^3*d^2*e^3 - 2*a^3*b*c*d^5 - 2*a*b^4*d^4*e -
 4*a*b*c^3*d*e^4 - 6*a*b^3*c*d^3*e^2 + 7*a^2*b^2*c*d^4*e - 5*a*b^2*c^2*d^2*
e^3 + 8*a^2*b*c^2*d^3*e^2))/(c^2*d^2) + (a*e*x*(2*a^3*b^2*d^5 - 3*a^4*c*d^5
```

```
 + 2*b^3*c^2*e^5 + 2*b^5*d^2*e^3 - 2*a*b^4*d^3*e^2 - 2*a^2*b^3*d^4*e + 8*a^
2*c^3*d*e^4 - 8*a^3*c^2*d^3*e^2 - 8*a*b*c^3*e^5 + b^4*c*d*e^4 + 4*a^3*b*c*d
^4*e - 6*a*b^2*c^2*d*e^4 - 12*a*b^3*c*d^2*e^3 + 16*a^2*b*c^2*d^2*e^3 + 10*a
^2*b^2*c*d^3*e^2))/(c^2*d^2) + (a*e*(b^5*e + b^4*e*(b^2 - 4*a*c)^(1/2) - 4*
a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2) + 5*a^2*b^2
*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b*c*d*(b^2 -
 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2
*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*
x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c
*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*
d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/
(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)))*(b^5*e + b^4*e*(b^2 - 4*a*c)
^(1/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2)
+ 5*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b
*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)) + (a*e*x*(a^4*b^2*d^6 + 2*a^2*c^4*e^6 + b^4
*c^2*e^6 + b^6*d^2*e^4 - 4*a*b^2*c^3*e^6 - 6*a^3*c^3*d^2*e^4 + 2*a^4*c^2*d^
4*e^2 + 2*b^5*c*d*e^5 + 11*a^2*b^2*c^2*d^2*e^4 - 10*a*b^3*c^2*d*e^5 - 6*a*b
^4*c*d^2*e^4 + 10*a^2*b*c^3*d*e^5))/(c^4*d^4))*(b^5*e + b^4*e*(b^2 - 4*a*c)
^(1/2) - 4*a^3*c^2*d - a*b^4*d - 6*a*b^3*c*e - a*b^3*d*(b^2 - 4*a*c)^(1/2)
+ 5*a^2*b^2*c*d + 8*a^2*b*c^2*e + 2*a^2*c^2*e*(b^2 - 4*a*c)^(1/2) + 3*a^2*b
*c*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b^2 - 4...
```

3.70 $\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=343

$$-\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3c}{a^3\sqrt{b^2-4ac}\,(ad^2-}$$

```
[Out] -(2*a*d+b*e)*x/a^2/e^3+1/2*x^2/a/e^2+d^5/e^4/(a*d^2-e*(b*d-c*e))/(e*x+d)+d^
4*(3*a*d^2-e*(4*b*d-5*c*e))*ln(e*x+d)/e^4/(a*d^2-e*(b*d-c*e))^2+1/2*(b^4*d^
2-2*b^3*c*d*e+4*a*b*c^2*d*e+a*c^2*(a*d^2-c*e^2)-b^2*c*(3*a*d^2-c*e^2))*ln(a
*x^2+b*x+c)/a^3/(a*d^2-e*(b*d-c*e))^2+(b^5*d^2-2*b^4*c*d*e+8*a*b^2*c^2*d*e-
4*a^2*c^3*d*e+a*b*c^2*(5*a*d^2-3*c*e^2)-b^3*c*(5*a*d^2-c*e^2))*arctanh((2*a
*x+b)/(-4*a*c+b^2)^(1/2))/a^3/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 1642, 648, 632, 212, 642}

$$\frac{\left(-b^2c(3ad^2-ce^2)+4abc^2de+ac^2(ad^2-ce^2)+b^4d^2-2b^3cde\right)\log\left(ax^2+bx+c\right)}{2a^3\left(ad^2-e(bd-ce)\right)^2}-\frac{x(2ad+be)}{a^2e^3}+\frac{\left(-4a^2c^3de-b^3c(5ad^2-ce^2)+8ab^2c^2de+abc^2(5ad^2-3ce^2)+b^5d^2-2b^4cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^3\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}+\frac{d^5}{e^4(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{d^4\log(d+ex)\left(3ad^2-e(4bd-5ce)\right)}{e^4\left(ad^2-e(bd-ce)\right)^2}+\frac{x^2}{2ae^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 - e*(b*d -
 c*e))*(d + e*x)) + ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^3*d
*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) - b^3*c*(5*a*d^2 - c*e^2))*ArcTanh[(b + 2*
a*x)/Sqrt[b^2 - 4*a*c]])/(a^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2)
+ (d^4*(3*a*d^2 - e*(4*b*d - 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 - e*(b*d - c
*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^2 - c*e^2) -
 b^2*c*(3*a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a^3*(a*d^2 - e*(b*d - c*
e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^3}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^5}{(d+ex)^2\,(c+bx+ax^2)}\,dx$$

$$= \int \left(\frac{-2ad-be}{a^2e^3} + \frac{x}{ae^2} + \frac{d^5}{e^3\,(-ad^2+e(bd-ce))\,(d+ex)^2} + \frac{d^4(3ad^2-e(}{e^3\,(ad^2-e(bd-}\right.$$

$$= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\,(ad^2-e(bd-ce))\,(d+ex)} + \frac{d^4(3ad^2-e(4bd-}{e^4\,(ad^2-e(}$$

$$= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\,(ad^2-e(bd-ce))\,(d+ex)} + \frac{d^4(3ad^2-e(4bd-}{e^4\,(ad^2-e(}$$

$$= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\,(ad^2-e(bd-ce))\,(d+ex)} + \frac{d^4(3ad^2-e(4bd-}{e^4\,(ad^2-e(}$$

$$= -\frac{(2ad+be)x}{a^2e^3} + \frac{x^2}{2ae^2} + \frac{d^5}{e^4\,(ad^2-e(bd-ce))\,(d+ex)} + \frac{(b^5d^2-2b^4cde+8}{}$$

Mathematica [A]

time = 0.25, size = 338, normalized size = 0.99

$$-\frac{(2ad+be)x}{a^2e^3}+\frac{x^2}{2ae^2}+\frac{d^5}{e^4\,(ad^2+e(-bd+ce))\,(d+ex)}-\frac{(b^5d^2-2b^4cde+8ab^2c^2de-4a^2c^3de+abc^2(5ad^2-3ce^2)+b^3c(-5ad^2+ce^2))\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^3\sqrt{-b^2+4ac}\,(ad^2+e(-bd+ce))^2}+\frac{(3ad^6+d^4e(-4bd+5ce))\log(d+ex)}{e^4\,(ad^2+e(-bd+ce))^2}+\frac{(b^4d^2-2b^3cde+4abc^2de+ac^2(ad^2-ce^2)+b^2c(-3ad^2+ce^2))\log(c+x(b+ax))}{2a^3\,(ad^2+e(-bd+ce))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(((2*a*d + b*e)*x)/(a^2*e^3)) + x^2/(2*a*e^2) + d^5/(e^4*(a*d^2 + e*(-(b*d
) + c*e))*(d + e*x)) - ((b^5*d^2 - 2*b^4*c*d*e + 8*a*b^2*c^2*d*e - 4*a^2*c^
3*d*e + a*b*c^2*(5*a*d^2 - 3*c*e^2) + b^3*c*(-5*a*d^2 + c*e^2))*ArcTan[(b +
 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c
*e))^2) + ((3*a*d^6 + d^4*e*(-4*b*d + 5*c*e))*Log[d + e*x])/(e^4*(a*d^2 + e
*(-(b*d) + c*e))^2) + ((b^4*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + a*c^2*(a*d^
2 - c*e^2) + b^2*c*(-3*a*d^2 + c*e^2))*Log[c + x*(b + a*x)])/(2*a^3*(a*d^2
+ e*(-(b*d) + c*e))^2)
```

Maple [A]

time = 0.28, size = 360, normalized size = 1.05

method	result
default	$\frac{d^4\left(3a\,d^2-4deb+5c\,e^2\right)\ln(ex+d)}{e^4\left(a\,d^2-deb+c\,e^2\right)^2}+\frac{d^5}{e^4\left(a\,d^2-deb+c\,e^2\right)(ex+d)}-\frac{-\frac{1}{2}ae\,x^2+2adx+ebx}{a^2e^3}+\frac{\frac{\left(a^2c^2d^2-3a\,b^2c\,d^2+4ed\,c^2ba-a\,c^3e^2+b^4d^2-\right.}{2a}}{}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*d^4*(3*a*d^2-4*b*d*e+5*c*e^2)/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)+1/e^4*d
^5/(a*d^2-b*d*e+c*e^2)/(e*x+d)-1/a^2/e^3*(-1/2*a*e*x^2+2*a*d*x+e*b*x)+1/(a*
d^2-b*d*e+c*e^2)^2/a^2*(1/2*(a^2*c^2*d^2-3*a*b^2*c*d^2+4*a*b*c^2*d*e-a*c^3*
e^2+b^4*d^2-2*b^3*c*d*e+b^2*c^2*e^2)/a*ln(a*x^2+b*x+c)+2*(-2*a*b*c^2*d^2+2*
a*c^3*d*e+b^3*c*d^2-2*b^2*c^2*d*e+b*c^3*e^2-1/2*(a^2*c^2*d^2-3*a*b^2*c*d^2+
4*a*b*c^2*d*e-a*c^3*e^2+b^4*d^2-2*b^3*c*d*e+b^2*c^2*e^2)*b/a)/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1334 vs. $2(345) = 690$.
time = 62.84, size = 2689, normalized size = 7.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^4*b^2 - 4*a^5*c)*d^7 - ((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3*e^4
+ (b^3*c^2 - 3*a*b*c^3)*x*e^7 - (2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*x -
(b^3*c^2 - 3*a*b*c^3)*d)*e^6 + ((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2*x - 2*(
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^2)*e^5)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2
 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)
) + ((a^2*b^2*c^2 - 4*a^3*c^3)*x^3 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*x^2)*e^7 -
 (2*(a^2*b^3*c - 4*a^3*b*c^2)*d*x^3 - (4*a*b^4*c - 19*a^2*b^2*c^2 + 12*a^3*
c^3)*d*x^2 + 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*x)*e^6 + ((a^2*b^4 - 2*a^3*b^2*c
 - 8*a^4*c^2)*d^2*x^3 - 2*(a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*x^2 + 4*(
a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*x)*e^5 - (2*(a^3*b^3 - 4*a^4*b*c)*
```

d^3*x^3 - (a^2*b^4 - 10*a^3*b^2*c + 24*a^4*c^2)*d^3*x^2 + 2*(a*b^5 - 6*a^2*
b^3*c + 8*a^3*b*c^2)*d^3*x)*e^4 + ((a^4*b^2 - 4*a^5*c)*d^4*x^3 + 4*(a^3*b^3
- 4*a^4*b*c)*d^4*x^2 - 8*(a^3*b^2*c - 4*a^4*c^2)*d^4*x)*e^3 - (3*(a^4*b^2
- 4*a^5*c)*d^5*x^2 - 6*(a^3*b^3 - 4*a^4*b*c)*d^5*x - 2*(a^3*b^2*c - 4*a^4*c
^2)*d^5)*e^2 - 2*(2*(a^4*b^2 - 4*a^5*c)*d^6*x + (a^3*b^3 - 4*a^4*b*c)*d^6)*
e + ((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*e^4 + (b^4*c^2 - 5*
a*b^2*c^3 + 4*a^2*c^4)*x*e^7 - (2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*x -
(b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d)*e^6 + ((b^6 - 7*a*b^4*c + 13*a^2*b^
2*c^2 - 4*a^3*c^3)*d^2*x - 2*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d^2)*e^5)*
log(a*x^2 + b*x + c) + 2*(3*(a^4*b^2 - 4*a^5*c)*d^7 + 5*(a^3*b^2*c - 4*a^4*
c^2)*d^4*x*e^3 - (4*(a^3*b^3 - 4*a^4*b*c)*d^5*x - 5*(a^3*b^2*c - 4*a^4*c^2)
*d^5)*e^2 + (3*(a^4*b^2 - 4*a^5*c)*d^6*x - 4*(a^3*b^3 - 4*a^4*b*c)*d^6)*e)*
log(x*e + d))/((a^5*b^2 - 4*a^6*c)*d^5*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*x*e^
9 - (2*(a^3*b^3*c - 4*a^4*b*c^2)*d*x - (a^3*b^2*c^2 - 4*a^4*c^3)*d)*e^8 + (
(a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^2*x - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2
)*e^7 - (2*(a^4*b^3 - 4*a^5*b*c)*d^3*x - (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2
)*d^3)*e^6 + ((a^5*b^2 - 4*a^6*c)*d^4*x - 2*(a^4*b^3 - 4*a^5*b*c)*d^4)*e^5)
, 1/2*(2*(a^4*b^2 - 4*a^5*c)*d^7 + 2*((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3*e
^4 + (b^3*c^2 - 3*a*b*c^3)*x*e^7 - (2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*x
- (b^3*c^2 - 3*a*b*c^3)*d)*e^6 + ((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2*x -
2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d^2)*e^5)*sqrt(-b^2 + 4*a*c)*arctan(-sq
rt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + ((a^2*b^2*c^2 - 4*a^3*c^3)*x^
3 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*x^2)*e^7 - (2*(a^2*b^3*c - 4*a^3*b*c^2)*d*x
^3 - (4*a*b^4*c - 19*a^2*b^2*c^2 + 12*a^3*c^3)*d*x^2 + 2*(a*b^3*c^2 - 4*a^2
*b*c^3)*d*x)*e^6 + ((a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*x^3 - 2*(a*b^5
- 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*x^2 + 4*(a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c
^3)*d^2*x)*e^5 - (2*(a^3*b^3 - 4*a^4*b*c)*d^3*x^3 - (a^2*b^4 - 10*a^3*b^2*c
+ 24*a^4*c^2)*d^3*x^2 + 2*(a*b^5 - 6*a^2*b^3*c + 8*a^3*b*c^2)*d^3*x)*e^4 +
((a^4*b^2 - 4*a^5*c)*d^4*x^3 + 4*(a^3*b^3 - 4*a^4*b*c)*d^4*x^2 - 8*(a^3*b^
2*c - 4*a^4*c^2)*d^4*x)*e^3 - (3*(a^4*b^2 - 4*a^5*c)*d^5*x^2 - 6*(a^3*b^3 -
4*a^4*b*c)*d^5*x - 2*(a^3*b^2*c - 4*a^4*c^2)*d^5)*e^2 - 2*(2*(a^4*b^2 - 4*
a^5*c)*d^6*x + (a^3*b^3 - 4*a^4*b*c)*d^6)*e + ((b^6 - 7*a*b^4*c + 13*a^2*b^
2*c^2 - 4*a^3*c^3)*d^3*e^4 + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*x*e^7 - (2
*(b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d*x - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c
^4)*d)*e^6 + ((b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*x - 2*(b^5
*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*d^2)*e^5)*log(a*x^2 + b*x + c) + 2*(3*(a^4*
b^2 - 4*a^5*c)*d^7 + 5*(a^3*b^2*c - 4*a^4*c^2)*d^4*x*e^3 - (4*(a^3*b^3 - 4*
a^4*b*c)*d^5*x - 5*(a^3*b^2*c - 4*a^4*c^2)*d^5)*e^2 + (3*(a^4*b^2 - 4*a^5*c
)*d^6*x - 4*(a^3*b^3 - 4*a^4*b*c)*d^6)*e)*log(x*e + d))/((a^5*b^2 - 4*a^6*c
)*d^5*e^4 + (a^3*b^2*c^2 - 4*a^4*c^3)*x*e^9 - (2*(a^3*b^3*c - 4*a^4*b*c^2)*
d*x - (a^3*b^2*c^2 - 4*a^4*c^3)*d)*e^8 + ((a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^
2)*d^2*x - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^2)*e^7 - (2*(a^4*b^3 - 4*a^5*b*c)*
d^3*x - (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^3)*e^6 + ((a^5*b^2 - 4*a^6*c)
*d^4*x - 2*(a^4*b^3 - 4*a^5*b*c)*d^4)*e^5)]

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.54, size = 565, normalized size = 1.65

$$\frac{d^5e^4}{(ad^2e^8 - bde^9 + ce^{10})(xe+d)} + \frac{(b^5d^2e^2 - 5\,ab^3cd^2e^2 + 5\,a^2bc^2d^2e^2 - 2\,b^4cde^3 + 8\,ab^2c^2de^3 - 4\,a^2c^3de^3 + b^3c^2e^4 - 3\,abc^3e^4)\arctan\left(-\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{(a^5d^4 - 2\,a^4bd^3e + a^3b^2d^2e^2 + 2\,a^4cd^2e^2 - 2\,a^3bcde^3 + a^3c^2e^4)\sqrt{-b^2+4\,ac}} + \frac{\left(a^2 - \frac{2\,(3\,a^2de + abe^2)e^{(-1)}}{xe+d}\right)(xe+d)^2e^{(-4)}}{2\,a^3} + \frac{(b^4d^2 - 3\,ab^2cd^2 + a^2c^2d^2 - 2\,b^3cde + 4\,abc^2de + b^2c^2e^2 - ac^3e^2)\log\left(-a + \frac{2\,ad}{xe+d} - \frac{ad^2}{(xe+d)^2} - \frac{be}{xe+d} + \frac{bde}{(xe+d)^2} - \frac{ce^2}{(xe+d)^2}\right)}{2\,(a^5d^4 - 2\,a^4bd^3e + a^3b^2d^2e^2 + 2\,a^4cd^2e^2 - 2\,a^3bcde^3 + a^3c^2e^4)} - \frac{(3\,a^2d^2 + 2\,abde + b^2e^2 - ace^2)e^{(-4)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] d^5*e^4/((a*d^2*e^8 - b*d*e^9 + c*e^10)*(x*e + d)) + (b^5*d^2*e^2 - 5*a*b^3
*c*d^2*e^2 + 5*a^2*b*c^2*d^2*e^2 - 2*b^4*c*d*e^3 + 8*a*b^2*c^2*d*e^3 - 4*a^
2*c^3*d*e^3 + b^3*c^2*e^4 - 3*a*b*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e +
d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c)
)*e^(-2)/((a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 + 2*a^4*c*d^2*e^2 - 2*
a^3*b*c*d*e^3 + a^3*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(a^2 - 2*(3*a^2*d*e
+ a*b*e^2)*e^(-1)/(x*e + d))*(x*e + d)^2*e^(-4)/a^3 + 1/2*(b^4*d^2 - 3*a*b^
2*c*d^2 + a^2*c^2*d^2 - 2*b^3*c*d*e + 4*a*b*c^2*d*e + b^2*c^2*e^2 - a*c^3*e
^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x
*e + d)^2 - c*e^2/(x*e + d)^2)/(a^5*d^4 - 2*a^4*b*d^3*e + a^3*b^2*d^2*e^2 +
 2*a^4*c*d^2*e^2 - 2*a^3*b*c*d*e^3 + a^3*c^2*e^4) - (3*a^2*d^2 + 2*a*b*d*e
+ b^2*e^2 - a*c*e^2)*e^(-4)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a^3
```

Mupad [**B**]
time = 8.04, size = 2500, normalized size = 7.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(3*a*d^6 + 5*c*d^4*e^2 - 4*b*d^5*e))/(c^2*e^8 + a^2*d^4*e^4 +
 b^2*d^2*e^6 - 2*b*c*d*e^7 - 2*a*b*d^3*e^5 + 2*a*c*d^2*e^6) - (log(12*a^5*c
*d^8 - 2*a*c^5*e^8 - 3*a^4*b^2*d^8 + b^2*c^4*e^8 + b^6*d^4*e^4 + 4*a^3*b^3*
d^7*e - 4*b^3*c^3*d*e^7 - 4*b^5*c*d^3*e^5 + b^5*d^4*e^4*(b^2 - 4*a*c)^(1/2)
 + 12*a^2*c^4*d^2*e^6 - 22*a^3*c^3*d^4*e^4 + 8*a^4*c^2*d^6*e^2 + 6*b^4*c^2*
d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^(1/2) + b*c^4*e^8*(b^2 - 4*a*c)^(1/2) -
```

6*a^5*d^8*x*(b^2 - 4*a*c)^(1/2) + 12*a*b*c^4*d*e^7 - 16*a^4*b*c*d^7*e - 4*
a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 20*a^3*c^2*d^5*e^3*(b^2 - 4*a*c)^(1/2
) + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^(1/2) + a*b*c^4*e^8*x + 24*a^5*c*d^7*e*
x + 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 - 4*a*c)^(1/2) + 12*a^4*c*d
^7*e*(b^2 - 4*a*c)^(1/2) + a*c^4*e^8*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^4*c*d^4*
e^4 + a*b^5*d^4*e^4*x - 6*a^4*b^2*d^7*e*x + 8*a^2*c^4*d*e^7*x + 4*a^3*b^2*d
^7*e*(b^2 - 4*a*c)^(1/2) - 4*b^2*c^3*d*e^7*(b^2 - 4*a*c)^(1/2) - 4*b^4*c*d^
3*e^5*(b^2 - 4*a*c)^(1/2) - 24*a*b^2*c^3*d^2*e^6 + 20*a*b^3*c^2*d^3*e^5 - 2
0*a^2*b*c^3*d^3*e^5 - 4*a^2*b^3*c*d^5*e^3 + 16*a^3*b*c^2*d^5*e^3 - 2*a^3*b^
2*c*d^6*e^2 - 4*a^2*b^4*d^5*e^3*x + 11*a^3*b^3*d^6*e^2*x - 8*a^3*c^3*d^3*e^
5*x + 40*a^4*c^2*d^5*e^3*x - 12*a*b*c^3*d^2*e^6*(b^2 - 4*a*c)^(1/2) - 4*a*b
^3*c*d^4*e^4*(b^2 - 4*a*c)^(1/2) - 24*a^3*b*c*d^6*e^2*(b^2 - 4*a*c)^(1/2) +
a*b^4*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 4*a^4*c*d^6*e^2*x*(b^2 - 4*a*c)^(1/2
) + 6*a*b^3*c^2*d^2*e^6*x - 18*a^2*b*c^3*d^2*e^6*x - 15*a^3*b*c^2*d^4*e^4*x
+ 6*a^3*b^2*c*d^5*e^3*x + 12*a*b^2*c^2*d^3*e^5*(b^2 - 4*a*c)^(1/2) - 2*a^2
*b*c^2*d^4*e^4*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^2*c*d^5*e^3*(b^2 - 4*a*c)^(1/2
) + 4*a^2*b^3*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) - 11*a^3*b^2*d^6*e^2*x*(b^2 - 4
*a*c)^(1/2) - 6*a^2*c^3*d^2*e^6*x*(b^2 - 4*a*c)^(1/2) + 11*a^3*c^2*d^4*e^4*
x*(b^2 - 4*a*c)^(1/2) + 16*a^2*b^2*c^2*d^3*e^5*x + 14*a^4*b*d^7*e*x*(b^2 -
4*a*c)^(1/2) - 4*a*b^2*c^3*d*e^7*x - 4*a*b^4*c*d^3*e^5*x - 44*a^4*b*c*d^6*e
^2*x - 4*a*b*c^3*d*e^7*x*(b^2 - 4*a*c)^(1/2) - 4*a*b^3*c*d^3*e^5*x*(b^2 - 4
*a*c)^(1/2) + 2*a^3*b*c*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d^2*e^6
x(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d^3*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a^2*
b^2*c*d^4*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^6*d^2 + b^5*d^2*(b^2 - 4*a*c)^(1/2)
- 4*a^3*c^3*d^2 + 4*a^2*c^4*e^2 + b^4*c^2*e^2 - 5*a*b^2*c^3*e^2 + b^3*c^2*
e^2*(b^2 - 4*a*c)^(1/2) - 2*b^5*c*d*e + 13*a^2*b^2*c^2*d^2 - 7*a*b^4*c*d^2
+ 12*a*b^3*c^2*d*e - 16*a^2*b*c^3*d*e - 5*a*b^3*c*d^2*(b^2 - 4*a*c)^(1/2) -
3*a*b*c^3*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^2*c^3*d*e*(b^2 - 4*a*c)^(1/2) + 5*
a^2*b*c^2*d^2*(b^2 - 4*a*c)^(1/2) - 2*b^4*c*d*e*(b^2 - 4*a*c)^(1/2) + 8*a*b
^2*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^6*c*d^4 - a^5*b^2*d^4 + 4*a^4*c^3*
e^4 + 2*a^4*b^3*d^3*e - a^3*b^2*c^2*e^4 - a^3*b^4*d^2*e^2 + 8*a^5*c^2*d^2*e
^2 - 8*a^5*b*c*d^3*e + 2*a^3*b^3*c*d*e^3 - 8*a^4*b*c^2*d*e^3 + 2*a^4*b^2*c*
d^2*e^2)) - (log(2*a*c^5*e^8 - 12*a^5*c*d^8 + 3*a^4*b^2*d^8 - b^2*c^4*e^8 -
b^6*d^4*e^4 - 4*a^3*b^3*d^7*e + 4*b^3*c^3*d*e^7 + 4*b^5*c*d^3*e^5 + b^5*d^
4*e^4*(b^2 - 4*a*c)^(1/2) - 12*a^2*c^4*d^2*e^6 + 22*a^3*c^3*d^4*e^4 - 8*a^4
*c^2*d^6*e^2 - 6*b^4*c^2*d^2*e^6 - 3*a^4*b*d^8*(b^2 - 4*a*c)^(1/2) + b*c^4*
e^8*(b^2 - 4*a*c)^(1/2) - 6*a^5*d^8*x*(b^2 - 4*a*c)^(1/2) - 12*a*b*c^4*d*e^
7 + 16*a^4*b*c*d^7*e - 4*a^2*c^3*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 20*a^3*c^2*d
^5*e^3*(b^2 - 4*a*c)^(1/2) + 6*b^3*c^2*d^2*e^6*(b^2 - 4*a*c)^(1/2) - a*b*c^
4*e^8*x - 24*a^5*c*d^7*e*x - 14*a^2*b^2*c^2*d^4*e^4 + 4*a*c^4*d*e^7*(b^2 -
4*a*c)^(1/2) + 12*a^4*c*d^7*e*(b^2 - 4*a*c)^(1/2) + a*c^4*e^8*x*(b^2 - 4*a*
c)^(1/2) + 6*a*b^4*c*d^4*e^4 - a*b^5*d^4*e^4*x + 6*a^4*b^2*d^7*e*x - 8*a^2*
c^4*d*e^7*x + 4*a^3*b^2*d^7*e*(b^2 - 4*a*c)^(1/2) - 4*b^2*c^3*d*e^7*(b^2 -
4*a*c)^(1/2) - 4*b^4*c*d^3*e^5*(b^2 - 4*a*c)^(1/2) + 24*a*b^2*c^3*d^2*e^6 -
20*a*b^3*c^2*d^3*e^5 + 20*a^2*b*c^3*d^3*e^5 + 4*a^2*b^3*c*d^5*e^3 - 16*a^3

```
*b*c^2*d^5*e^3 + 2*a^3*b^2*c*d^6*e^2 + 4*a^2*b^4*d^5*e^3*x - 11*a^3*b^3*d^6
*e^2*x + 8*a^3*c^3*d^3*e^5*x - 40*a^4*c^2*d^5*e^3*x - 12*a*b*c^3*d^2*e^6*(b
^2 - 4*a*c)^(1/2) - 4*a*b^3*c*d^4*e^4*(b^2 - 4*a*c)^(1/2) - 24*a^3*b*c*d^6*
e^2*(b^2 - 4*a*c)^(1/2) + a*b^4*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 4*a^4*c*d^6
*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a*b^3*c^2*d^2*e^6*x + 18*a^2*b*c^3*d^2*e^6*x
 + 15*a^3*b*c^2*d^4*e^4*x - 6*a^3*b^2*c*d^5*e^3*x + 12*a*b^2*c^2*d^3*e^5*(b
^2 - 4*a*c)^(1/2) - 2*a^2*b*c^2*d^4*e^4*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^2*c*d
^5*e^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*b^3*d^5*e^3*x*(b^2 - 4*a*c)^(1/2) - 11*a
^3*b^2*d^6*e^2*x*(b^2 - 4*a*c)^(1/2) - 6*a^2*c^3*d^2*e^6*x*(b^2 - 4*a*c)^(1
/2) + 11*a^3*c^2*d^4*e^4*x*(b^2 - 4*a*c)^(1/2) - 16*a^2*b^2*c^2*d^3*e^5*x +
 14*a^4*b*d^7*e*x*(b^2 - 4*a*c)^(1/2) + 4*a*b^2*c^3*d*e^7*x + 4*a*b^4*c*d^3
*e^5*x + 44*a^4*b*c*d^6*e^2*x - 4*a*b*c^3*d*e^7*x*(b^2 - 4*a*c)^(1/2) - 4*a
*b^3*c*d^3*e^5*x*(b^2 - 4*a*c)^(1/2) + 2*a^3*b*c*d^5*e^3*x*(b^2 - 4*a*c)^(1
/2) + 6*a*b^2*c^2*d^2*e^6*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d^3*e^5*x*(b^
2 - 4*a*c)^(1/2) - 8*a^2*b^2*c*d^4*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^6*d^2 - b^
5*d^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^3*d^2 + 4*a...
```

3.71 $\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=274

$$\frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{\left(b^4d^2-2b^3cde+6abc^2de+2ac^2(ad^2-ce^2)-b^2c(4ad^2-ce^2)\right)\tanh^{-1}}{a^2\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}$$

```
[Out] x/a/e^2-d^4/e^3/(a*d^2-e*(b*d-c*e))/(e*x+d)-d^3*(2*a*d^2-e*(3*b*d-4*c*e))*l
n(e*x+d)/e^3/(a*d^2-e*(b*d-c*e))^2-1/2*(b*d-c*e)*(-2*a*c*d+b^2*d-b*c*e)*ln(
a*x^2+b*x+c)/a^2/(a*d^2-e*(b*d-c*e))^2-(b^4*d^2-2*b^3*c*d*e+6*a*b*c^2*d*e+2
*a*c^2*(a*d^2-c*e^2)-b^2*c*(4*a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^
(1/2))/a^2/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 1642, 648, 632, 212, 642}

$$-\frac{(bd-ce)\left(-2acd+b^2d-bce\right)\log\left(ax^2+bx+c\right)}{2a^2\left(ad^2-e(bd-ce)\right)^2}-\frac{\left(-b^2c(4ad^2-ce^2)+6abc^2de+2ac^2(ad^2-ce^2)+b^4d^2-2b^3cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a^2\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}-\frac{d^4}{e^3(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{d^3\log(d+ex)\left(2ad^2-e(3bd-4ce)\right)}{e^3\left(ad^2-e(bd-ce)\right)^2}+\frac{x}{ae^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 - e*(b*d - c*e))*(d + e*x)) - ((b^4*d^2 - 2*b^3
*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) - b^2*c*(4*a*d^2 - c*e^2))
*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(a^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*
(b*d - c*e))^2) - (d^3*(2*a*d^2 - e*(3*b*d - 4*c*e))*Log[d + e*x])/(e^3*(a*
d^2 - e*(b*d - c*e))^2) - ((b*d - c*e)*(b^2*d - 2*a*c*d - b*c*e)*Log[c + b*
x + a*x^2])/(2*a^2*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^4}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{ae^2}+\frac{d^4}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)^2}+\frac{d^3(-2ad^2+e(3bd-4ce))}{e^2\left(ad^2-e(bd-ce)\right)^2(d+ex}\right.$$

$$= \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{d^3(2ad^2-e(3bd-4ce))\log(d+ex}{e^3\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{d^3(2ad^2-e(3bd-4ce))\log(d+ex}{e^3\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{d^3(2ad^2-e(3bd-4ce))\log(d+ex}{e^3\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2-e(bd-ce)\right)(d+ex)}-\frac{(b^4d^2-2b^3cde+6abc^2de+2ac^2(ac}{a^2\sqrt{b^2-}}$$

Mathematica [A]
time = 0.20, size = 269, normalized size = 0.98

$$\frac{x}{ae^2}-\frac{d^4}{e^3\left(ad^2+e(-bd+ce)\right)(d+ex)}+\frac{\left(b^4d^2-2b^3cde+6abc^2de+2ac^2(ad^2-ce^2)+b^2c(-4ad^2+ce^2)\right)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a^2\sqrt{-b^2+4ac}\,\left(ad^2+e(-bd+ce)\right)^2}-\frac{\left(2ad^5+d^3e(-3bd+4ce)\right)\log(d+ex)}{e^3\left(ad^2+e(-bd+ce)\right)^2}+\frac{(bd-ce)\left(-b^2d+2acd+bce\right)\log(c+x(b+ax))}{2a^2\left(ad^2+e(-bd+ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] x/(a*e^2) - d^4/(e^3*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((b^4*d^2 - 2*
b^3*c*d*e + 6*a*b*c^2*d*e + 2*a*c^2*(a*d^2 - c*e^2) + b^2*c*(-4*a*d^2 + c*e
^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(a^2*Sqrt[-b^2 + 4*a*c]*(a*d^2
 + e*(-(b*d) + c*e))^2) - ((2*a*d^5 + d^3*e*(-3*b*d + 4*c*e))*Log[d + e*x])
/(e^3*(a*d^2 + e*(-(b*d) + c*e))^2) + ((b*d - c*e)*(-(b^2*d) + 2*a*c*d + b*
c*e)*Log[c + x*(b + a*x)])/(2*a^2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [A]
time = 0.31, size = 290, normalized size = 1.06

method	result
default	$-\frac{d^4}{e^3(a\,d^2-deb+c\,e^2)(ex+d)}-\frac{d^3\left(2a\,d^2-3deb+4c\,e^2\right)\ln(ex+d)}{e^3(a\,d^2-deb+c\,e^2)^2}+\frac{x}{a\,e^2}+\frac{\frac{\left(2abc\,d^2-2a\,c^2de-b^3d^2+2b^2cde-c^2e^2b\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\cdots}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^3*d^4/(a*d^2-b*d*e+c*e^2)/(e*x+d)-1/e^3*d^3*(2*a*d^2-3*b*d*e+4*c*e^2)/
(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)+x/a/e^2+1/(a*d^2-b*d*e+c*e^2)^2/a*(1/2*(2*a
*b*c*d^2-2*a*c^2*d*e-b^3*d^2+2*b^2*c*d*e-b*c^2*e^2)/a*ln(a*x^2+b*x+c)+2*(a*
c^2*d^2-b^2*c*d^2+2*e*d*c^2*b-c^3*e^2-1/2*(2*a*b*c*d^2-2*a*c^2*d*e-b^3*d^2+
2*b^2*c*d*e-b*c^2*e^2)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^
(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see 'assume?' for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(279) = 558$.
time = 26.56, size = 2131, normalized size = 7.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a^3*b^2 - 4*a^4*c)*d^6 - 2*(a*b^2*c^2 - 4*a^2*c^3)*x^2*e^6 + ((b^
4 - 4*a*b^2*c + 2*a^2*c^2)*d^3*e^3 + (b^2*c^2 - 2*a*c^3)*x*e^6 - (2*(b^3*c
- 3*a*b*c^2)*d*x - (b^2*c^2 - 2*a*c^3)*d)*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*c
^2)*d^2*x - 2*(b^3*c - 3*a*b*c^2)*d^2)*e^4)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^
2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c
)) + 2*(2*(a*b^3*c - 4*a^2*b*c^2)*d*x^2 - (a*b^2*c^2 - 4*a^2*c^3)*d*x)*e^5
- 2*((a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*x^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*
d^2*x)*e^4 + 2*(2*(a^2*b^3 - 4*a^3*b*c)*d^3*x^2 - (a*b^4 - 2*a^2*b^2*c - 8*
a^3*c^2)*d^3*x)*e^3 - 2*((a^3*b^2 - 4*a^4*c)*d^4*x^2 - 2*(a^2*b^3 - 4*a^3*b
*c)*d^4*x - (a^2*b^2*c - 4*a^3*c^2)*d^4)*e^2 - 2*((a^3*b^2 - 4*a^4*c)*d^5*x
 + (a^2*b^3 - 4*a^3*b*c)*d^5)*e + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^3*e^3
+ (b^3*c^2 - 4*a*b*c^3)*x*e^6 - (2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*x -
(b^3*c^2 - 4*a*b*c^3)*d)*e^5 + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d^2*x - 2*(
b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2)*e^4)*log(a*x^2 + b*x + c) + 2*(2*(a^3
*b^2 - 4*a^4*c)*d^6 + 4*(a^2*b^2*c - 4*a^3*c^2)*d^3*x*e^3 - (3*(a^2*b^3 - 4
*a^3*b*c)*d^4*x - 4*(a^2*b^2*c - 4*a^3*c^2)*d^4)*e^2 + (2*(a^3*b^2 - 4*a^4*
c)*d^5*x - 3*(a^2*b^3 - 4*a^3*b*c)*d^5)*e)*log(x*e + d))/((a^4*b^2 - 4*a^5*
c)*d^5*e^3 + (a^2*b^2*c^2 - 4*a^3*c^3)*x*e^8 - (2*(a^2*b^3*c - 4*a^3*b*c^2)
*d*x - (a^2*b^2*c^2 - 4*a^3*c^3)*d)*e^7 + ((a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c
^2)*d^2*x - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2)*e^6 - (2*(a^3*b^3 - 4*a^4*b*c)
*d^3*x - (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3)*e^5 + ((a^4*b^2 - 4*a^5*c
)*d^4*x - 2*(a^3*b^3 - 4*a^4*b*c)*d^4)*e^4), -1/2*(2*(a^3*b^2 - 4*a^4*c)*d^
6 - 2*(a*b^2*c^2 - 4*a^2*c^3)*x^2*e^6 + 2*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^
3*e^3 + (b^2*c^2 - 2*a*c^3)*x*e^6 - (2*(b^3*c - 3*a*b*c^2)*d*x - (b^2*c^2 -
 2*a*c^3)*d)*e^5 + ((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^2*x - 2*(b^3*c - 3*a*b*
c^2)*d^2)*e^4)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b
^2 - 4*a*c)) + 2*(2*(a*b^3*c - 4*a^2*b*c^2)*d*x^2 - (a*b^2*c^2 - 4*a^2*c^3)
*d*x)*e^5 - 2*((a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*x^2 - 2*(a*b^3*c - 4*a
^2*b*c^2)*d^2*x)*e^4 + 2*(2*(a^2*b^3 - 4*a^3*b*c)*d^3*x^2 - (a*b^4 - 2*a^2*
b^2*c - 8*a^3*c^2)*d^3*x)*e^3 - 2*((a^3*b^2 - 4*a^4*c)*d^4*x^2 - 2*(a^2*b^3
 - 4*a^3*b*c)*d^4*x - (a^2*b^2*c - 4*a^3*c^2)*d^4)*e^2 - 2*((a^3*b^2 - 4*a^
4*c)*d^5*x + (a^2*b^3 - 4*a^3*b*c)*d^5)*e + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2
)*d^3*e^3 + (b^3*c^2 - 4*a*b*c^3)*x*e^6 - (2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c
```

```
^3)*d*x - (b^3*c^2 - 4*a*b*c^3)*d)*e^5 + ((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d
^2*x - 2*(b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d^2)*e^4)*log(a*x^2 + b*x + c) +
 2*(2*(a^3*b^2 - 4*a^4*c)*d^6 + 4*(a^2*b^2*c - 4*a^3*c^2)*d^3*x*e^3 - (3*(a
^2*b^3 - 4*a^3*b*c)*d^4*x - 4*(a^2*b^2*c - 4*a^3*c^2)*d^4)*e^2 + (2*(a^3*b^
2 - 4*a^4*c)*d^5*x - 3*(a^2*b^3 - 4*a^3*b*c)*d^5)*e)*log(x*e + d))/((a^4*b^
2 - 4*a^5*c)*d^5*e^3 + (a^2*b^2*c^2 - 4*a^3*c^3)*x*e^8 - (2*(a^2*b^3*c - 4*
a^3*b*c^2)*d*x - (a^2*b^2*c^2 - 4*a^3*c^3)*d)*e^7 + ((a^2*b^4 - 2*a^3*b^2*c
 - 8*a^4*c^2)*d^2*x - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^2)*e^6 - (2*(a^3*b^3 -
4*a^4*b*c)*d^3*x - (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^3)*e^5 + ((a^4*b^2
 - 4*a^5*c)*d^4*x - 2*(a^3*b^3 - 4*a^4*b*c)*d^4)*e^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.69, size = 476, normalized size = 1.74

$$-\frac{d^4e^3}{(ad^2e^6-bde^7+ce^8)(xe+d)}-\frac{(b^4d^2e^2-4\,ab^2cd^2e^2+2\,a^2c^2d^2e^2-2\,b^3cde^3+6\,abc^2de^3+b^2c^2e^4-2\,ac^3e^4)\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{(a^4d^4-2\,a^3bd^3e+a^2b^2d^2e^2+2\,a^3cd^2e^2-2\,a^2bcde^3+a^2c^2e^4)\sqrt{-b^2+4\,ac}}+\frac{(xe+d)e^{(-3)}}{a}-\frac{(b^3d^2-2\,abcd^2-2\,b^2cde+2\,ac^2de+bc^2e^2)\log\left(-a+\frac{2\,ad}{xe+d}-\frac{ad^2}{(xe+d)^2}-\frac{be}{xe+d}+\frac{bde}{(xe+d)^2}-\frac{ce^2}{(xe+d)^2}\right)}{2\,(a^4d^4-2\,a^3bd^3e+a^2b^2d^2e^2+2\,a^3cd^2e^2-2\,a^2bcde^3+a^2c^2e^4)}+\frac{(2\,ad+be)e^{(-3)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^2}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -d^4*e^3/((a*d^2*e^6 - b*d*e^7 + c*e^8)*(x*e + d)) - (b^4*d^2*e^2 - 4*a*b^2
*c*d^2*e^2 + 2*a^2*c^2*d^2*e^2 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3 + b^2*c^2*
e^4 - 2*a*c^3*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e
+ d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^4*d^4 - 2*a
^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a^2*c^2*
e^4)*sqrt(-b^2 + 4*a*c)) + (x*e + d)*e^(-3)/a - 1/2*(b^3*d^2 - 2*a*b*c*d^2
- 2*b^2*c*d*e + 2*a*c^2*d*e + b*c^2*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(
x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^4*d^
4 - 2*a^3*b*d^3*e + a^2*b^2*d^2*e^2 + 2*a^3*c*d^2*e^2 - 2*a^2*b*c*d*e^3 + a
^2*c^2*e^4) + (2*a*d + b*e)*e^(-3)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a^2
```

Mupad [**B**]
time = 6.00, size = 2495, normalized size = 9.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x)^2*(a + b/x + c/x^2)),x)

[Out] x/(a*e^2) - (log(d + e*x)*(2*a*d^5 + 4*c*d^3*e^2 - 3*b*d^4*e))/(c^2*e^7 + a
^2*d^4*e^3 + b^2*d^2*e^5 - 2*b*c*d*e^6 - 2*a*b*d^3*e^4 + 2*a*c*d^2*e^5) + (
log(8*a^4*c*d^7 + b*c^4*e^7 + c^4*e^7*(b^2 - 4*a*c)^(1/2) - 2*a^3*b^2*d^7 +
 b^5*d^4*e^3 + 3*a^2*b^3*d^6*e - 4*b^2*c^3*d*e^6 - 4*b^4*c*d^3*e^4 + b^4*d^
4*e^3*(b^2 - 4*a*c)^(1/2) - 24*a^2*c^3*d^3*e^4 + 8*a^3*c^2*d^5*e^2 + 6*b^3*
c^2*d^2*e^5 + 8*a*c^4*d*e^6 + 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*c)^(1/
2) - 4*a^4*d^7*x*(b^2 - 4*a*c)^(1/2) - 12*a^3*b*c*d^6*e + 17*a^2*c^2*d^4*e^
3*(b^2 - 4*a*c)^(1/2) + 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) + 16*a^4*c*d^
6*e*x + 8*a^3*c*d^6*e*(b^2 - 4*a*c)^(1/2) - 4*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/
2) - 18*a*b*c^3*d^2*e^5 - 8*a*b^3*c*d^4*e^3 - 2*a*b^4*d^4*e^3*x - 4*a^3*b^2
*d^6*e*x + 3*a^2*b^2*d^6*e*(b^2 - 4*a*c)^(1/2) - 6*a*c^3*d^2*e^5*(b^2 - 4*a
*c)^(1/2) - 4*b^3*c*d^3*e^4*(b^2 - 4*a*c)^(1/2) + 20*a*b^2*c^2*d^3*e^4 + 17
*a^2*b*c^2*d^4*e^3 - 2*a^2*b^2*c*d^5*e^2 + 8*a^2*b^3*d^5*e^2*x - 12*a^2*c^3
*d^2*e^5*x + 34*a^3*c^2*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) -
 18*a^2*b*c*d^5*e^2*(b^2 - 4*a*c)^(1/2) + 4*a*b^3*d^4*e^3*x*(b^2 - 4*a*c)^(
1/2) - 4*a^3*c*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 6*a*b^2*c^2*d^2*e^5*x - 4*a^
2*b*c^2*d^3*e^4*x - 8*a^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^3*d*e
^6*x + 12*a^2*c^2*d^3*e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a^3*b*d^6*e*x*(b^2 - 4
*a*c)^(1/2) - 4*a*c^3*d*e^6*x*(b^2 - 4*a*c)^(1/2) - 32*a^3*b*c*d^5*e^2*x +
6*a*b*c^2*d^2*e^5*x*(b^2 - 4*a*c)^(1/2) - 8*a*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)
^(1/2))*(b^5*d^2 + b^4*d^2*(b^2 - 4*a*c)^(1/2) + b^3*c^2*e^2 + 8*a^2*b*c^2*
d^2 + 2*a^2*c^2*d^2*(b^2 - 4*a*c)^(1/2) + b^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) -
 2*b^4*c*d*e - 6*a*b^3*c*d^2 - 4*a*b*c^3*e^2 - 8*a^2*c^3*d*e - 2*a*c^3*e^2*
(b^2 - 4*a*c)^(1/2) + 10*a*b^2*c^2*d*e - 4*a*b^2*c*d^2*(b^2 - 4*a*c)^(1/2)
- 2*b^3*c*d*e*(b^2 - 4*a*c)^(1/2) + 6*a*b*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*
(4*a^5*c*d^4 - a^4*b^2*d^4 + 4*a^3*c^3*e^4 + 2*a^3*b^3*d^3*e - a^2*b^2*c^2*
e^4 - a^2*b^4*d^2*e^2 + 8*a^4*c^2*d^2*e^2 - 8*a^4*b*c*d^3*e + 2*a^2*b^3*c*d
*e^3 - 8*a^3*b*c^2*d*e^3 + 2*a^3*b^2*c*d^2*e^2)) - (log(c^4*e^7*(b^2 - 4*a*
c)^(1/2) - b*c^4*e^7 - 8*a^4*c*d^7 + 2*a^3*b^2*d^7 - b^5*d^4*e^3 - 3*a^2*b^
3*d^6*e + 4*b^2*c^3*d*e^6 + 4*b^4*c*d^3*e^4 + b^4*d^4*e^3*(b^2 - 4*a*c)^(1/
2) + 24*a^2*c^3*d^3*e^4 - 8*a^3*c^2*d^5*e^2 - 6*b^3*c^2*d^2*e^5 - 8*a*c^4*d
*e^6 - 2*a*c^4*e^7*x - 2*a^3*b*d^7*(b^2 - 4*a*c)^(1/2) - 4*a^4*d^7*x*(b^2 -
 4*a*c)^(1/2) + 12*a^3*b*c*d^6*e + 17*a^2*c^2*d^4*e^3*(b^2 - 4*a*c)^(1/2) +
 6*b^2*c^2*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 16*a^4*c*d^6*e*x + 8*a^3*c*d^6*e*(
b^2 - 4*a*c)^(1/2) - 4*b*c^3*d*e^6*(b^2 - 4*a*c)^(1/2) + 18*a*b*c^3*d^2*e^5
 + 8*a*b^3*c*d^4*e^3 + 2*a*b^4*d^4*e^3*x + 4*a^3*b^2*d^6*e*x + 3*a^2*b^2*d^
6*e*(b^2 - 4*a*c)^(1/2) - 6*a*c^3*d^2*e^5*(b^2 - 4*a*c)^(1/2) - 4*b^3*c*d^3
*e^4*(b^2 - 4*a*c)^(1/2) - 20*a*b^2*c^2*d^3*e^4 - 17*a^2*b*c^2*d^4*e^3 + 2*
a^2*b^2*c*d^5*e^2 - 8*a^2*b^3*d^5*e^2*x + 12*a^2*c^3*d^2*e^5*x - 34*a^3*c^2
*d^4*e^3*x + 4*a*b*c^2*d^3*e^4*(b^2 - 4*a*c)^(1/2) - 18*a^2*b*c*d^5*e^2*(b^
2 - 4*a*c)^(1/2) + 4*a*b^3*d^4*e^3*x*(b^2 - 4*a*c)^(1/2) - 4*a^3*c*d^5*e^2*
x*(b^2 - 4*a*c)^(1/2) - 6*a*b^2*c^2*d^2*e^5*x + 4*a^2*b*c^2*d^3*e^4*x - 8*a
^2*b^2*d^5*e^2*x*(b^2 - 4*a*c)^(1/2) + 4*a*b*c^3*d*e^6*x + 12*a^2*c^2*d^3*e
```

```
^4*x*(b^2 - 4*a*c)^(1/2) + 10*a^3*b*d^6*e*x*(b^2 - 4*a*c)^(1/2) - 4*a*c^3*d
*e^6*x*(b^2 - 4*a*c)^(1/2) + 32*a^3*b*c*d^5*e^2*x + 6*a*b*c^2*d^2*e^5*x*(b^
2 - 4*a*c)^(1/2) - 8*a*b^2*c*d^3*e^4*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2*(b^2 -
 4*a*c)^(1/2) - b^5*d^2 - b^3*c^2*e^2 - 8*a^2*b*c^2*d^2 + 2*a^2*c^2*d^2*(b^
2 - 4*a*c)^(1/2) + b^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 2*b^4*c*d*e + 6*a*b^3*
c*d^2 + 4*a*b*c^3*e^2 + 8*a^2*c^3*d*e - 2*a*c^3*e^2*(b^2 - 4*a*c)^(1/2) - 1
0*a*b^2*c^2*d*e - 4*a*b^2*c*d^2*(b^2 - 4*a*c)^(1/2) - 2*b^3*c*d*e*(b^2 - 4*
a*c)^(1/2) + 6*a*b*c^2*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a^5*c*d^4 - a^4*b^2*
d^4 + 4*a^3*c^3*e^4 + 2*a^3*b^3*d^3*e - a^2*b^2*c^2*e^4 - a^2*b^4*d^2*e^2 +
 8*a^4*c^2*d^2*e^2 - 8*a^4*b*c*d^3*e + 2*a^2*b^3*c*d*e^3 - 8*a^3*b*c^2*d*e^
3 + 2*a^3*b^2*c*d^2*e^2)) - (a*d^4)/(e*(a*d*e^2 + a*e^3*x)*(a*d^2 + c*e^2 -
 b*d*e))
```

3.72 $\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=246

$$\frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{\left(b^3d^2-2b^2cde+4ac^2de-bc(3ad^2-ce^2)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}+\frac{d^2(ad^2 \cdots}{e \cdots}$$

```
[Out] d^3/e^2/(a*d^2-e*(b*d-c*e))/(e*x+d)+d^2*(a*d^2-e*(2*b*d-3*c*e))*ln(e*x+d)/e
^2/(a*d^2-e*(b*d-c*e))^2+1/2*(b^2*d^2-2*b*c*d*e-c*(a*d^2-c*e^2))*ln(a*x^2+b
*x+c)/a/(a*d^2-e*(b*d-c*e))^2+(b^3*d^2-2*b^2*c*d*e+4*a*c^2*d*e-b*c*(3*a*d^2
-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/a/(a*d^2-e*(b*d-c*e))^2/(-4*
a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1583, 1642, 648, 632, 212, 642}

$$\frac{\left(-c(ad^2-ce^2)+b^2d^2-2bcde\right)\log\left(ax^2+bx+c\right)}{2a\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-bc(3ad^2-ce^2)+4ac^2de+b^3d^2-2b^2cde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{a\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}+\frac{d^2\log(d+ex)\left(ad^2-e(2bd-3ce)\right)}{e^2\left(ad^2-e(bd-ce)\right)^2}+\frac{d^3}{e^2(d+ex)\left(ad^2-e(bd-ce)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] d^3/(e^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*d^2 - 2*b^2*c*d*e + 4*a
*c^2*d*e - b*c*(3*a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(
a*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + (d^2*(a*d^2 - e*(2*b*d - 3
*c*e))*Log[d + e*x])/(e^2*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^2 - 2*b*c*d*
e - c*(a*d^2 - c*e^2))*Log[c + b*x + a*x^2])/(2*a*(a*d^2 - e*(b*d - c*e))^2
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^3}{(d+ex)^2\,(c+bx+ax^2)}\,dx$$

$$= \int \left(\frac{d^3}{e\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{d^2(ad^2-e(2bd-3ce))}{e\left(ad^2-e(bd-ce)\right)^2(d+ex)} + \frac{cd(b$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \int \frac{cd(b$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \frac{(b^2d^2$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{d^2(ad^2-e(2bd-3ce))\log(d+ex)}{e^2\left(ad^2-e(bd-ce)\right)^2} + \frac{(b^2d^2$$

$$= \frac{d^3}{e^2\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{(b^3d^2-2b^2cde+4ac^2de-bc(3ad^2-ce^2))\,\mathrm{t}}{a\sqrt{b^2-4ac}\,\left(ad^2-e(bd-\right.}$$

Mathematica [A]
time = 0.16, size = 207, normalized size = 0.84

$$\frac{\frac{2d^3(ad^2+e(-bd+ce))}{e^2(d+ex)}-\frac{2(b^3d^2-2b^2cde+4ac^2de+bc(-3ad^2+ce^2))\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{a\sqrt{-b^2+4ac}}+\frac{2(ad^4+d^2e(-2bd+3ce))\log(d+ex)}{e^2}+\frac{(b^2d^2-2bcde+c(-ad^2+ce^2))\log(c+x(b+ax))}{a}}{2\left(ad^2+e(-bd+ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((2*d^3*(a*d^2 + e*(-(b*d) + c*e)))/(e^2*(d + e*x)) - (2*(b^3*d^2 - 2*b^2*c
*d*e + 4*a*c^2*d*e + b*c*(-3*a*d^2 + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 +
 4*a*c]])/(a*Sqrt[-b^2 + 4*a*c]) + (2*(a*d^4 + d^2*e*(-2*b*d + 3*c*e))*Log[
d + e*x])/e^2 + ((b^2*d^2 - 2*b*c*d*e + c*(-(a*d^2) + c*e^2))*Log[c + x*(b
+ a*x)])/a)/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [A]
time = 0.27, size = 228, normalized size = 0.93

method	result
default	$\frac{d^2(a\,d^2-2deb+3c\,e^2)\ln(ex+d)}{(a\,d^2-deb+c\,e^2)^2e^2}+\frac{d^3}{e^2(a\,d^2-deb+c\,e^2)(ex+d)}+\frac{\frac{(-ac\,d^2+b^2d^2-2bcde+c^2e^2)\ln(a\,x^2+bx+c)}{2a}+\frac{2\left(bc\,d^2-2c^2de-\frac{(-ac\,\cdots}{\cdots}\right.}{\cdots}}{(a\,d^2-deb+c\,e^2)\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^2*(a*d^2-2*b*d*e+3*c*e^2)/(a*d^2-b*d*e+c*e^2)^2/e^2*ln(e*x+d)+d^3/e^2/(a*
d^2-b*d*e+c*e^2)/(e*x+d)+1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(-a*c*d^2+b^2*d^2-2*b
*c*d*e+c^2*e^2)/a*ln(a*x^2+b*x+c)+2*(b*c*d^2-2*c^2*d*e-1/2*(-a*c*d^2+b^2*d^
2-2*b*c*d*e+c^2*e^2)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1
/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 710 vs. $2(251) = 502$.
time = 11.04, size = 1439, normalized size = 5.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2*c
- 4*a^2*c^2)*d^3*e^2 + (b*c^2*x*e^5 + (b^3 - 3*a*b*c)*d^3*e^2 + (b*c^2*d -
2*(b^2*c - 2*a*c^2)*d*x)*e^4 + ((b^3 - 3*a*b*c)*d^2*x - 2*(b^2*c - 2*a*c^2)
*d^2)*e^3)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(
b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^
2)*d^3*e^2 + (b^2*c^2 - 4*a*c^3)*x*e^5 - (2*(b^3*c - 4*a*b*c^2)*d*x - (b^2*
c^2 - 4*a*c^3)*d)*e^4 + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^2*x - 2*(b^3*c - 4
*a*b*c^2)*d^2)*e^3)*log(a*x^2 + b*x + c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 + 3*(
a*b^2*c - 4*a^2*c^2)*d^2*x*e^3 - (2*(a*b^3 - 4*a^2*b*c)*d^3*x - 3*(a*b^2*c
- 4*a^2*c^2)*d^3)*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*x - 2*(a*b^3 - 4*a^2*b*c)*
d^4)*e)*log(x*e + d))/((a^3*b^2 - 4*a^4*c)*d^5*e^2 + (a*b^2*c^2 - 4*a^2*c^3
)*x*e^7 - (2*(a*b^3*c - 4*a^2*b*c^2)*d*x - (a*b^2*c^2 - 4*a^2*c^3)*d)*e^6 +
 ((a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*x - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2)*
e^5 - (2*(a^2*b^3 - 4*a^3*b*c)*d^3*x - (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^
3)*e^4 + ((a^3*b^2 - 4*a^4*c)*d^4*x - 2*(a^2*b^3 - 4*a^3*b*c)*d^4)*e^3), 1/
2*(2*(a^2*b^2 - 4*a^3*c)*d^5 - 2*(a*b^3 - 4*a^2*b*c)*d^4*e + 2*(a*b^2*c - 4
*a^2*c^2)*d^3*e^2 + 2*(b*c^2*x*e^5 + (b^3 - 3*a*b*c)*d^3*e^2 + (b*c^2*d - 2
*(b^2*c - 2*a*c^2)*d*x)*e^4 + ((b^3 - 3*a*b*c)*d^2*x - 2*(b^2*c - 2*a*c^2)*
d^2)*e^3)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 -
4*a*c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3*e^2 + (b^2*c^2 - 4*a*c^3)*x*e^
5 - (2*(b^3*c - 4*a*b*c^2)*d*x - (b^2*c^2 - 4*a*c^3)*d)*e^4 + ((b^4 - 5*a*b
^2*c + 4*a^2*c^2)*d^2*x - 2*(b^3*c - 4*a*b*c^2)*d^2)*e^3)*log(a*x^2 + b*x +
 c) + 2*((a^2*b^2 - 4*a^3*c)*d^5 + 3*(a*b^2*c - 4*a^2*c^2)*d^2*x*e^3 - (2*(
a*b^3 - 4*a^2*b*c)*d^3*x - 3*(a*b^2*c - 4*a^2*c^2)*d^3)*e^2 + ((a^2*b^2 - 4
*a^3*c)*d^4*x - 2*(a*b^3 - 4*a^2*b*c)*d^4)*e)*log(x*e + d))/((a^3*b^2 - 4*a
^4*c)*d^5*e^2 + (a*b^2*c^2 - 4*a^2*c^3)*x*e^7 - (2*(a*b^3*c - 4*a^2*b*c^2)*
d*x - (a*b^2*c^2 - 4*a^2*c^3)*d)*e^6 + ((a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d
^2*x - 2*(a*b^3*c - 4*a^2*b*c^2)*d^2)*e^5 - (2*(a^2*b^3 - 4*a^3*b*c)*d^3*x
- (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^3)*e^4 + ((a^3*b^2 - 4*a^4*c)*d^4*x -
 2*(a^2*b^3 - 4*a^3*b*c)*d^4)*e^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.93, size = 412, normalized size = 1.67

$$\frac{1}{2}\left(\frac{2\,d^{3}e^{2}}{(ad^{2}e^{3}-bde^{4}+ce^{5})(xe+d)}+\frac{2\,(b^{3}d^{2}e^{3}-3\,abcd^{2}e^{3}-2\,b^{2}cde^{4}+4\,ac^{2}de^{4}+bc^{2}e^{5})\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^{2}}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^{2}}{xe+d}\right)e^{(-1)}}{\sqrt{-b^{2}+4\,ac}}\right)e^{(-2)}}{(a^{3}d^{4}-2\,a^{2}bd^{3}e+ab^{2}d^{2}e^{2}+2\,a^{2}cd^{2}e^{2}-2\,abcde^{3}+ac^{2}e^{4})\sqrt{-b^{2}+4\,ac}}+\frac{(b^{2}d^{2}e-acd^{2}e-2\,bcde^{2}+c^{2}e^{3})\log\left(-a+\frac{2\,ad}{xe+d}-\frac{ad^{2}}{(xe+d)^{2}}-\frac{be}{xe+d}+\frac{bde}{(xe+d)^{2}}-\frac{ce^{2}}{(xe+d)^{2}}\right)}{a^{3}d^{4}-2\,a^{2}bd^{3}e+ab^{2}d^{2}e^{2}+2\,a^{2}cd^{2}e^{2}-2\,abcde^{3}+ac^{2}e^{4}}-\frac{2\,e^{(-1)}\log\left(\frac{|xe+d|e^{(-1)}}{(xe+d)^{2}}\right)}{a}\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*d^3*e^2/((a*d^2*e^3 - b*d*e^4 + c*e^5)*(x*e + d)) + 2*(b^3*d^2*e^3 -
 3*a*b*c*d^2*e^3 - 2*b^2*c*d*e^4 + 4*a*c^2*d*e^4 + b*c^2*e^5)*arctan(-(2*a*
d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)
/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^3*d^4 - 2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a
^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4)*sqrt(-b^2 + 4*a*c)) + (b^2*d^2*e
- a*c*d^2*e - 2*b*c*d*e^2 + c^2*e^3)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e
+ d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^3*d^4 -
2*a^2*b*d^3*e + a*b^2*d^2*e^2 + 2*a^2*c*d^2*e^2 - 2*a*b*c*d*e^3 + a*c^2*e^4
) - 2*e^(-1)*log(abs(x*e + d)*e^(-1)/(x*e + d)^2)/a)*e^(-1)
```

Mupad [B]
time = 5.11, size = 2037, normalized size = 8.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(a*d^4 + 3*c*d^2*e^2 - 2*b*d^3*e))/(c^2*e^6 + a^2*d^4*e^2 + b
^2*d^2*e^4 - 2*b*c*d*e^5 - 2*a*b*d^3*e^3 + 2*a*c*d^2*e^4) - (log(a^2*b^2*d^
6 - 4*a^3*c*d^6 - 2*c^4*e^6 - b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^(1/2) +
 24*a*c^3*d^2*e^4 + 6*b^3*c*d^3*e^3 + 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2 -
4*a*c)^(1/2) - 10*a^2*c^2*d^4*e^2 - 9*b^2*c^2*d^2*e^4 - 2*a*b^3*d^5*e + 4*b
*c^3*d*e^5 - b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^(1/2) + 4*c^3*d*e^5*(b^2
 - 4*a*c)^(1/2) + 2*a^3*d^6*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d^5*e + 8*a*c
^3*d*e^5*x - 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c*
d^5*e*(b^2 - 4*a*c)^(1/2) - 20*a*b*c^2*d^3*e^3 + 6*a*b^2*c*d^4*e^2 - 6*a*b^
3*d^4*e^2*x + 2*a^2*b^2*d^5*e*x - 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b^2
 - 4*a*c)^(1/2) - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e^3*(b^
2 - 4*a*c)^(1/2) - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) - 32*a^2*c^2*d^3*e^3
*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 12*a*c^2*d^2*e^4*x*(b^2 - 4*a*
c)^(1/2) + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^4*x*(b^2 -
```

```
4*a*c)^(1/2) + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b*d^5*e*x*(b^2
 - 4*a*c)^(1/2) + 6*a*b*c^2*d^2*e^4*x + 2*a*b^2*c*d^3*e^3*x + 23*a^2*b*c*d^
4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2 - 4*a*c^3*e^2 + b
^3*d^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e - 5*
a*b^2*c*d^2 + b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^2*d*e - 3*a*b*c*d^2*(
b^2 - 4*a*c)^(1/2) + 4*a*c^2*d*e*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d*e*(b^2 - 4
*a*c)^(1/2)))/(2*(4*a^4*c*d^4 - a^3*b^2*d^4 + 4*a^2*c^3*e^4 - a*b^2*c^2*e^4
 - a*b^4*d^2*e^2 + 2*a^2*b^3*d^3*e + 8*a^3*c^2*d^2*e^2 + 2*a*b^3*c*d*e^3 -
8*a^3*b*c*d^3*e - 8*a^2*b*c^2*d*e^3 + 2*a^2*b^2*c*d^2*e^2)) - (log(2*c^4*e^
6 + 4*a^3*c*d^6 - a^2*b^2*d^6 + b^4*d^4*e^2 + c^3*e^6*x*(b^2 - 4*a*c)^(1/2)
 - 24*a*c^3*d^2*e^4 - 6*b^3*c*d^3*e^3 - 2*b^4*d^3*e^3*x - b^3*d^4*e^2*(b^2
- 4*a*c)^(1/2) + 10*a^2*c^2*d^4*e^2 + 9*b^2*c^2*d^2*e^4 + 2*a*b^3*d^5*e - 4
*b*c^3*d*e^5 + b*c^3*e^6*x + a^2*b*d^6*(b^2 - 4*a*c)^(1/2) + 4*c^3*d*e^5*(b
^2 - 4*a*c)^(1/2) + 2*a^3*d^6*x*(b^2 - 4*a*c)^(1/2) - 8*a^2*b*c*d^5*e - 8*a
*c^3*d*e^5*x + 8*a^3*c*d^5*e*x - 2*a*b^2*d^5*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*
c*d^5*e*(b^2 - 4*a*c)^(1/2) + 20*a*b*c^2*d^3*e^3 - 6*a*b^2*c*d^4*e^2 + 6*a*
b^3*d^4*e^2*x - 2*a^2*b^2*d^5*e*x + 3*b^3*c*d^2*e^4*x - 16*a*c^2*d^3*e^3*(b
^2 - 4*a*c)^(1/2) - 3*b*c^2*d^2*e^4*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d^3*e^3*(
b^2 - 4*a*c)^(1/2) - 2*b^3*d^3*e^3*x*(b^2 - 4*a*c)^(1/2) + 32*a^2*c^2*d^3*e
^3*x + 4*a*b^2*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) - 12*a*c^2*d^2*e^4*x*(b^2 - 4*
a*c)^(1/2) + 5*a^2*c*d^4*e^2*x*(b^2 - 4*a*c)^(1/2) + 3*b^2*c*d^2*e^4*x*(b^2
 - 4*a*c)^(1/2) + 14*a*b*c*d^4*e^2*(b^2 - 4*a*c)^(1/2) - 6*a^2*b*d^5*e*x*(b
^2 - 4*a*c)^(1/2) - 6*a*b*c^2*d^2*e^4*x - 2*a*b^2*c*d^3*e^3*x - 23*a^2*b*c*
d^4*e^2*x + 2*a*b*c*d^3*e^3*x*(b^2 - 4*a*c)^(1/2))*(b^4*d^2 - 4*a*c^3*e^2 -
 b^3*d^2*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d^2 + b^2*c^2*e^2 - 2*b^3*c*d*e -
5*a*b^2*c*d^2 - b*c^2*e^2*(b^2 - 4*a*c)^(1/2) + 8*a*b*c^2*d*e + 3*a*b*c*d^2
*(b^2 - 4*a*c)^(1/2) - 4*a*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 2*b^2*c*d*e*(b^2 -
 4*a*c)^(1/2)))/(2*(4*a^4*c*d^4 - a^3*b^2*d^4 + 4*a^2*c^3*e^4 - a*b^2*c^2*e
^4 - a*b^4*d^2*e^2 + 2*a^2*b^3*d^3*e + 8*a^3*c^2*d^2*e^2 + 2*a*b^3*c*d*e^3
- 8*a^3*b*c*d^3*e - 8*a^2*b*c^2*d*e^3 + 2*a^2*b^2*c*d^2*e^2)) + d^3/(e^2*(d
 + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

3.73 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx$

Optimal. Leaf size=194

$$-\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)}-\frac{\left(b^2d^2-2bcde-2c\left(ad^2-ce^2\right)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}+\frac{d(bd-2ce)\log(d+}{(ad^2-e(bd-ce}$$

```
[Out] -d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d*(b*d-2*c*e)*ln(e*x+d)/(a*d^2-e*(b*d-c*
e))^2-1/2*d*(b*d-2*c*e)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2-(b^2*d^2-2*b*
c*d*e-2*c*(a*d^2-c*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*
d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1459, 1642, 648, 632, 212, 642}

$$-\frac{\left(-2c\left(ad^2-ce^2\right)+b^2d^2-2bcde\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}-\frac{d^2}{e(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{d(bd-2ce)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] -(d^2/(e*(a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((b^2*d^2 - 2*b*c*d*e - 2*c*
(a*d^2 - c*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]
*(a*d^2 - e*(b*d - c*e))^2) + (d*(b*d - 2*c*e)*Log[d + e*x])/(a*d^2 - e*(b*
d - c*e))^2 - (d*(b*d - 2*c*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c
*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1459

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x
_)^(n_.))^(q_.), x_Symbol] :> Int[((d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p)
/x^(2*n*p), x] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[mn, -n] && EqQ[mn2
, 2*mn] && IntegerQ[p]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)(d+ex)^2}\,dx = \int \frac{x^2}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$
$$= \int \left(\frac{d^2}{\left(ad^2-e(bd-ce)\right)(d+ex)^2}+\frac{de(bd-2ce)}{\left(ad^2-e(bd-ce)\right)^2(d+ex)}+\frac{-c(ad^2 \ldots}{(ad^2-e \ldots}\right.$$
$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}+\frac{\int \frac{-c(ad^2-ce^2)-ad(b \ldots}{c+bx+ax^2}}{(ad^2-e(bd- \ldots}$$
$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}-\frac{(d(bd-2ce))\int \ldots}{2\left(ad^2-e(bd \ldots\right.}$$
$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{d(bd-2ce)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}-\frac{d(bd-2ce)\log(\ldots}{2\left(ad^2-e(b \ldots\right.}$$
$$= -\frac{d^2}{e\left(ad^2-bde+ce^2\right)(d+ex)}-\frac{\left(b^2d^2-2bcde-2c(ad^2-ce^2)\right)\tanh^{-1}\left(\frac{\ldots}{\sqrt{b \ldots}}\right.}{\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}$$

Mathematica [A]

time = 0.16, size = 159, normalized size = 0.82

$$\frac{-\frac{2d^2(ad^2+e(-bd+ce))}{e(d+ex)}+\frac{2(b^2d^2-2bcde+2c(-ad^2+ce^2))\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+2d(bd-2ce)\log(d+ex)-d(bd-2ce)\log(c+x(b+ax))}{2\left(ad^2+e(-bd+ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*(d + e*x)^2),x]
```

```
[Out] ((-2*d^2*(a*d^2 + e*(-(b*d) + c*e)))/(e*(d + e*x)) + (2*(b^2*d^2 - 2*b*c*d*
e + 2*c*(-(a*d^2) + c*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b
^2 + 4*a*c] + 2*d*(b*d - 2*c*e)*Log[d + e*x] - d*(b*d - 2*c*e)*Log[c + x*(b
 + a*x)])/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [A]
time = 0.29, size = 188, normalized size = 0.97

method	result
default	$-\frac{d^2}{e(a\,d^2-deb+c\,e^2)(ex+d)}+\frac{d(bd-2ce)\ln(ex+d)}{(a\,d^2-deb+c\,e^2)^2}+\frac{\frac{\left(-d^2ab+2acde\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(-ac\,d^2+c^2e^2-\frac{\left(-d^2ab+2acde\right)b}{2a}\right)\arctan\left(\cdots\right.}{\sqrt{4ac-b^2}}}{(a\,d^2-deb+c\,e^2)^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^2/e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+d*(b*d-2*c*e)/(a*d^2-b*d*e+c*e^2)^2*ln(e
*x+d)+1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(-a*b*d^2+2*a*c*d*e)/a*ln(a*x^2+b*x+c)+2
*(-a*c*d^2+c^2*e^2-1/2*(-a*b*d^2+2*a*c*d*e)*b/a)/(4*a*c-b^2)^(1/2)*arctan((
2*a*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(197) = 394$.
time = 3.86, size = 1099, normalized size = 5.66

$$\left[-\frac{2\,(ab^2-4\,a^2c)d^4-2\,(b^3-4\,abc)d^3e+2\,(b^2c-4\,ac^2)d^2e^2-((b^2-2\,ac)d^3e+2\,c^2xe^4-2\,(bcdx-c^2d)e^3-(2\,bcd^2-(b^2-2\,ac)d^2x)e^2)\sqrt{b^2-4\,ac}\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,ax+b)}{ax^2+bx+c}\right)+((b^3-4\,abc)d^3e-2\,(b^2c-4\,ac^2)dxe^3+((b^3-4\,abc)d^2x-2\,(b^2c-4\,ac^2)d^2)e^2)\log(ax^2+bx+c)-2\,((b^3-4\,abc)d^3e-2\,(b^2c-4\,ac^2)dxe^3+((b^3-4\,abc)d^2x-2\,(b^2c-4\,ac^2)d^2)e^2)\log(xe+d)}{2\,((a^2b^2-4\,a^3c)d^5e+(b^2c^2-4\,ac^3)xe^6-(2\,(b^3c-4\,abc^2)dx-(b^2c^2-4\,ac^3)d)e^5+((b^4-2\,ab^2c-8\,a^2c^2)d^2x-2\,(b^3c-4\,abc^2)d^2)e^4-(2\,(ab^3-4\,a^2bc)d^3x-(b^4-2\,ab^2c-8\,a^2c^2)d^3)e^3+((a^2b^2-4\,a^3c)d^4x-2\,(ab^3-4\,a^2bc)d^4)e^2)},\ -\frac{2\,(ab^2-4\,a^2c)d^4-2\,(b^3-4\,abc)d^3e+2\,(b^2c-4\,ac^2)d^2e^2+2\,((b^2-2\,ac)d^3e+2\,c^2xe^4-2\,(bcdx-c^2d)e^3-(2\,bcd^2-(b^2-2\,ac)d^2x)e^2)\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax+b)}{b^2-4\,ac}\right)+((b^3-4\,abc)d^3e-2\,(b^2c-4\,ac^2)dxe^3+((b^3-4\,abc)d^2x-2\,(b^2c-4\,ac^2)d^2)e^2)\log(ax^2+bx+c)-2\,((b^3-4\,abc)d^3e-2\,(b^2c-4\,ac^2)dxe^3+((b^3-4\,abc)d^2x-2\,(b^2c-4\,ac^2)d^2)e^2)\log(xe+d)}{2\,((a^2b^2-4\,a^3c)d^5e+(b^2c^2-4\,ac^3)xe^6-(2\,(b^3c-4\,abc^2)dx-(b^2c^2-4\,ac^3)d)e^5+((b^4-2\,ab^2c-8\,a^2c^2)d^2x-2\,(b^3c-4\,abc^2)d^2)e^4-(2\,(ab^3-4\,a^2bc)d^3x-(b^4-2\,ab^2c-8\,a^2c^2)d^3)e^3+((a^2b^2-4\,a^3c)d^4x-2\,(ab^3-4\,a^2bc)d^4)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c
^2)*d^2*e^2 - ((b^2 - 2*a*c)*d^3*e + 2*c^2*x*e^4 - 2*(b*c*d*x - c^2*d)*e^3
- (2*b*c*d^2 - (b^2 - 2*a*c)*d^2*x)*e^2)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 +
 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c))
+ ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d*x*e^3 + ((b^3 - 4*a*b*c)*d
^2*x - 2*(b^2*c - 4*a*c^2)*d^2)*e^2)*log(a*x^2 + b*x + c) - 2*((b^3 - 4*a*b
*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d*x*e^3 + ((b^3 - 4*a*b*c)*d^2*x - 2*(b^2*c
 - 4*a*c^2)*d^2)*e^2)*log(x*e + d))/((a^2*b^2 - 4*a^3*c)*d^5*e + (b^2*c^2 -
 4*a*c^3)*x*e^6 - (2*(b^3*c - 4*a*b*c^2)*d*x - (b^2*c^2 - 4*a*c^3)*d)*e^5 +
 ((b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*x - 2*(b^3*c - 4*a*b*c^2)*d^2)*e^4 - (2
*(a*b^3 - 4*a^2*b*c)*d^3*x - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3)*e^3 + ((a^2
*b^2 - 4*a^3*c)*d^4*x - 2*(a*b^3 - 4*a^2*b*c)*d^4)*e^2), -1/2*(2*(a*b^2 - 4
*a^2*c)*d^4 - 2*(b^3 - 4*a*b*c)*d^3*e + 2*(b^2*c - 4*a*c^2)*d^2*e^2 + 2*((b
^2 - 2*a*c)*d^3*e + 2*c^2*x*e^4 - 2*(b*c*d*x - c^2*d)*e^3 - (2*b*c*d^2 - (b
^2 - 2*a*c)*d^2*x)*e^2)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a*
x + b)/(b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d*x*e^
3 + ((b^3 - 4*a*b*c)*d^2*x - 2*(b^2*c - 4*a*c^2)*d^2)*e^2)*log(a*x^2 + b*x
+ c) - 2*((b^3 - 4*a*b*c)*d^3*e - 2*(b^2*c - 4*a*c^2)*d*x*e^3 + ((b^3 - 4*a
*b*c)*d^2*x - 2*(b^2*c - 4*a*c^2)*d^2)*e^2)*log(x*e + d))/((a^2*b^2 - 4*a^3
*c)*d^5*e + (b^2*c^2 - 4*a*c^3)*x*e^6 - (2*(b^3*c - 4*a*b*c^2)*d*x - (b^2*c
^2 - 4*a*c^3)*d)*e^5 + ((b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*x - 2*(b^3*c - 4*
a*b*c^2)*d^2)*e^4 - (2*(a*b^3 - 4*a^2*b*c)*d^3*x - (b^4 - 2*a*b^2*c - 8*a^2
*c^2)*d^3)*e^3 + ((a^2*b^2 - 4*a^3*c)*d^4*x - 2*(a*b^3 - 4*a^2*b*c)*d^4)*e^
2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.46, size = 331, normalized size = 1.71

$$\frac{(b^2d^2e^2-2\,acd^2e^2-2\,bcde^3+2\,c^2e^4)\arctan\left(\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)\sqrt{-b^2+4\,ac}}-\frac{d^2e}{(ad^2e^2-bde^3+ce^4)(xe+d)}-\frac{(bd^2-2\,cde)\log\left(a-\frac{2\,ad}{xe+d}+\frac{ad^2}{(xe+d)^2}+\frac{be}{xe+d}-\frac{bde}{(xe+d)^2}+\frac{ce^2}{(xe+d)^2}\right)}{2\,(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (b^2*d^2*e^2 - 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + 2*c^2*e^4)*arctan((2*a*d - 2*a
*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-
b^2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2
- 2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - d^2*e/((a*d^2*e^2 - b*d*e^3
+ c*e^4)*(x*e + d)) - 1/2*(b*d^2 - 2*c*d*e)*log(a - 2*a*d/(x*e + d) + a*d^2
/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2*
d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^3 + c^2*e^4)
```

Mupad [B]
time = 6.09, size = 1585, normalized size = 8.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(2*a*b^3*d^4 + b*c^3*e^4 - c^3*e^4*(b^2 - 4*a*c)^(1/2) + 16*a^2*c^2*d^3
*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4*x + b^2*c^2*e^4*x - b^4*
d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3*c*d^4*x - 2*a*c^3*e^4*x
+ 2*a*b^2*d^4*(b^2 - 4*a*c)^(1/2) - a^2*c*d^4*(b^2 - 4*a*c)^(1/2) - 6*a*b^2
*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x - 2*b*c^2*d*e^3*(b^2 - 4*a*c)^
(1/2) + 3*a^2*b*d^4*x*(b^2 - 4*a*c)^(1/2) - b*c^2*e^4*x*(b^2 - 4*a*c)^(1/2)
 + 10*a*b*c^2*d^2*e^2 + 14*a*c^2*d^2*e^2*(b^2 - 4*a*c)^(1/2) + b^2*c*d^2*e^
2*(b^2 - 4*a*c)^(1/2) + b^3*d^2*e^2*x*(b^2 - 4*a*c)^(1/2) + 28*a^2*c^2*d^2*
e^2*x - 10*a*b*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 12*a*b*c^2*d*e^3*x - 12*a^2*b*
c*d^3*e*x - 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^(1/2) + 8*a*c^2*d*e^3*x*(b^2 - 4*
a*c)^(1/2) - 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^(1/2) - 2*b^2*c*d*e^3*x*(b^2 - 4
*a*c)^(1/2) + 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2))*(d^2*(b^3/2 + (b^2*(b^
2 - 4*a*c)^(1/2))/2) - c*(d^2*(2*a*b + a*(b^2 - 4*a*c)^(1/2)) + d*(b^2*e +
b*e*(b^2 - 4*a*c)^(1/2))) + c^2*(e^2*(b^2 - 4*a*c)^(1/2) + 4*a*d*e)))/(4*a^
3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2
*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*
e + 2*a*b^2*c*d^2*e^2) - (log(2*a*b^3*d^4 + b*c^3*e^4 + c^3*e^4*(b^2 - 4*a*
c)^(1/2) + 16*a^2*c^2*d^3*e + 2*b^2*c^2*d*e^3 - b^3*c*d^2*e^2 + a^2*b^2*d^4
*x + b^2*c^2*e^4*x - b^4*d^2*e^2*x - 7*a^2*b*c*d^4 - 16*a*c^3*d*e^3 - 2*a^3
*c*d^4*x - 2*a*c^3*e^4*x - 2*a*b^2*d^4*(b^2 - 4*a*c)^(1/2) + a^2*c*d^4*(b^2
 - 4*a*c)^(1/2) - 6*a*b^2*c*d^3*e + 2*a*b^3*d^3*e*x + 2*b^3*c*d*e^3*x + 2*b
*c^2*d*e^3*(b^2 - 4*a*c)^(1/2) - 3*a^2*b*d^4*x*(b^2 - 4*a*c)^(1/2) + b*c^2*
e^4*x*(b^2 - 4*a*c)^(1/2) + 10*a*b*c^2*d^2*e^2 - 14*a*c^2*d^2*e^2*(b^2 - 4*
a*c)^(1/2) - b^2*c*d^2*e^2*(b^2 - 4*a*c)^(1/2) - b^3*d^2*e^2*x*(b^2 - 4*a*c
)^(1/2) + 28*a^2*c^2*d^2*e^2*x + 10*a*b*c*d^3*e*(b^2 - 4*a*c)^(1/2) - 12*a*
b*c^2*d*e^3*x - 12*a^2*b*c*d^3*e*x + 2*a*b^2*d^3*e*x*(b^2 - 4*a*c)^(1/2) -
```

```
8*a*c^2*d*e^3*x*(b^2 - 4*a*c)^(1/2) + 8*a^2*c*d^3*e*x*(b^2 - 4*a*c)^(1/2) +
 2*b^2*c*d*e^3*x*(b^2 - 4*a*c)^(1/2) - 2*a*b*c*d^2*e^2*x*(b^2 - 4*a*c)^(1/2
))*(c*(d^2*(2*a*b - a*(b^2 - 4*a*c)^(1/2)) + d*(b^2*e - b*e*(b^2 - 4*a*c)^(
1/2))) - d^2*(b^3/2 - (b^2*(b^2 - 4*a*c)^(1/2))/2) + c^2*(e^2*(b^2 - 4*a*c)
^(1/2) - 4*a*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4
- b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c
^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) + (log(d + e*x)*(b*d^2 - 2*
c*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a*
c*d^2*e^2) - d^2/(e*(d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

3.74 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx$

Optimal. Leaf size=183

$$\frac{d}{(ad^2-bde+ce^2)(d+ex)}+\frac{(bce^2+ad(bd-4ce))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}-\frac{(ad^2-ce^2)\log(d+ex)}{(ad^2-e(bd-ce))^2}+\frac{(ad^2-}{2}$$

```
[Out] d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-(a*d^2-c*e^2)*ln(e*x+d)/(a*d^2-e*(b*d-c*e))^2
+1/2*(a*d^2-c*e^2)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2+(b*c*e^2+a*d*(b*d-
4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e))^2/(-4*a*c
+b^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 814, 648, 632, 212, 642}

$$\frac{(ad(bd-4ce)+bce^2)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}+\frac{(ad^2-ce^2)\log\left(ax^2+bx+c\right)}{2\left(ad^2-e(bd-ce)\right)^2}+\frac{d}{(d+ex)\left(ad^2-bde+ce^2\right)}-\frac{(ad^2-ce^2)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] d/((a*d^2 - b*d*e + c*e^2)*(d + e*x)) + ((b*c*e^2 + a*d*(b*d - 4*c*e))*ArcT
anh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*
e))^2) - ((a*d^2 - c*e^2)*Log[d + e*x])/(a*d^2 - e*(b*d - c*e))^2 + ((a*d^2
 - c*e^2)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x(d+ex)^2}\,dx &= \int \frac{x}{(d+ex)^2\left(c+bx+ax^2\right)}\,dx \\
&= \int \left(\frac{de}{\left(-ad^2+e(bd-ce)\right)(d+ex)^2}+\frac{e\left(-ad^2+ce^2\right)}{\left(ad^2-e(bd-ce)\right)^2(d+ex)}+\frac{ce(\cdots}{(ad^2\cdots}\right. \\
&= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)}-\frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}+\frac{\int \frac{ce(2ad-be)+a\left(ad^2-ce\cdots\right.}{c+bx+ax^2}}{\left(ad^2-e(bd-ce\cdots\right.} \\
&= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)}-\frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(ad^2-ce^2\right)\int \frac{b+2a\cdots}{c+bx+\cdots}}{2\left(ad^2-e(bd-c\cdots\right.} \\
&= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)}-\frac{\left(ad^2-ce^2\right)\log(d+ex)}{\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(ad^2-ce^2\right)\log(c+\cdots}{2\left(ad^2-e(bd-\cdots\right.} \\
&= \frac{d}{\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{\left(bce^2+ad(bd-4ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\left(ad^2-e(bd-ce)\right)^2}
\end{aligned}$$

Mathematica [A]

time = 0.17, size = 148, normalized size = 0.81

$$\frac{\frac{2d(ad^2+e(-bd+ce))}{d+ex}-\frac{2(bce^2+ad(bd-4ce))\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-2ad^2+2ce^2)\log(d+ex)+(ad^2-ce^2)\log(c+x(b+ax))}{2\left(ad^2+e(-bd+ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x*(d + e*x)^2),x]
```

```
[Out] ((2*d*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) - (2*(b*c*e^2 + a*d*(b*d - 4*c*
e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-2*a*d^2
+ 2*c*e^2)*Log[d + e*x] + (a*d^2 - c*e^2)*Log[c + x*(b + a*x)])/(2*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

Maple [A]
time = 0.26, size = 187, normalized size = 1.02

method	result
default	$\frac{d}{(a\,d^2-deb+c\,e^2)(ex+d)}-\frac{(a\,d^2-c\,e^2)\ln(ex+d)}{(a\,d^2-deb+c\,e^2)^2}+\frac{\frac{(a^2d^2-ac\,e^2)\ln(a\,x^2+bx+c)}{2a}+\frac{2\left(2acde-bc\,e^2-\frac{(a^2d^2-ac\,e^2)b}{2a}\right)\arctan\left(\frac{2ax}{\sqrt{4ac}}\right.}{\sqrt{4ac-b^2}}}{(a\,d^2-deb+c\,e^2)^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-(a*d^2-c*e^2)/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d)
+1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(a^2*d^2-a*c*e^2)/a*ln(a*x^2+b*x+c)+2*(2*a*c*
d*e-b*c*e^2-1/2*(a^2*d^2-a*c*e^2)*b/a)/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(
4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(183) = 366$.
time = 3.50, size = 1057, normalized size = 5.78

$$\left[\frac{2\,(ab^2 - 4\,a^2c)d^3 - 2\,(b^3 - 4\,abc)d^2e + 2\,(b^2c - 4\,ac^2)de^2 + (abd^3 + bcxe^3 - (4\,acdx - bcd)e^2 + (abd^2x - 4\,acd^2)e)\sqrt{b^2 - 4\,ac}\log\left(\frac{2\,a^2x^2 + 2\,abx + b^2 - 2\,ac + \sqrt{b^2 - 4\,ac}\,(2\,ax + b)}{ax^2 + bx + c}\right) + ((ab^2 - 4\,a^2c)d^2xe + (ab^2 - 4\,a^2c)d^3 - (b^2c - 4\,ac^2)xe^3 - (b^2c - 4\,ac^2)de^2)\log(ax^2 + bx + c) - 2\,((ab^2 - 4\,a^2c)d^2xe + (ab^2 - 4\,a^2c)d^3 - (b^2c - 4\,ac^2)xe^3 - (b^2c - 4\,ac^2)de^2)\log(xe + d)}{2\,((a^2b^2 - 4\,a^3c)d^5 + (b^2c^2 - 4\,ac^3)xe^5 - (2\,(b^3c - 4\,abc^2)dx - (b^2c^2 - 4\,ac^3)d)e^4 + ((b^4 - 2\,ab^2c - 8\,a^2c^2)d^2x - 2\,(b^3c - 4\,abc^2)d^2)e^3 - (2\,(ab^3 - 4\,a^2bc)d^3x - (b^4 - 2\,ab^2c - 8\,a^2c^2)d^3)e^2 + ((a^2b^2 - 4\,a^3c)d^4x - 2\,(ab^3 - 4\,a^2bc)d^4)e)}, \frac{2\,(ab^2 - 4\,a^2c)d^3 - 2\,(b^3 - 4\,abc)d^2e + 2\,(b^2c - 4\,ac^2)de^2 + 2\,(abd^3 + bcxe^3 - (4\,acdx - bcd)e^2 + (abd^2x - 4\,acd^2)e)\sqrt{-b^2 + 4\,ac}\arctan\left(-\frac{\sqrt{-b^2 + 4\,ac}\,(2\,ax + b)}{b^2 - 4\,ac}\right) + ((ab^2 - 4\,a^2c)d^2xe + (ab^2 - 4\,a^2c)d^3 - (b^2c - 4\,ac^2)xe^3 - (b^2c - 4\,ac^2)de^2)\log(ax^2 + bx + c) - 2\,((ab^2 - 4\,a^2c)d^2xe + (ab^2 - 4\,a^2c)d^3 - (b^2c - 4\,ac^2)xe^3 - (b^2c - 4\,ac^2)de^2)\log(xe + d)}{2\,((a^2b^2 - 4\,a^3c)d^5 + (b^2c^2 - 4\,ac^3)xe^5 - (2\,(b^3c - 4\,abc^2)dx - (b^2c^2 - 4\,ac^3)d)e^4 + ((b^4 - 2\,ab^2c - 8\,a^2c^2)d^2x - 2\,(b^3c - 4\,abc^2)d^2)e^3 - (2\,(ab^3 - 4\,a^2bc)d^3x - (b^4 - 2\,ab^2c - 8\,a^2c^2)d^3)e^2 + ((a^2b^2 - 4\,a^3c)d^4x - 2\,(ab^3 - 4\,a^2bc)d^4)e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2*c - 4*a*c^
2)*d*e^2 + (a*b*d^3 + b*c*x*e^3 - (4*a*c*d*x - b*c*d)*e^2 + (a*b*d^2*x - 4*
a*c*d^2)*e)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt
(b^2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + ((a*b^2 - 4*a^2*c)*d^2*x*e
+ (a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*x*e^3 - (b^2*c - 4*a*c^2)*d*e^2
)*log(a*x^2 + b*x + c) - 2*((a*b^2 - 4*a^2*c)*d^2*x*e + (a*b^2 - 4*a^2*c)*d
^3 - (b^2*c - 4*a*c^2)*x*e^3 - (b^2*c - 4*a*c^2)*d*e^2)*log(x*e + d))/((a^2
*b^2 - 4*a^3*c)*d^5 + (b^2*c^2 - 4*a*c^3)*x*e^5 - (2*(b^3*c - 4*a*b*c^2)*d*
x - (b^2*c^2 - 4*a*c^3)*d)*e^4 + ((b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*x - 2*(
b^3*c - 4*a*b*c^2)*d^2)*e^3 - (2*(a*b^3 - 4*a^2*b*c)*d^3*x - (b^4 - 2*a*b^2
*c - 8*a^2*c^2)*d^3)*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*x - 2*(a*b^3 - 4*a^2*b*
c)*d^4)*e), 1/2*(2*(a*b^2 - 4*a^2*c)*d^3 - 2*(b^3 - 4*a*b*c)*d^2*e + 2*(b^2
*c - 4*a*c^2)*d*e^2 + 2*(a*b*d^3 + b*c*x*e^3 - (4*a*c*d*x - b*c*d)*e^2 + (a
*b*d^2*x - 4*a*c*d^2)*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*a
*x + b)/(b^2 - 4*a*c)) + ((a*b^2 - 4*a^2*c)*d^2*x*e + (a*b^2 - 4*a^2*c)*d^3
 - (b^2*c - 4*a*c^2)*x*e^3 - (b^2*c - 4*a*c^2)*d*e^2)*log(a*x^2 + b*x + c)
- 2*((a*b^2 - 4*a^2*c)*d^2*x*e + (a*b^2 - 4*a^2*c)*d^3 - (b^2*c - 4*a*c^2)*
x*e^3 - (b^2*c - 4*a*c^2)*d*e^2)*log(x*e + d))/((a^2*b^2 - 4*a^3*c)*d^5 + (
b^2*c^2 - 4*a*c^3)*x*e^5 - (2*(b^3*c - 4*a*b*c^2)*d*x - (b^2*c^2 - 4*a*c^3)
*d)*e^4 + ((b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*x - 2*(b^3*c - 4*a*b*c^2)*d^2)
*e^3 - (2*(a*b^3 - 4*a^2*b*c)*d^3*x - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3)*e^
2 + ((a^2*b^2 - 4*a^3*c)*d^4*x - 2*(a*b^3 - 4*a^2*b*c)*d^4)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.19, size = 323, normalized size = 1.77

$$-\frac{1}{2}\left(\frac{2\,(abd^2e - 4\,acde^2 + bce^3)\arctan\left(\frac{\left(2\,ad - \frac{2\,ad^2}{xe+d} - be + \frac{2\,bde}{xe+d} - \frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2 + 4\,ac}}\right)e^{(-2)}}{(a^2d^4 - 2\,abd^3e + b^2d^2e^2 + 2\,acd^2e^2 - 2\,bcde^3 + c^2e^4)\sqrt{-b^2 + 4\,ac}} - \frac{(ad^2 - ce^2)\log\left(a - \frac{2\,ad}{xe+d} + \frac{ad^2}{(xe+d)^2} + \frac{be}{xe+d} - \frac{bde}{(xe+d)^2} + \frac{ce^2}{(xe+d)^2}\right)}{a^2d^4e - 2\,abd^3e^2 + b^2d^2e^3 + 2\,acd^2e^3 - 2\,bcde^4 + c^2e^5} - \frac{2\,de}{(ad^2e^2 - bde^3 + ce^4)(xe + d)}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*(a*b*d^2*e - 4*a*c*d*e^2 + b*c*e^3)*arctan((2*a*d - 2*a*d^2/(x*e +
d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c)
)*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2*b*c*d*e^
3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - (a*d^2 - c*e^2)*log(a - 2*a*d/(x*e + d)
+ a*d^2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2
)/(a^2*d^4*e - 2*a*b*d^3*e^2 + b^2*d^2*e^3 + 2*a*c*d^2*e^3 - 2*b*c*d*e^4 +
c^2*e^5) - 2*d*e/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d)))*e
```

Mupad [B]
time = 8.07, size = 1768, normalized size = 9.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] d/((d + e*x)*(a*d^2 + c*e^2 - b*d*e)) - (log(56*a^3*b^2*c*d^4 - 96*a^4*c^2*
d^4 - 96*a^2*c^4*e^4 - 8*b^4*c^2*e^4 - 8*a^2*b^4*d^4 + 56*a*b^2*c^3*e^4 - 4
*a^3*b^3*d^4*x + 320*a^3*c^3*d^2*e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8*c*
d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e^4*x*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4*x +
 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2) - 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 12*a^3
*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*d*e^3*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c*d
^4*x - 112*a^2*b^2*c^2*d^2*e^2 - 8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) + 8*b^2*
c*d*e^3*(b^2 - 4*a*c)^(3/2) + 10*a*d^2*e^2*x*(b^2 - 4*a*c)^(5/2) - 5*b^2*c*
e^4*x*(b^2 - 4*a*c)^(3/2) + 6*b^3*d*e^3*x*(b^2 - 4*a*c)^(3/2) + 16*a*b^3*c^
2*d*e^3 + 8*a*b^4*c*d^2*e^2 - 64*a^2*b*c^3*d*e^3 + 16*a^2*b^3*c*d^3*e - 64*
a^3*b*c^2*d^3*e + 60*a*b^3*c^2*e^4*x - 112*a^2*b*c^3*e^4*x + 4*a*b^5*d^2*e^
2*x - 8*a^2*b^4*d^3*e*x + 256*a^3*c^3*d*e^3*x - 256*a^4*c^2*d^3*e*x - 6*a*b
^2*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) - 160*a^2*b^2*c^2*d*e^3*x - 56*a^2*b^3*c*d
^2*e^2*x + 160*a^3*b*c^2*d^2*e^2*x + 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(
b^2 - 4*a*c)^(3/2) + 96*a^3*b^2*c*d^3*e*x)*(b^2*((a*d^2)/2 - (c*e^2)/2) - b
*((a*d^2*(b^2 - 4*a*c)^(1/2))/2 + (c*e^2*(b^2 - 4*a*c)^(1/2))/2) - 2*a^2*c*
d^2 + 2*a*c^2*e^2 + 2*a*c*d*e*(b^2 - 4*a*c)^(1/2)))/(4*a^3*c*d^4 + 4*a*c^3*
e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3
*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*
e^2) - (log(8*a^2*b^4*d^4 + 96*a^4*c^2*d^4 + 96*a^2*c^4*e^4 + 8*b^4*c^2*e^4
 - 56*a^3*b^2*c*d^4 - 56*a*b^2*c^3*e^4 + 4*a^3*b^3*d^4*x - 320*a^3*c^3*d^2*
e^2 + 8*a*d^3*e*(b^2 - 4*a*c)^(5/2) - 8*c*d*e^3*(b^2 - 4*a*c)^(5/2) - 3*c*e
^4*x*(b^2 - 4*a*c)^(5/2) + 8*b^5*c*e^4*x + 8*a^2*b*d^4*(b^2 - 4*a*c)^(3/2)
- 8*b*c^2*e^4*(b^2 - 4*a*c)^(3/2) + 12*a^3*d^4*x*(b^2 - 4*a*c)^(3/2) - 6*b*
d*e^3*x*(b^2 - 4*a*c)^(5/2) - 16*a^4*b*c*d^4*x + 112*a^2*b^2*c^2*d^2*e^2 -
8*a*b^2*d^3*e*(b^2 - 4*a*c)^(3/2) + 8*b^2*c*d*e^3*(b^2 - 4*a*c)^(3/2) + 10*
```

```
a*d^2*e^2*x*(b^2 - 4*a*c)^(5/2) - 5*b^2*c*e^4*x*(b^2 - 4*a*c)^(3/2) + 6*b^3
*d*e^3*x*(b^2 - 4*a*c)^(3/2) - 16*a*b^3*c^2*d*e^3 - 8*a*b^4*c*d^2*e^2 + 64*
a^2*b*c^3*d*e^3 - 16*a^2*b^3*c*d^3*e + 64*a^3*b*c^2*d^3*e - 60*a*b^3*c^2*e^
4*x + 112*a^2*b*c^3*e^4*x - 4*a*b^5*d^2*e^2*x + 8*a^2*b^4*d^3*e*x - 256*a^3
*c^3*d*e^3*x + 256*a^4*c^2*d^3*e*x - 6*a*b^2*d^2*e^2*x*(b^2 - 4*a*c)^(3/2)
+ 160*a^2*b^2*c^2*d*e^3*x + 56*a^2*b^3*c*d^2*e^2*x - 160*a^3*b*c^2*d^2*e^2*
x - 24*a*b^4*c*d*e^3*x - 8*a^2*b*d^3*e*x*(b^2 - 4*a*c)^(3/2) - 96*a^3*b^2*c
*d^3*e*x)*(b*((a*d^2*(b^2 - 4*a*c)^(1/2))/2 + (c*e^2*(b^2 - 4*a*c)^(1/2))/2
) + b^2*((a*d^2)/2 - (c*e^2)/2) - 2*a^2*c*d^2 + 2*a*c^2*e^2 - 2*a*c*d*e*(b^
2 - 4*a*c)^(1/2)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^2*e^4 -
 b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8*a*b*c^
2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (log(d + e*x)*(a*d^2 - c*e
^2))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3 + 2*a*c*d
^2*e^2)
```

3.75 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx$

Optimal. Leaf size=189

$$-\frac{e}{(ad^2-bde+ce^2)(d+ex)}-\frac{(2a^2d^2+b^2e^2-2ae(bd+ce))\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}+\frac{e(2ad-be)\log(d+e\ldots}{(ad^2-e(bd-ce))^2}$$

```
[Out] -e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e*(2*a*d-b*e)*ln(e*x+d)/(a*d^2-e*(b*d-c*e))^
2-1/2*e*(2*a*d-b*e)*ln(a*x^2+b*x+c)/(a*d^2-e*(b*d-c*e))^2-(2*a^2*d^2+b^2*e^
2-2*a*e*(b*d+c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/(a*d^2-e*(b*d-c*e)
)^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1583, 723, 814, 648, 632, 212, 642}

$$-\frac{(2a^2d^2-2ae(bd+ce)+b^2e^2)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))^2}-\frac{e}{(d+ex)(ad^2-bde+ce^2)}-\frac{e(2ad-be)\log(ax^2+bx+c)}{2(ad^2-e(bd-ce))^2}+\frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] -(e/((a*d^2 - b*d*e + c*e^2)*(d + e*x))) - ((2*a^2*d^2 + b^2*e^2 - 2*a*e*(b
*d + c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4*a*c]*(a*d^
2 - e*(b*d - c*e))^2) + (e*(2*a*d - b*e)*Log[d + e*x])/(a*d^2 - e*(b*d - c*
e))^2 - (e*(2*a*d - b*e)*Log[c + b*x + a*x^2])/(2*(a*d^2 - e*(b*d - c*e))^2
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 723

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x,
 x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^2(d+ex)^2}\,dx = \int \frac{1}{(d+ex)^2\,(c+bx+ax^2)}\,dx$$

$$= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{\int \frac{ad-be-aex}{(d+ex)(c+bx+ax^2)}\,dx}{ad^2-bde+ce^2}$$

$$= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{\int \Big(\frac{e^2(2ad-be)}{(ad^2-e(bd-ce))(d+ex)} + \frac{a^2d^2+b^2e^2-ae(2bd+ce}{(ad^2-e(bd-ce))(c}}{ad^2-bde+ce^2}$$

$$= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2} + \frac{\int \frac{a^2d^2+b^2e^2-ae(2}{c+}}{(ad^2-}$$

$$= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2} - \frac{(e(2ad-be))}{2\,(ad^2-e(}$$

$$= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} + \frac{e(2ad-be)\log(d+ex)}{(ad^2-e(bd-ce))^2} - \frac{e(2ad-be)\log}{2\,(ad^2-e}$$

$$= -\frac{e}{(ad^2-bde+ce^2)\,(d+ex)} - \frac{(2a^2d^2+b^2e^2-2ae(bd+ce))\tanh^{-1}\Big(}{\sqrt{b^2-4ac}\,(ad^2-e(bd-ce))}$$

Mathematica [A]
time = 0.15, size = 151, normalized size = 0.80

$$\frac{-\frac{2e(ad^2+e(-bd+ce))}{d+ex}+\frac{2(2a^2d^2+b^2e^2-2ae(bd+ce))\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}-2e(-2ad+be)\log(d+ex)+e(-2ad+be)\log(c+x(b+ax))}{2\,(ad^2+e(-bd+ce))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^2*(d + e*x)^2),x]
```

```
[Out] ((-2*e*(a*d^2 + e*(-(b*d) + c*e)))/(d + e*x) + (2*(2*a^2*d^2 + b^2*e^2 - 2*
a*e*(b*d + c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
 - 2*e*(-2*a*d + b*e)*Log[d + e*x] + e*(-2*a*d + b*e)*Log[c + x*(b + a*x)])
/(2*(a*d^2 + e*(-(b*d) + c*e))^2)
```

Maple [A]
time = 0.30, size = 197, normalized size = 1.04

method	result
default	$-\frac{e}{(a\,d^2-deb+c\,e^2)(ex+d)}+\frac{e(2ad-eb)\ln(ex+d)}{(a\,d^2-deb+c\,e^2)^2}+\frac{\frac{(-2de\,a^2+ab\,e^2)\ln(a\,x^2+bx+c)}{2a}+\frac{2\left(a^2d^2-2abde-ac\,e^2+e^2b^2-\frac{(-2de\,a^2+ab\,e^2)}{2a}\right.}{\sqrt{4ac-b^2}}}{(a\,d^2-deb+c\,e^2)^2}$

risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e*(2*a*d-b*e)/(a*d^2-b*d*e+c*e^2)^2*ln(e*x+d
)+1/(a*d^2-b*d*e+c*e^2)^2*(1/2*(-2*a^2*d*e+a*b*e^2)/a*ln(a*x^2+b*x+c)+2*(a^
2*d^2-2*a*b*d*e-a*c*e^2+e^2*b^2-1/2*(-2*a^2*d*e+a*b*e^2)*b/a)/(4*a*c-b^2)^(
1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(197) = 394$.
time = 1.94, size = 1071, normalized size = 5.67

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e - 2*(b^3 - 4*a*b*c)*d*e^2 + (2*a^2*d^3 + (
b^2 - 2*a*c)*x*e^3 - (2*a*b*d*x - (b^2 - 2*a*c)*d)*e^2 + 2*(a^2*d^2*x - a*b
*d^2)*e)*sqrt(b^2 - 4*a*c)*log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c + sqrt(b^
2 - 4*a*c)*(2*a*x + b))/(a*x^2 + b*x + c)) + 2*(b^2*c - 4*a*c^2)*e^3 + (2*(
a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*x*e^3 + (2*(a*b^2 - 4*a^2*c)*d*x -
 (b^3 - 4*a*b*c)*d)*e^2)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a^2*c)*d^2*
e - (b^3 - 4*a*b*c)*x*e^3 + (2*(a*b^2 - 4*a^2*c)*d*x - (b^3 - 4*a*b*c)*d)*e
^2)*log(x*e + d))/((a^2*b^2 - 4*a^3*c)*d^5 + (b^2*c^2 - 4*a*c^3)*x*e^5 - (2
*(b^3*c - 4*a*b*c^2)*d*x - (b^2*c^2 - 4*a*c^3)*d)*e^4 + ((b^4 - 2*a*b^2*c -
 8*a^2*c^2)*d^2*x - 2*(b^3*c - 4*a*b*c^2)*d^2)*e^3 - (2*(a*b^3 - 4*a^2*b*c)
*d^3*x - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3)*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*
x - 2*(a*b^3 - 4*a^2*b*c)*d^4)*e), -1/2*(2*(a*b^2 - 4*a^2*c)*d^2*e - 2*(b^3
 - 4*a*b*c)*d*e^2 + 2*(2*a^2*d^3 + (b^2 - 2*a*c)*x*e^3 - (2*a*b*d*x - (b^2
```

```
- 2*a*c)*d)*e^2 + 2*(a^2*d^2*x - a*b*d^2)*e)*sqrt(-b^2 + 4*a*c)*arctan(-sqr
t(-b^2 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c)) + 2*(b^2*c - 4*a*c^2)*e^3 + (2*(
a*b^2 - 4*a^2*c)*d^2*e - (b^3 - 4*a*b*c)*x*e^3 + (2*(a*b^2 - 4*a^2*c)*d*x -
 (b^3 - 4*a*b*c)*d)*e^2)*log(a*x^2 + b*x + c) - 2*(2*(a*b^2 - 4*a^2*c)*d^2*
e - (b^3 - 4*a*b*c)*x*e^3 + (2*(a*b^2 - 4*a^2*c)*d*x - (b^3 - 4*a*b*c)*d)*e
^2)*log(x*e + d))/((a^2*b^2 - 4*a^3*c)*d^5 + (b^2*c^2 - 4*a*c^3)*x*e^5 - (2
*(b^3*c - 4*a*b*c^2)*d*x - (b^2*c^2 - 4*a*c^3)*d)*e^4 + ((b^4 - 2*a*b^2*c -
 8*a^2*c^2)*d^2*x - 2*(b^3*c - 4*a*b*c^2)*d^2)*e^3 - (2*(a*b^3 - 4*a^2*b*c)
*d^3*x - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^3)*e^2 + ((a^2*b^2 - 4*a^3*c)*d^4*
x - 2*(a*b^3 - 4*a^2*b*c)*d^4)*e)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**2/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.13, size = 331, normalized size = 1.75

$$-\frac{\left(2\,a^2d^2e^2-2\,abde^3+b^2e^4-2\,ace^4\right)\arctan\left(-\frac{\left(2\,ad-\frac{2\,ad^2}{xe+d}-be+\frac{2\,bde}{xe+d}-\frac{2\,ce^2}{xe+d}\right)e^{(-1)}}{\sqrt{-b^2+4\,ac}}\right)e^{(-2)}}{\left(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4\right)\sqrt{-b^2+4\,ac}}-\frac{\left(2\,ade-be^2\right)\log\left(-a+\frac{2\,ad}{xe+d}-\frac{ad^2}{(xe+d)^2}-\frac{be}{xe+d}+\frac{bde}{(xe+d)^2}-\frac{ce^2}{(xe+d)^2}\right)}{2\left(a^2d^4-2\,abd^3e+b^2d^2e^2+2\,acd^2e^2-2\,bcde^3+c^2e^4\right)}-\frac{e^3}{\left(ad^2e^2-bde^3+ce^4\right)(xe+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^2/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(2*a^2*d^2*e^2 - 2*a*b*d*e^3 + b^2*e^4 - 2*a*c*e^4)*arctan(-(2*a*d - 2*a*d
^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^
2 + 4*a*c))*e^(-2)/((a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 -
2*b*c*d*e^3 + c^2*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*a*d*e - b*e^2)*log(-a +
 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 -
c*e^2/(x*e + d)^2)/(a^2*d^4 - 2*a*b*d^3*e + b^2*d^2*e^2 + 2*a*c*d^2*e^2 - 2
*b*c*d*e^3 + c^2*e^4) - e^3/((a*d^2*e^2 - b*d*e^3 + c*e^4)*(x*e + d))
```

Mupad [**B**]
time = 8.11, size = 1782, normalized size = 9.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(c*e^4*(b^2 - 4*a*c)^(5/2) - 8*b^5*c*e^4 - 8*b^6*e^4*x - 4*a^3*d^4*(b^2
 - 4*a*c)^(3/2) - 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a*c)^(3/2) + 60*a*b^
3*c^2*e^4 - 112*a^2*b*c^3*e^4 + 4*a*b^5*d^2*e^2 - 8*a^2*b^4*d^3*e + 256*a^3
*c^3*d*e^3 - 256*a^4*c^2*d^3*e - 8*a^4*b^2*d^4*x + 32*a^3*c^3*e^4*x + 10*b*
d*e^3*(b^2 - 4*a*c)^(5/2) + 4*b*e^4*x*(b^2 - 4*a*c)^(5/2) + 16*a^4*b*c*d^4
+ 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^(5/2) + 7*b^2*c*e^4*(b^2 - 4*
a*c)^(3/2) - 10*b^3*d*e^3*(b^2 - 4*a*c)^(3/2) - 8*a*d*e^3*x*(b^2 - 4*a*c)^(
5/2) + 24*a*b^4*c*d*e^3 + 64*a*b^4*c*e^4*x + 32*a*b^5*d*e^3*x - 8*a^2*b*d^3
*e*(b^2 - 4*a*c)^(3/2) - 32*a^3*d^3*e*x*(b^2 - 4*a*c)^(3/2) + 96*a^3*b^2*c*
d^3*e + 16*a^3*b^3*d^3*e*x + 18*a*b^2*d^2*e^2*(b^2 - 4*a*c)^(3/2) - 160*a^2
*b^2*c^2*d*e^3 - 56*a^2*b^3*c*d^2*e^2 + 160*a^3*b*c^2*d^2*e^2 - 136*a^2*b^2
*c^2*e^4*x - 40*a^2*b^4*d^2*e^2*x - 448*a^4*c^2*d^2*e^2*x + 48*a^2*b*d^2*e^
2*x*(b^2 - 4*a*c)^(3/2) + 272*a^3*b^2*c*d^2*e^2*x - 64*a^4*b*c*d^3*e*x - 24
*a*b^2*d*e^3*x*(b^2 - 4*a*c)^(3/2) - 240*a^2*b^3*c*d*e^3*x + 448*a^3*b*c^2*
d*e^3*x)*(a*(e^2*(2*b*c - c*(b^2 - 4*a*c)^(1/2)) + e*(b^2*d - b*d*(b^2 - 4*
a*c)^(1/2))) - e^2*(b^3/2 - (b^2*(b^2 - 4*a*c)^(1/2))/2) + a^2*(d^2*(b^2 -
4*a*c)^(1/2) - 4*c*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a^2*b^2*d^4 - b^2*c^
2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e + 2*b^3*c*d*e^3 - 8
*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) - (log(d + e*x)*(b*e^
2 - 2*a*d*e))/(a^2*d^4 + c^2*e^4 + b^2*d^2*e^2 - 2*a*b*d^3*e - 2*b*c*d*e^3
+ 2*a*c*d^2*e^2) - (log(c*e^4*(b^2 - 4*a*c)^(5/2) + 8*b^5*c*e^4 + 8*b^6*e^4
*x - 4*a^3*d^4*(b^2 - 4*a*c)^(3/2) + 4*a^3*b^3*d^4 + 4*b^3*e^4*x*(b^2 - 4*a
*c)^(3/2) - 60*a*b^3*c^2*e^4 + 112*a^2*b*c^3*e^4 - 4*a*b^5*d^2*e^2 + 8*a^2*
b^4*d^3*e - 256*a^3*c^3*d*e^3 + 256*a^4*c^2*d^3*e + 8*a^4*b^2*d^4*x - 32*a^
3*c^3*e^4*x + 10*b*d*e^3*(b^2 - 4*a*c)^(5/2) + 4*b*e^4*x*(b^2 - 4*a*c)^(5/2
) - 16*a^4*b*c*d^4 - 32*a^5*c*d^4*x - 14*a*d^2*e^2*(b^2 - 4*a*c)^(5/2) + 7*
b^2*c*e^4*(b^2 - 4*a*c)^(3/2) - 10*b^3*d*e^3*(b^2 - 4*a*c)^(3/2) - 8*a*d*e^
3*x*(b^2 - 4*a*c)^(5/2) - 24*a*b^4*c*d*e^3 - 64*a*b^4*c*e^4*x - 32*a*b^5*d*
e^3*x - 8*a^2*b*d^3*e*(b^2 - 4*a*c)^(3/2) - 32*a^3*d^3*e*x*(b^2 - 4*a*c)^(3
/2) - 96*a^3*b^2*c*d^3*e - 16*a^3*b^3*d^3*e*x + 18*a*b^2*d^2*e^2*(b^2 - 4*a
*c)^(3/2) + 160*a^2*b^2*c^2*d*e^3 + 56*a^2*b^3*c*d^2*e^2 - 160*a^3*b*c^2*d^
2*e^2 + 136*a^2*b^2*c^2*e^4*x + 40*a^2*b^4*d^2*e^2*x + 448*a^4*c^2*d^2*e^2*
x + 48*a^2*b*d^2*e^2*x*(b^2 - 4*a*c)^(3/2) - 272*a^3*b^2*c*d^2*e^2*x + 64*a
^4*b*c*d^3*e*x - 24*a*b^2*d*e^3*x*(b^2 - 4*a*c)^(3/2) + 240*a^2*b^3*c*d*e^3
*x - 448*a^3*b*c^2*d*e^3*x)*(e^2*(b^3/2 + (b^2*(b^2 - 4*a*c)^(1/2))/2) - a*
(e^2*(2*b*c + c*(b^2 - 4*a*c)^(1/2)) + e*(b^2*d + b*d*(b^2 - 4*a*c)^(1/2)))
 + a^2*(d^2*(b^2 - 4*a*c)^(1/2) + 4*c*d*e)))/(4*a^3*c*d^4 + 4*a*c^3*e^4 - a
^2*b^2*d^4 - b^2*c^2*e^4 - b^4*d^2*e^2 + 8*a^2*c^2*d^2*e^2 + 2*a*b^3*d^3*e
+ 2*b^3*c*d*e^3 - 8*a*b*c^2*d*e^3 - 8*a^2*b*c*d^3*e + 2*a*b^2*c*d^2*e^2) -
e/((d + e*x)*(a*d^2 + c*e^2 - b*d*e))
```

3.76 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx$

Optimal. Leaf size=248

$$\frac{e^2}{d\left(ad^2-bde+ce^2\right)(d+ex)}+\frac{\left(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce)\right)\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)}{cd^2}-e^2($$

```
[Out] e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+d)+ln(x)/c/d^2-e^2*(3*a*d^2-e*(2*b*d-c*e))*l
n(e*x+d)/d^2/(a*d^2-e*(b*d-c*e))^2-1/2*(a^2*d^2+b^2*e^2-a*e*(2*b*d+c*e))*ln
(a*x^2+b*x+c)/c/(a*d^2-e*(b*d-c*e))^2+(b^3*e^2-a*b*e*(2*b*d+3*c*e)+a^2*d*(b
*d+4*c*e))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c/(a*d^2-e*(b*d-c*e))^2/(-
4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 907, 648, 632, 212, 642}

$$-\frac{\left(a^2d^2-ae(2bd+ce)+b^2e^2\right)\log\left(ax^2+bx+c\right)}{2c\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(a^2d(bd+4ce)-abe(2bd+3ce)+b^3e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}+\frac{e^2}{d(d+ex)\left(ad^2-e(bd-ce)\right)}-\frac{e^2\log(d+ex)\left(3ad^2-e(2bd-ce)\right)}{d^2\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((b^3*e^2 - a*b*e*(2*b*d + 3*c*
e) + a^2*d*(b*d + 4*c*e))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c*Sqrt[b
^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*d^2 - e
*(2*b*d - c*e))*Log[d + e*x])/(d^2*(a*d^2 - e*(b*d - c*e))^2) - ((a^2*d^2 +
 b^2*e^2 - a*e*(2*b*d + c*e))*Log[c + b*x + a*x^2])/(2*c*(a*d^2 - e*(b*d -
c*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^3(d+ex)^2}\,dx = \int \frac{1}{x(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x} + \frac{e^3}{d\left(-ad^2+e(bd-ce)\right)(d+ex)^2} + \frac{e^3(-3ad^2+e(2bd-c}{d^2\left(ad^2-e(bd-ce)\right)^2(d}\right.$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2-e(2bd-ce))\log(d+}{d^2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2-e(2bd-ce))\log(d+}{d^2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2-e(2bd-ce))\log(d+}{d^2\left(ad^2-e(bd-ce)\right)^2}$$

$$= \frac{e^2}{d\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce))}{c\sqrt{b^2-4ac}\,(ad^2-e(bd}$$

Mathematica [A]

time = 0.18, size = 246, normalized size = 0.99

$$\frac{e^2}{d\left(ad^2+e(-bd+ce)\right)(d+ex)} - \frac{\left(b^3e^2-abe(2bd+3ce)+a^2d(bd+4ce)\right)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c\sqrt{-b^2+4ac}\,\left(ad^2+e(-bd+ce)\right)^2} + \frac{\log(x)}{cd^2} - \frac{e^2(3ad^2+e(-2bd+ce))\log(d+ex)}{\left(ad^3+de(-bd+ce)\right)^2} + \frac{\left(-a^2d^2-b^2e^2+ae(2bd+ce)\right)\log(c+x(b+ax))}{2c\left(ad^2+e(-bd+ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^3*(d + e*x)^2),x]
```

```
[Out] e^2/(d*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) - ((b^3*e^2 - a*b*e*(2*b*d + 3
*c*e) + a^2*d*(b*d + 4*c*e))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c*Sqr
t[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2) + Log[x]/(c*d^2) - (e^2*(3*a*
d^2 + e*(-2*b*d + c*e))*Log[d + e*x])/(a*d^3 + d*e*(-(b*d) + c*e))^2 + ((-(
a^2*d^2) - b^2*e^2 + a*e*(2*b*d + c*e))*Log[c + x*(b + a*x)])/(2*c*(a*d^2 +
 e*(-(b*d) + c*e))^2)
```

Maple [A]

time = 0.34, size = 281, normalized size = 1.13

method	result
default	$\frac{e^2}{d(a\,d^2-deb+c\,e^2)(ex+d)} - \frac{e^2\left(3a\,d^2-2deb+c\,e^2\right)\ln(ex+d)}{d^2(a\,d^2-deb+c\,e^2)^2} + \frac{\frac{\left(-a^3d^2+2a^2bde+a^2c\,e^2-a\,b^2e^2\right)\ln\left(a\,x^2+bx+c\right)}{2a} + \frac{2\left(-a^2b\,d^2-2a^2cde-\right.}{}}{(}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^2/d/(a*d^2-b*d*e+c*e^2)/(e*x+d)-e^2*(3*a*d^2-2*b*d*e+c*e^2)/d^2/(a*d^2-b*
d*e+c*e^2)^2*ln(e*x+d)+1/(a*d^2-b*d*e+c*e^2)^2/c*(1/2*(-a^3*d^2+2*a^2*b*d*e
+a^2*c*e^2-a*b^2*e^2)/a*ln(a*x^2+b*x+c)+2*(-a^2*b*d^2-2*a^2*c*d*e+2*a*b^2*d
*e+2*a*b*c*e^2-b^3*e^2-1/2*(-a^3*d^2+2*a^2*b*d*e+a^2*c*e^2-a*b^2*e^2)*b/a)/
(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))+ln(x)/c/d^2
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**3/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.07, size = 391, normalized size = 1.58

$$-\frac{{\left(a^{2} b d^{2} e^{2} - 2 \, a b^{2} d e^{3} + 4 \, a^{2} c d e^{3} + b^{3} e^{4} - 3 \, a b c e^{4}\right)} \arctan\left(\frac{{\left(2 \, a d - \frac{2 \, a d^{2}}{x e + d} - b e + \frac{2 \, b d e}{x e + d} - \frac{2 \, c e^{2}}{x e + d}\right)} e^{\left(-1\right)}}{\sqrt{-b^{2} + 4 \, a c}}\right) e^{\left(-2\right)}}{{\left(a^{2} c d^{4} - 2 \, a b c d^{3} e + b^{2} c d^{2} e^{2} + 2 \, a c^{2} d^{2} e^{2} - 2 \, b c^{2} d e^{3} + c^{3} e^{4}\right)} \sqrt{-b^{2} + 4 \, a c}} - \frac{{\left(a^{2} d^{2} - 2 \, a b d e + b^{2} e^{2} - a c e^{2}\right)} \log\left(a - \frac{2 \, a d}{x e + d} + \frac{a d^{2}}{{\left(x e + d\right)}^{2}} + \frac{b e}{x e + d} - \frac{b d e}{{\left(x e + d\right)}^{2}} + \frac{c e^{2}}{{\left(x e + d\right)}^{2}}\right)}{2 \, {\left(a^{2} c d^{4} - 2 \, a b c d^{3} e + b^{2} c d^{2} e^{2} + 2 \, a c^{2} d^{2} e^{2} - 2 \, b c^{2} d e^{3} + c^{3} e^{4}\right)}} + \frac{e^{5}}{{\left(a d^{3} e^{3} - b d^{2} e^{4} + c d e^{5}\right)} {\left(x e + d\right)}} + \frac{\log\left({\left| -\frac{d}{x e + d} + 1 \right|}\right)}{c d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^3/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a^2*b*d^2*e^2 - 2*a*b^2*d*e^3 + 4*a^2*c*d*e^3 + b^3*e^4 - 3*a*b*c*e^4)*ar
ctan((2*a*d - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e +
d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d
^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 + c^3*e^4)*sqrt(-b^2 + 4*a*c)) - 1
/2*(a^2*d^2 - 2*a*b*d*e + b^2*e^2 - a*c*e^2)*log(a - 2*a*d/(x*e + d) + a*d^
2/(x*e + d)^2 + b*e/(x*e + d) - b*d*e/(x*e + d)^2 + c*e^2/(x*e + d)^2)/(a^2
*c*d^4 - 2*a*b*c*d^3*e + b^2*c*d^2*e^2 + 2*a*c^2*d^2*e^2 - 2*b*c^2*d*e^3 +
c^3*e^4) + e^5/((a*d^3*e^3 - b*d^2*e^4 + c*d*e^5)*(x*e + d)) + log(abs(-d/(
x*e + d) + 1))/(c*d^2)
```

Mupad [B]
time = 25.28, size = 2500, normalized size = 10.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log((a^4*e^4)/(d*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^5*x)/(d^2*(a*d^2 + c*
e^2 - b*d*e)^2) - (((a*e^3*(3*a^3*b*d^4 + b^3*c*e^4 - b^4*d*e^3 + 5*a*b^3*d
^2*e^2 - 7*a^2*b^2*d^3*e + 8*a^2*c^2*d*e^3 - 3*a*b*c^2*e^4 + 9*a^3*c*d^3*e
- a*b^2*c*d*e^3 - 8*a^2*b*c*d^2*e^2))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) + (((
a*e*(a^3*b*d^5 - 4*a*c^3*e^5 + b^2*c^2*e^5 - b^4*d^2*e^3 + 3*a*b^3*d^3*e^2
- 3*a^2*b^2*d^4*e - 8*a^2*c^2*d^2*e^3 + 4*a^3*c*d^4*e - b^3*c*d*e^4 + 4*a*b
*c^2*d*e^4 + 6*a*b^2*c*d^2*e^3 - 9*a^2*b*c*d^3*e^2))/(a*d^3 - b*d^2*e + c*d
*e^2) + (a*e*x*(3*a^4*d^5 + 2*b^3*c*e^5 - 4*b^4*d*e^4 + 9*a*b^3*d^2*e^3 + 4
*a^2*c^2*d*e^4 + 19*a^3*c*d^3*e^2 - 3*a^2*b^2*d^3*e^2 - 8*a*b*c^2*e^5 - 5*a
^3*b*d^4*e + 15*a*b^2*c*d*e^4 - 36*a^2*b*c*d^2*e^3))/(a*d^3 - b*d^2*e + c*d
*e^2) - (a*e*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2
*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 + a^2*b*d^2*(b^2 - 4*a*c
)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e*(b^
2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*
c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e
^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^
2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c
^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x
))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 +
b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5
*a*b^2*c*e^2 + a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*
(b^2 - 4*a*c)^(1/2) - 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 -
4*a*c)^(1/2)))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e^3*x*(9*
a^4*d^4 + b^4*e^4 + 2*a^2*c^2*e^4 - 6*a^3*c*d^2*e^2 + 8*a^2*b^2*d^2*e^2 - 4
```

```
*a*b^2*c*e^4 - 4*a*b^3*d*e^3 - 12*a^3*b*d^3*e + 10*a^2*b*c*d*e^3))/(d^2*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2 - 4*a*c)^(1/
2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 + a^2*b*d^2*
(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a
*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*
c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 + b^3*e^2*(b^2
- 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2
+ a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e - 3*a*b*c*e^2*(b^2 - 4*a*c)
^(1/2) - 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2))
)/(2*(4*a*c^4*e^4 + 4*a^3*c^2*d^4 - b^2*c^3*e^4 - a^2*b^2*c*d^4 + 2*b^3*c^2
*d*e^3 - b^4*c*d^2*e^2 + 8*a^2*c^3*d^2*e^2 - 8*a*b*c^3*d*e^3 + 2*a*b^3*c*d^
3*e - 8*a^2*b*c^2*d^3*e + 2*a*b^2*c^2*d^2*e^2)) + (log((a^4*e^4)/(d*(a*d^2
+ c*e^2 - b*d*e)^2) + (a^4*e^5*x)/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e^
3*(3*a^3*b*d^4 + b^3*c*e^4 - b^4*d*e^3 + 5*a*b^3*d^2*e^2 - 7*a^2*b^2*d^3*e
+ 8*a^2*c^2*d*e^3 - 3*a*b*c^2*e^4 + 9*a^3*c*d^3*e - a*b^2*c*d*e^3 - 8*a^2*b
*c*d^2*e^2))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2) + (((a*e*(a^3*b*d^5 - 4*a*c^3*
e^5 + b^2*c^2*e^5 - b^4*d^2*e^3 + 3*a*b^3*d^3*e^2 - 3*a^2*b^2*d^4*e - 8*a^2
*c^2*d^2*e^3 + 4*a^3*c*d^4*e - b^3*c*d*e^4 + 4*a*b*c^2*d*e^4 + 6*a*b^2*c*d^
2*e^3 - 9*a^2*b*c*d^3*e^2))/(a*d^3 - b*d^2*e + c*d*e^2) + (a*e*x*(3*a^4*d^5
 + 2*b^3*c*e^5 - 4*b^4*d*e^4 + 9*a*b^3*d^2*e^3 + 4*a^2*c^2*d*e^4 + 19*a^3*c
*d^3*e^2 - 3*a^2*b^2*d^3*e^2 - 8*a*b*c^2*e^5 - 5*a^3*b*d^4*e + 15*a*b^2*c*d
*e^4 - 36*a^2*b*c*d^2*e^3))/(a*d^3 - b*d^2*e + c*d*e^2) - (a*e*(b^4*e^2 - 4
*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*
a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e +
 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*
c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2
 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*
c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e
*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*
d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c*(4*a*c - b^2)*(a*
d^2 + c*e^2 - b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/
2) + a^2*b^2*d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*
(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a
*b^2*d*e*(b^2 - 4*a*c)^(1/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*
c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e^3*x*(9*a^4*d^4 + b^4*e^4 + 2*a^2
*c^2*e^4 - 6*a^3*c*d^2*e^2 + 8*a^2*b^2*d^2*e^2 - 4*a*b^2*c*e^4 - 4*a*b^3*d*
e^3 - 12*a^3*b*d^3*e + 10*a^2*b*c*d*e^3))/(d^2*(a*d^2 + c*e^2 - b*d*e)^2))*
(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*d^2 + 4*a^2*
c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e^2 - a^2*b*d^2*(b^2 - 4*a*c)^(1/2) + 8*a
^2*b*c*d*e + 3*a*b*c*e^2*(b^2 - 4*a*c)^(1/2) + 2*a*b^2*d*e*(b^2 - 4*a*c)^(1
/2) - 4*a^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c*(4*a*c - b^2)*(a*d^2 + c*e^2 -
 b*d*e)^2))*(b^4*e^2 - 4*a^3*c*d^2 - b^3*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^2*
d^2 + 4*a^2*c^2*e^2 - 2*a*b^3*d*e - 5*a*b^2*c*e...
```

3.77 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx$

Optimal. Leaf size=291

$$-\frac{1}{cd^2x}-\frac{e^3}{d^2\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{\left(2a^3cd^2-b^4e^2+2ab^2e(bd+2ce)-a^2(b^2d^2+6bcde+2c^2e^2)\right)\tanh$$

[Out] -1/c/d^2/x-e^3/d^2/(a*d^2-e*(b*d-c*e))/(e*x+d)-(b*d+2*c*e)*ln(x)/c^2/d^3+e^3*(4*a*d^2-e*(3*b*d-2*c*e))*ln(e*x+d)/d^3/(a*d^2-e*(b*d-c*e))^2+1/2*(a*d-b*e)*(a*b*d+2*a*c*e-b^2*e)*ln(a*x^2+b*x+c)/c^2/(a*d^2-e*(b*d-c*e))^2+(2*a^3*c*d^2-b^4*e^2+2*a*b^2*e*(b*d+2*c*e)-a^2*(b^2*d^2+6*b*c*d*e+2*c^2*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*d^2-e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)

Rubi [A]
time = 0.38, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 907, 648, 632, 212, 642}

$$\frac{\left(2a^3cd^2-a^2(b^2d^2+6bcde+2c^2e^2)+2ab^2e(bd+2ce)+b^4(-e^2)\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^2\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}+\frac{(ad-be)\left(abd+2ace+b^2(-e)\right)\log\left(ax^2+bx+c\right)}{2c^2\left(ad^2-e(bd-ce)\right)^2}-\frac{e^3}{d^2(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{e^3\log(d+ex)\left(4ad^2-e(3bd-2ce)\right)}{d^3\left(ad^2-e(bd-ce)\right)^2}-\frac{\log(x)(bd+2ce)}{c^2d^3}-\frac{1}{cd^2x}$$

Antiderivative was successfully verified.

[In] Int[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]

[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 - e*(b*d - c*e))*(d + e*x)) + ((2*a^3*c*d^2 - b^4*e^2 + 2*a*b^2*e*(b*d + 2*c*e) - a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/(c^2*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d^2 - e*(3*b*d - 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 - e*(b*d - c*e))^2) + ((a*d - b*e)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + b*x + a*x^2])/(2*c^2*(a*d^2 - e*(b*d - c*e))^2)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^4(d+ex)^2}\,dx = \int \frac{1}{x^2(d+ex)^2\,(c+bx+ax^2)}\,dx$$

$$= \int \left(\frac{1}{cd^2x^2} + \frac{-bd-2ce}{c^2d^3x} + \frac{e^4}{d^2\,(ad^2-e(bd-ce))\,(d+ex)^2} + \frac{e^4(4ad^2}{d^3\,(ad^2-e(}\right.$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\,(ad^2-e(bd-ce))\,(d+ex)} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3(4ad^2-}{d^3}$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\,(ad^2-e(bd-ce))\,(d+ex)} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3(4ad^2-}{d^3}$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\,(ad^2-e(bd-ce))\,(d+ex)} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3(4ad^2-}{d^3}$$

$$= -\frac{1}{cd^2x} - \frac{e^3}{d^2\,(ad^2-e(bd-ce))\,(d+ex)} + \frac{(2a^3cd^2-b^4e^2+2ab^2e(bd+2}{c^2\sqrt{}}$$

Mathematica [A]

time = 0.24, size = 287, normalized size = 0.99

$$-\frac{1}{cd^2x} - \frac{e^3}{d^2\,(ad^2+e(-bd+ce))\,(d+ex)} + \frac{(-2a^3cd^2+b^4e^2-2ab^2e(bd+2ce)+a^2(b^2d^2+6bcde+2c^2e^2))\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^2\sqrt{-b^2+4ac}\,(ad^2+e(-bd+ce))^2} - \frac{(bd+2ce)\log(x)}{c^2d^3} + \frac{e^3(4ad^2+e(-3bd+2ce))\log(d+ex)}{d^3\,(ad^2+e(-bd+ce))^2} + \frac{(ad-be)\,(abd-b^2e+2ace)\log(c+x(b+ax))}{2c^2\,(ad^2+e(-bd+ce))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^4*(d + e*x)^2),x]
```

```
[Out] -(1/(c*d^2*x)) - e^3/(d^2*(a*d^2 + e*(-(b*d) + c*e))*(d + e*x)) + ((-2*a^3*
c*d^2 + b^4*e^2 - 2*a*b^2*e*(b*d + 2*c*e) + a^2*(b^2*d^2 + 6*b*c*d*e + 2*c^
2*e^2))*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/(c^2*Sqrt[-b^2 + 4*a*c]*(a*
d^2 + e*(-(b*d) + c*e))^2) - ((b*d + 2*c*e)*Log[x])/(c^2*d^3) + (e^3*(4*a*d
^2 + e*(-3*b*d + 2*c*e))*Log[d + e*x])/(d^3*(a*d^2 + e*(-(b*d) + c*e))^2) +
 ((a*d - b*e)*(a*b*d - b^2*e + 2*a*c*e)*Log[c + x*(b + a*x)])/(2*c^2*(a*d^2
 + e*(-(b*d) + c*e))^2)
```

Maple [A]

time = 0.28, size = 346, normalized size = 1.19

method	result
default	$-\frac{e^3}{d^2(a\,d^2-deb+c\,e^2)(ex+d)} + \frac{e^3\left(4a\,d^2-3deb+2c\,e^2\right)\ln(ex+d)}{d^3(a\,d^2-deb+c\,e^2)^2} + \frac{\frac{\left(a^3b\,d^2+2a^3cde-2a^2b^2de-2a^2bc\,e^2+a\,b^3e^2\right)\ln\left(a\,x^2+bx+c\right)}{2a}+\frac{2\left(\right.}{}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e^3/d^2/(a*d^2-b*d*e+c*e^2)/(e*x+d)+e^3*(4*a*d^2-3*b*d*e+2*c*e^2)/d^3/(a*d
^2-b*d*e+c*e^2)^2*ln(e*x+d)+1/(a*d^2-b*d*e+c*e^2)^2/c^2*(1/2*(a^3*b*d^2+2*a
^3*c*d*e-2*a^2*b^2*d*e-2*a^2*b*c*e^2+a*b^3*e^2)/a*ln(a*x^2+b*x+c)+2*(-a^3*c
*d^2+a^2*b^2*d^2+4*a^2*b*c*d*e+e^2*c^2*a^2-2*a*b^3*d*e-3*a*b^2*c*e^2+b^4*e^
2-1/2*(a^3*b*d^2+2*a^3*c*d*e-2*a^2*b^2*d*e-2*a^2*b*c*e^2+a*b^3*e^2)*b/a)/(4
*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-1/c/d^2/x+1/c^2/d^3*(-
b*d-2*c*e)*ln(x)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**4/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.84, size = 487, normalized size = 1.67

$$-\frac{\left(a^{2} b^{2} d^{2} e^{2} - 2 \, a^{3} c d^{2} e^{2} - 2 \, a b^{3} d e^{3} + 6 \, a^{2} b c d e^{3} + b^{4} e^{4} - 4 \, a b^{2} c e^{4} + 2 \, a^{2} c^{2} e^{4}\right) \arctan\left(-\frac{\left(2 \, a d - \frac{2 \, a d^{2}}{x e + d} - b e + \frac{2 \, b d e}{x e + d} - \frac{2 \, c e^{2}}{x e + d}\right) e^{\left(-1\right)}}{\sqrt{-b^{2} + 4 \, a c}}\right) e^{\left(-2\right)}}{\left(a^{2} c^{2} d^{4} - 2 \, a b c^{2} d^{3} e + b^{2} c^{2} d^{2} e^{2} + 2 \, a c^{3} d^{2} e^{2} - 2 \, b c^{3} d e^{3} + c^{4} e^{4}\right) \sqrt{-b^{2} + 4 \, a c}} + \frac{\left(a^{2} b d^{2} - 2 \, a b^{2} d e + 2 \, a^{2} c d e + b^{3} e^{2} - 2 \, a b c e^{2}\right) \log\left(-a + \frac{2 \, a d}{x e + d} - \frac{a d^{2}}{{\left(x e + d\right)}^{2}} - \frac{b e}{x e + d} + \frac{b d e}{{\left(x e + d\right)}^{2}} - \frac{c e^{2}}{{\left(x e + d\right)}^{2}}\right)}{2 \, \left(a^{2} c^{2} d^{4} - 2 \, a b c^{2} d^{3} e + b^{2} c^{2} d^{2} e^{2} + 2 \, a c^{3} d^{2} e^{2} - 2 \, b c^{3} d e^{3} + c^{4} e^{4}\right)} - \frac{e^{7}}{\left(a d^{4} e^{4} - b d^{3} e^{5} + c d^{2} e^{6}\right)\left(x e + d\right)} - \frac{\left(b d e + 2 \, c e^{2}\right) e^{\left(-1\right)} \log\left(\left|-\frac{d}{x e + d} + 1\right|\right)}{c^{2} d^{3}} + \frac{e}{c d^{3} \left(\frac{d}{x e + d} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^4/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] -(a^2*b^2*d^2*e^2 - 2*a^3*c*d^2*e^2 - 2*a*b^3*d*e^3 + 6*a^2*b*c*d*e^3 + b^4
*e^4 - 4*a*b^2*c*e^4 + 2*a^2*c^2*e^4)*arctan(-(2*a*d - 2*a*d^2/(x*e + d) -
b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/sqrt(-b^2 + 4*a*c))*e^(
-2)/((a^2*c^2*d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2
*b*c^3*d*e^3 + c^4*e^4)*sqrt(-b^2 + 4*a*c)) + 1/2*(a^2*b*d^2 - 2*a*b^2*d*e
+ 2*a^2*c*d*e + b^3*e^2 - 2*a*b*c*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*
e + d)^2 - b*e/(x*e + d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*c^2*
d^4 - 2*a*b*c^2*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*b*c^3*d*e^3 +
 c^4*e^4) - e^7/((a*d^4*e^4 - b*d^3*e^5 + c*d^2*e^6)*(x*e + d)) - (b*d*e +
2*c*e^2)*e^(-1)*log(abs(-d/(x*e + d) + 1))/(c^2*d^3) + e/(c*d^3*(d/(x*e + d
) - 1))
```

Mupad [B]
time = 31.16, size = 2500, normalized size = 8.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] (log(d + e*x)*(2*c*e^5 + 4*a*d^2*e^3 - 3*b*d*e^4))/(a^2*d^7 + b^2*d^5*e^2 +
 c^2*d^3*e^4 - 2*a*b*d^6*e + 2*a*c*d^5*e^2 - 2*b*c*d^4*e^3) - (1/(c*d) + (x
*(2*c*e^3 + a*d^2*e - b*d*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))/(d*x + e*x
^2) - (log((((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*e^5 - 2*a*b^5*d^4*e^
4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^7 - b^5*c*d^2*e^6 +
a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5 + a^4*c^2*d^5*e^3 -
12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5 + 4*a*b^2*c^3*d*e^7
 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*c^3*d^2*e^6 + a^2*b^
3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) - (
((a*e*(a^4*c*d^6 + 8*a*c^4*e^6 - a^3*b^2*d^6 - 2*b^2*c^3*e^6 + b^5*d^3*e^3
- 3*a*b^4*d^4*e^2 + 3*a^2*b^3*d^5*e + b^3*c^2*d*e^5 + b^4*c*d^2*e^4 + 8*a^2
*c^3*d^2*e^4 - 7*a^3*c^2*d^4*e^2 - 4*a*b*c^3*d*e^5 - 7*a^3*b*c*d^5*e - 7*a*
b^3*c*d^3*e^3 - 6*a*b^2*c^2*d^2*e^4 + 12*a^2*b*c^2*d^3*e^3 + 12*a^2*b^2*c*d
^4*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(b^5*e^2 + b^4*e^2*(b^2 - 4
*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/
2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*
b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) + 10*a^2*b^2*c*
```

```
d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) +
 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*
c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d
^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*
b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8
*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x))/(2*c^2*(4*a*c
 - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (2*a*e*x*(a*d - b*e)*(a^3*b*d^5 + 8*a*
c^3*e^5 - 2*b^2*c^2*e^5 + b^4*d^2*e^3 - a*b^3*d^3*e^2 - a^2*b^2*d^4*e + 16*
a^2*c^2*d^2*e^3 + 2*a^3*c*d^4*e + 2*b^3*c*d*e^4 - 8*a*b*c^2*d*e^4 - 8*a*b^2
*c*d^2*e^3 + 4*a^2*b*c*d^3*e^2))/(c*d^2*(a*d^2 + c*e^2 - b*d*e)))*(b^5*e^2
 + b^4*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2
*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*
a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/
2) + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^
2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c - b^2)
*(a*d^2 + c*e^2 - b*d*e)^2) + (a*e*x*(a^6*d^8 + 8*a^2*c^4*e^8 + 4*b^4*c^2*e
^8 + b^6*d^2*e^6 - 16*a*b^2*c^3*e^8 - 2*a*b^5*d^3*e^5 + 2*a^5*c*d^6*e^2 + a
^2*b^4*d^4*e^4 + a^4*b^2*d^6*e^2 + 8*a^3*c^3*d^2*e^6 + 18*a^4*c^2*d^4*e^4 -
 2*a^5*b*d^7*e - 4*b^5*c*d*e^7 - 26*a^2*b^2*c^2*d^2*e^6 + 8*a*b^3*c^2*d*e^7
 + 4*a*b^4*c*d^2*e^6 + 16*a^2*b*c^3*d*e^7 + 6*a^4*b*c*d^5*e^3 + 10*a^2*b^3*
c*d^3*e^5 - 18*a^3*b^2*c*d^4*e^4))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2))*(b^
5*e^2 + b^4*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b
^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*
e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*
c)^(1/2) + 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d
*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^2*(4*a*c
 - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e
^2 - 3*b*d*e))/(c^2*d^4*(a*d^2 + c*e^2 - b*d*e)^2) + (4*a^5*e^4*x*(a*d - b*
e))/(c^2*d^2*(a*d^2 + c*e^2 - b*d*e)^2))*(b^5*e^2 + b^4*e^2*(b^2 - 4*a*c)^(
1/2) + a^2*b^3*d^2 + 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*
a^2*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e - 4*a^3*b*c*d^2 - 6*a*b^3*c*e
^2 - 8*a^3*c^2*d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) + 10*a^2*b^2*c*d*e - 4
*a*b^2*c*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*
b*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*(4*a*c^5*e^4 + 4*a^3*c^3*d^4 - b^2*c^4*e^4
 + 2*b^3*c^3*d*e^3 - a^2*b^2*c^2*d^4 + 8*a^2*c^4*d^2*e^2 - b^4*c^2*d^2*e^2
 - 8*a*b*c^4*d*e^3 + 2*a*b^3*c^2*d^3*e - 8*a^2*b*c^3*d^3*e + 2*a*b^2*c^3*d^2
*e^2)) + (log((a^4*e^4*(b*d + 2*c*e)*(3*a*d^2 + 2*c*e^2 - 3*b*d*e))/(c^2*d^
4*(a*d^2 + c*e^2 - b*d*e)^2) - (((a*e*(a^5*b*d^8 + 4*b^3*c^3*e^8 + b^6*d^3*
e^5 - 2*a*b^5*d^4*e^4 - 2*a^4*b^2*d^7*e + 16*a^2*c^4*d*e^7 - 4*b^4*c^2*d*e^
7 - b^5*c*d^2*e^6 + a^2*b^4*d^5*e^3 + a^3*b^3*d^6*e^2 + 16*a^3*c^3*d^3*e^5
 + a^4*c^2*d^5*e^3 - 12*a*b*c^4*e^8 + 2*a^5*c*d^7*e - 16*a^2*b^2*c^2*d^3*e^5
 + 4*a*b^2*c^3*d*e^7 - 2*a^4*b*c*d^6*e^2 + 13*a*b^3*c^2*d^2*e^6 - 20*a^2*b*
c^3*d^2*e^6 + a^2*b^3*c*d^4*e^4 + 8*a^3*b*c^2*d^4*e^4))/(c^2*d^4*(a*d^2 + c
*e^2 - b*d*e)^2) - (((a*e*(b^4*e^2*(b^2 - 4*a*c)^(1/2) - b^5*e^2 - a^2*b^3*
d^2 - 8*a^2*b*c^2*e^2 + a^2*b^2*d^2*(b^2 - 4*a*c)^(1/2) + 2*a^2*c^2*e^2*(b^
```

```
2 - 4*a*c)^(1/2) + 2*a*b^4*d*e + 4*a^3*b*c*d^2 + 6*a*b^3*c*e^2 + 8*a^3*c^2*
d*e - 2*a^3*c*d^2*(b^2 - 4*a*c)^(1/2) - 10*a^2*b^2*c*d*e - 4*a*b^2*c*e^2*(b
^2 - 4*a*c)^(1/2) - 2*a*b^3*d*e*(b^2 - 4*a*c)^(1/2) + 6*a^2*b*c*d*e*(b^2 -
4*a*c)^(1/2))*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*
d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2...
```

3.78 $\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx$

Optimal. Leaf size=372

$$-\frac{1}{2cd^2x^2}+\frac{bd+2ce}{c^2d^3x}+\frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)}+\frac{\left(b^5e^2-a^3cd(3bd+4ce)-ab^3e(2bd+5ce)+a^2b(b^2d^2-\right.}{c^3\sqrt{b^2-4ac}\,\left(ad^2-e(b\right.}$$

```
[Out] -1/2/c/d^2/x^2+(b*d+2*c*e)/c^2/d^3/x+e^4/d^3/(a*d^2-e*(b*d-c*e))/(e*x+d)+(b
^2*d^2+2*b*c*d*e-c*(a*d^2-3*c*e^2))*ln(x)/c^3/d^4-e^4*(5*a*d^2-e*(4*b*d-3*c
*e))*ln(e*x+d)/d^4/(a*d^2-e*(b*d-c*e))^2+1/2*(a^3*c*d^2-b^4*e^2+a*b^2*e*(2*
b*d+3*c*e)-a^2*(b^2*d^2+4*b*c*d*e+c^2*e^2))*ln(a*x^2+b*x+c)/c^3/(a*d^2-e*(b
*d-c*e))^2+(b^5*e^2-a^3*c*d*(3*b*d+4*c*e)-a*b^3*e*(2*b*d+5*c*e)+a^2*b*(b^2*
d^2+8*b*c*d*e+5*c^2*e^2))*arctanh((2*a*x+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*d^2-
e*(b*d-c*e))^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 372, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1583, 907, 648, 632, 212, 642}

$$\frac{\left(a^3cd^2-a^2\left(b^2d^2+4bcde+c^2e^2\right)+ab^2e(2bd+3ce)+b^4\left(-e^2\right)\right)\log\left(ax^2+bx+c\right)}{2c^3\left(ad^2-e(bd-ce)\right)^2}+\frac{\left(-a^3cd(3bd+4ce)+a^2b\left(b^2d^2+8bcde+5c^2e^2\right)-ab^3e(2bd+5ce)+b^5e^2\right)\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{c^3\sqrt{b^2-4ac}\,\left(ad^2-e(bd-ce)\right)^2}+\frac{\log(x)\left(-c\left(ad^2-3ce^2\right)+b^2d^2+2bcde\right)}{c^3d^4}-\frac{e^4\log(d+ex)\left(5ad^2-e(4bd-3ce)\right)}{d^4\left(ad^2-e(bd-ce)\right)^2}+\frac{e^4}{d^3(d+ex)\left(ad^2-e(bd-ce)\right)}+\frac{bd+2ce}{c^2d^3x}-\frac{1}{2cd^2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/2*1/(c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 - e*(b*d -
 c*e))*(d + e*x)) + ((b^5*e^2 - a^3*c*d*(3*b*d + 4*c*e) - a*b^3*e*(2*b*d +
5*c*e) + a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTanh[(b + 2*a*x)/Sqrt[
b^2 - 4*a*c]])/(c^3*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))^2) + ((b^2*d^
2 + 2*b*c*d*e - c*(a*d^2 - 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*(5*a*d^2 - e*
(4*b*d - 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 - e*(b*d - c*e))^2) + ((a^3*c*d^
2 - b^4*e^2 + a*b^2*e*(2*b*d + 3*c*e) - a^2*(b^2*d^2 + 4*b*c*d*e + c^2*e^2)
)*Log[c + b*x + a*x^2])/(2*c^3*(a*d^2 - e*(b*d - c*e))^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1583

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_)
 + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p, x] /; FreeQ[{a, b, c, d, e, m, n, q}, x] && EqQ[mn
, -n] && EqQ[mn2, 2*mn] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a+\frac{c}{x^2}+\frac{b}{x}\right)x^5(d+ex)^2}\,dx = \int \frac{1}{x^3(d+ex)^2\left(c+bx+ax^2\right)}\,dx$$

$$= \int \left(\frac{1}{cd^2x^3} + \frac{-bd-2ce}{c^2d^3x^2} + \frac{b^2d^2+2bcde-c(ad^2-3ce^2)}{c^3d^4x} + \frac{}{d^3\left(-ad^2+e(b\right.}\right.$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\right.}{c}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\right.}{c}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^2d^2+2bcde-c\right.}{c}$$

$$= -\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2-e(bd-ce)\right)(d+ex)} + \frac{\left(b^5e^2-a^3cd(3bd\right.}{}$$

Mathematica [A]
time = 0.30, size = 370, normalized size = 0.99

$$-\frac{1}{2cd^2x^2} + \frac{bd+2ce}{c^2d^3x} + \frac{e^4}{d^3\left(ad^2+e(-bd+ce)\right)(d+ex)} + \frac{\left(-b^5e^2+a^3cd(3bd+4ce)+ab^3e(2bd+5ce)-a^2b(b^2d^2+8bcde+5c^2e^2)\right)\tan^{-1}\left(\frac{b+2ax}{\sqrt{-b^2+4ac}}\right)}{c^3\sqrt{-b^2+4ac}\left(ad^2+e(-bd+ce)\right)^2} + \frac{\left(b^2d^2+2bcde+c(-ad^2+3ce^2)\right)\log(x)}{c^3d^4} - \frac{e^4\left(5ad^2+e(-4bd+3ce)\right)\log(d+ex)}{d^4\left(ad^2+e(-bd+ce)\right)^2} - \frac{\left(-a^3cd^2+b^4e^2-ab^2e(2bd+3ce)+a^2(b^2d^2+4bcde+c^2e^2)\right)\log(c+x(b+ax))}{2c^3\left(ad^2+e(-bd+ce)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + c/x^2 + b/x)*x^5*(d + e*x)^2),x]
```

```
[Out] -1/2*1/(c*d^2*x^2) + (b*d + 2*c*e)/(c^2*d^3*x) + e^4/(d^3*(a*d^2 + e*(-(b*d
) + c*e))*(d + e*x)) + ((-(b^5*e^2) + a^3*c*d*(3*b*d + 4*c*e) + a*b^3*e*(2*
b*d + 5*c*e) - a^2*b*(b^2*d^2 + 8*b*c*d*e + 5*c^2*e^2))*ArcTan[(b + 2*a*x)/
Sqrt[-b^2 + 4*a*c]])/(c^3*Sqrt[-b^2 + 4*a*c]*(a*d^2 + e*(-(b*d) + c*e))^2)
+ ((b^2*d^2 + 2*b*c*d*e + c*(-(a*d^2) + 3*c*e^2))*Log[x])/(c^3*d^4) - (e^4*
(5*a*d^2 + e*(-4*b*d + 3*c*e))*Log[d + e*x])/(d^4*(a*d^2 + e*(-(b*d) + c*e)
)^2) - ((-(a^3*c*d^2) + b^4*e^2 - a*b^2*e*(2*b*d + 3*c*e) + a^2*(b^2*d^2 +
4*b*c*d*e + c^2*e^2))*Log[c + x*(b + a*x)])/(2*c^3*(a*d^2 + e*(-(b*d) + c*e
))^2)
```

Maple [A]
time = 0.33, size = 455, normalized size = 1.22

method	result

default	$\frac{e^4}{d^3(a\,d^2-deb+c\,e^2)(ex+d)}-\frac{e^4(5a\,d^2-4deb+3c\,e^2)\ln(ex+d)}{d^4(a\,d^2-deb+c\,e^2)^2}+\frac{\frac{(a^4c\,d^2-a^3b^2d^2-4a^3bcde-a^3c^2e^2+2a^2b^3de+3a^2b^2c\,e^2-a\,b^4e^2)\ln(a}{2a}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^4/d^3/(a*d^2-b*d*e+c*e^2)/(e*x+d)-e^4*(5*a*d^2-4*b*d*e+3*c*e^2)/d^4/(a*d^
2-b*d*e+c*e^2)^2*ln(e*x+d)+1/(a*d^2-b*d*e+c*e^2)^2/c^3*(1/2*(a^4*c*d^2-a^3*
b^2*d^2-4*a^3*b*c*d*e-a^3*c^2*e^2+2*a^2*b^3*d*e+3*a^2*b^2*c*e^2-a*b^4*e^2)/
a*ln(a*x^2+b*x+c)+2*(2*a^3*b*c*d^2+2*a^3*c^2*d*e-a^2*b^3*d^2-6*a^2*b^2*c*d*
e-3*e^2*c^2*a^2*b+2*a*b^4*d*e+4*a*c*e^2*b^3-e^2*b^5-1/2*(a^4*c*d^2-a^3*b^2*
d^2-4*a^3*b*c*d*e-a^3*c^2*e^2+2*a^2*b^3*d*e+3*a^2*b^2*c*e^2-a*b^4*e^2)*b/a)
/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2)))-1/2/c/d^2/x^2-(-b*d
-2*c*e)/c^2/d^3/x+1/c^3/d^4*(-a*c*d^2+b^2*d^2+2*b*c*d*e+3*c^2*e^2)*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x**2+b/x)/x**5/(e*x+d)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.93, size = 587, normalized size = 1.58

$$\frac{\left(a^{2} b^{3} d^{2} e^{2} - 3 \, a^{3} b c d^{2} e^{2} - 2 \, a b^{4} d e^{3} + 8 \, a^{2} b^{2} c d e^{3} - 4 \, a^{3} c^{2} d e^{3} + b^{5} e^{4} - 5 \, a b^{3} c e^{4} + 5 \, a^{2} b c^{2} e^{4}\right) \arctan\left(-\frac{{\left(2 \, a d - \frac{2 \, a d^{2}}{x e + d} - b e + \frac{2 \, b d e}{x e + d} - \frac{2 \, c e^{2}}{x e + d}\right)} e^{\left(-1\right)}}{\sqrt{-b^{2} + 4 \, a c}}\right) e^{\left(-2\right)}}{{\left(a^{2} c^{3} d^{4} - 2 \, a b c^{3} d^{3} e + b^{2} c^{3} d^{2} e^{2} + 2 \, a c^{4} d^{2} e^{2} - 2 \, b c^{4} d e^{3} + c^{5} e^{4}\right)} \sqrt{-b^{2} + 4 \, a c}} - \frac{{\left(a^{2} b^{2} d^{2} - a^{3} c d^{2} - 2 \, a b^{3} d e + 4 \, a^{2} b c d e + b^{4} e^{2} - 3 \, a b^{2} c e^{2} + a^{2} c^{2} e^{2}\right)} \log\left(-a + \frac{2 \, a d}{x e + d} - \frac{a d^{2}}{{\left(x e + d\right)}^{2}} - \frac{b e}{x e + d} + \frac{b d e}{{\left(x e + d\right)}^{2}} - \frac{c e^{2}}{{\left(x e + d\right)}^{2}}\right)}{2 \, {\left(a^{2} c^{3} d^{4} - 2 \, a b c^{3} d^{3} e + b^{2} c^{3} d^{2} e^{2} + 2 \, a c^{4} d^{2} e^{2} - 2 \, b c^{4} d e^{3} + c^{5} e^{4}\right)}} + \frac{e^{9}}{{\left(a d^{5} e^{5} - b d^{4} e^{6} + c d^{3} e^{7}\right)} {\left(x e + d\right)}} + \frac{{\left(b^{2} d^{2} e - a c d^{2} e + 2 \, b c d e^{2} + 3 \, c^{2} e^{3}\right)} e^{\left(-1\right)} \log\left({\left| -\frac{d}{x e + d} + 1 \right|}\right)}{c^{3} d^{4}} + \frac{2 \, b c d e + 5 \, c^{2} e^{2} - \frac{2 \, {\left(b c d^{2} e^{2} + 3 \, c^{2} d e^{3}\right)} e^{\left(-1\right)}}{x e + d}}{2 \, c^{3} d^{4} {\left(\frac{d}{x e + d} - 1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+c/x^2+b/x)/x^5/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] (a^2*b^3*d^2*e^2 - 3*a^3*b*c*d^2*e^2 - 2*a*b^4*d*e^3 + 8*a^2*b^2*c*d*e^3 -
4*a^3*c^2*d*e^3 + b^5*e^4 - 5*a*b^3*c*e^4 + 5*a^2*b*c^2*e^4)*arctan(-(2*a*d
 - 2*a*d^2/(x*e + d) - b*e + 2*b*d*e/(x*e + d) - 2*c*e^2/(x*e + d))*e^(-1)/
sqrt(-b^2 + 4*a*c))*e^(-2)/((a^2*c^3*d^4 - 2*a*b*c^3*d^3*e + b^2*c^3*d^2*e^
2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4)*sqrt(-b^2 + 4*a*c)) - 1/2*(a
^2*b^2*d^2 - a^3*c*d^2 - 2*a*b^3*d*e + 4*a^2*b*c*d*e + b^4*e^2 - 3*a*b^2*c*
e^2 + a^2*c^2*e^2)*log(-a + 2*a*d/(x*e + d) - a*d^2/(x*e + d)^2 - b*e/(x*e
+ d) + b*d*e/(x*e + d)^2 - c*e^2/(x*e + d)^2)/(a^2*c^3*d^4 - 2*a*b*c^3*d^3*
e + b^2*c^3*d^2*e^2 + 2*a*c^4*d^2*e^2 - 2*b*c^4*d*e^3 + c^5*e^4) + e^9/((a*
d^5*e^5 - b*d^4*e^6 + c*d^3*e^7)*(x*e + d)) + (b^2*d^2*e - a*c*d^2*e + 2*b*
c*d*e^2 + 3*c^2*e^3)*e^(-1)*log(abs(-d/(x*e + d) + 1))/(c^3*d^4) + 1/2*(2*b
*c*d*e + 5*c^2*e^2 - 2*(b*c*d^2*e^2 + 3*c^2*d*e^3)*e^(-1)/(x*e + d))/(c^3*d
^4*(d/(x*e + d) - 1)^2)
```

Mupad [B]
time = 45.61, size = 2500, normalized size = 6.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(d + e*x)^2*(a + b/x + c/x^2)),x)
```

```
[Out] ((x*(2*b*d + 3*c*e))/(2*c^2*d^2) - 1/(2*c*d) + (x^2*(3*c^2*e^4 - b^2*d^2*e^
2 + a*b*d^3*e - b*c*d*e^3 + 2*a*c*d^2*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e
)))/(d*x^2 + e*x^3) - (log(d + e*x)*(3*c*e^6 + 5*a*d^2*e^4 - 4*b*d*e^5))/(a
^2*d^8 + b^2*d^6*e^2 + c^2*d^4*e^4 - 2*a*b*d^7*e + 2*a*c*d^6*e^2 - 2*b*c*d^
5*e^3) + (log((((27*a^2*b*c^6*e^11 - 9*a*b^3*c^5*e^11 - a*b^8*d^5*e^6 - a^6
*b^3*d^10*e - 36*a^3*c^6*d*e^10 + 2*a^2*b^7*d^6*e^5 - a^3*b^6*d^7*e^4 - a^4
*b^5*d^8*e^3 + 2*a^5*b^4*d^9*e^2 - 36*a^4*c^5*d^3*e^8 + 4*a^5*c^4*d^5*e^6 +
 3*a^6*c^3*d^7*e^4 + a^7*b*c*d^10*e - 39*a^2*b^3*c^4*d^2*e^9 - 15*a^2*b^4*c
^3*d^3*e^8 + 7*a^2*b^5*c^2*d^4*e^7 + 53*a^3*b^2*c^4*d^3*e^8 + 7*a^3*b^3*c^3
*d^4*e^7 - 33*a^3*b^4*c^2*d^5*e^6 + 20*a^4*b^2*c^3*d^5*e^6 + 33*a^4*b^3*c^2
```

*d^6*e^5 - 9*a^5*b^2*c^2*d^7*e^4 + 6*a*b^4*c^4*d*e^10 - 2*a*b^7*c*d^4*e^7 + 5*a*b^5*c^3*d^2*e^9 + a*b^6*c^2*d^3*e^8 + 12*a^2*b^6*c*d^5*e^6 + 51*a^3*b*c^5*d^2*e^9 - 16*a^3*b^5*c*d^6*e^5 - 27*a^4*b*c^4*d^4*e^7 + 6*a^4*b^4*c*d^7*e^4 - 19*a^5*b*c^3*d^6*e^5 + 3*a^5*b^3*c*d^8*e^3 - a^6*b*c^2*d^8*e^3 - 4*a^6*b^2*c*d^9*e^2)/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) + (((a*e*(12*a*c^5*e^7 - a^3*b^3*d^7 - 3*b^2*c^4*e^7 + b^6*d^4*e^3 - 3*a*b^5*d^5*e^2 + 3*a^2*b^4*d^6*e + 4*a^4*c^2*d^6*e + b^3*c^3*d*e^6 + b^5*c*d^3*e^4 + 8*a^2*c^4*d^2*e^5 - 8*a^3*c^3*d^4*e^3 + b^4*c^2*d^2*e^5 + 2*a^4*b*c*d^7 - 4*a*b*c^4*d*e^6 + 18*a^2*b^2*c^2*d^4*e^3 - 8*a*b^4*c*d^4*e^3 - 10*a^3*b^2*c*d^6*e - 6*a*b^2*c^3*d^2*e^5 - 7*a*b^3*c^2*d^3*e^4 + 12*a^2*b*c^3*d^3*e^4 + 15*a^2*b^3*c*d^5*e^2 - 16*a^3*b*c^2*d^5*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e)) + (a*e*(4*a^2*c^2*d^3*e + b^2*c^2*d*e^3 + b^3*c*d^2*e^2 + 2*a^2*b^2*d^4*x + 2*b^2*c^2*e^4*x + 2*b^4*d^2*e^2*x + a^2*b*c*d^4 - 4*a*c^3*d*e^3 - 6*a^3*c*d^4*x - 8*a*c^3*e^4*x - 2*a*b^2*c*d^3*e - 4*a*b^3*d^3*e*x - 2*b^3*c*d*e^3*x - 3*a*b*c^2*d^2*e^2 - 6*a^2*c^2*d^2*e^2*x + 8*a*b*c^2*d*e^3*x + 14*a^2*b*c*d^3*e*x - 6*a*b^2*c*d^2*e^2*x)*(b^6*e^2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (a*e*x*(2*a^4*b^2*d^7 - 3*a^5*c*d^7 + 6*b^3*c^3*e^7 - 2*b^6*d^3*e^4 + 4*a*b^5*d^4*e^3 - 4*a^3*b^3*d^6*e + 24*a^2*c^4*d*e^6 - 5*b^4*c^2*d*e^6 - b^5*c*d^2*e^5 + 32*a^3*c^3*d^3*e^4 - 7*a^4*c^2*d^5*e^2 - 24*a*b*c^4*e^7 + 9*a^4*b*c*d^6*e - 36*a^2*b^2*c^2*d^3*e^4 + 14*a*b^2*c^3*d*e^6 + 15*a*b^4*c*d^3*e^4 + 16*a*b^3*c^2*d^2*e^5 - 48*a^2*b*c^3*d^2*e^5 - 24*a^2*b^3*c*d^4*e^3 + 32*a^3*b*c^2*d^4*e^3 + 4*a^3*b^2*c*d^5*e^2))/(c^2*d^3*(a*d^2 + c*e^2 - b*d*e)))*(b^6*e^2 + b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2 - 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^3*b*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c - b^2)*(a*d^2 + c*e^2 - b*d*e)^2) - (x*(18*a^3*c^6*e^11 + 9*a*b^4*c^4*e^11 + a*b^8*d^4*e^7 + a^7*b^2*d^10*e - 36*a^2*b^2*c^5*e^11 - 2*a^2*b^7*d^5*e^6 + a^3*b^6*d^6*e^5 + a^5*b^4*d^8*e^3 - 2*a^6*b^3*d^9*e^2 + 6*a^4*c^5*d^2*e^9 - 10*a^5*c^4*d^4*e^7 - 12*a^6*c^3*d^6*e^5 + 3*a^7*c^2*d^8*e^3 + 44*a^2*b^4*c^3*d^2*e^9 - 2*a^2*b^5*c^2*d^3*e^8 - 85*a^3*b^2*c^4*d^2*e^9 - 46*a^3*b^3*c^3*d^3*e^8 + 45*a^3*b^4*c^2*d^4*e^7 - 42*a^4*b^2*c^3*d^4*e^7 - 56*a^4*b^3*c^2*d^5*e^6 + 19*a^5*b^2*c^2*d^6*e^5 - 6*a*b^5*c^3*d*e^10 + 2*a*b^7*c*d^3*e^8 + 42*a^3*b*c^5*d*e^10 + 2*a^7*b*c*d^9*e^2 - 5*a*b^6*c^2*d^2*e^9 + 6*a^2*b^3*c^4*d*e^10 - 12*a^2*b^6*c*d^4*e^7 + 16*a^3*b^5*c*d^5*e^6 + 88*a^4*b*c^4*d^3*e^8 - 6*a^4*b^4*c*d^6*e^5 + 62*a^5*b*c^3*d^5*e^6 - 2*a^6*b*c^2*d^7*e^4 - 2*a^6*b^2*c*d^8*e^3))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2))*(b^6*e^2 +

```
 b^5*e^2*(b^2 - 4*a*c)^(1/2) + a^2*b^4*d^2 + 4*a^4*c^2*d^2 - 4*a^3*c^3*e^2
- 5*a^3*b^2*c*d^2 + a^2*b^3*d^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^5*d*e + 13*a^2*
b^2*c^2*e^2 - 7*a*b^4*c*e^2 + 12*a^2*b^3*c*d*e - 16*a^3*b*c^2*d*e - 3*a^3*b
*c*d^2*(b^2 - 4*a*c)^(1/2) - 5*a*b^3*c*e^2*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*
d*e*(b^2 - 4*a*c)^(1/2) + 5*a^2*b*c^2*e^2*(b^2 - 4*a*c)^(1/2) - 2*a*b^4*d*e
*(b^2 - 4*a*c)^(1/2) + 8*a^2*b^2*c*d*e*(b^2 - 4*a*c)^(1/2)))/(2*c^3*(4*a*c
- b^2)*(a*d^2 + c*e^2 - b*d*e)^2) + (a^4*e^4*(a^2*b^2*d^5 - 9*b*c^3*e^5 - a
^3*c*d^5 + 4*b^4*d^3*e^2 + 6*b^2*c^2*d*e^4 + 5*b^3*c*d^2*e^3 + 3*a^2*c^2*d^
3*e^2 - 5*a*b^3*d^4*e + 7*a^2*b*c*d^4*e - 12*a*b*c^2*d^2*e^3 - 14*a*b^2*c*d
^3*e^2))/(c^4*d^6*(a*d^2 + c*e^2 - b*d*e)^2) - (a^5*e^5*x*(9*c^3*e^4 + 4*a*
b^2*d^4 + a^2*c*d^4 - 4*b^3*d^3*e + 12*a*c^2*d^2*e^2 - 5*b^2*c*d^2*e^2 - 6*
b*c^2*d*e^3 + 8*a*b*c*d^3*e))/(c^4*d^6*(a*d^2 +...
```

3.79 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^4 \sqrt{d+ex}\, dx$

Optimal. Leaf size=981

$$-\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2(3b^2d^2-29bcde+50c^2e^2))\sqrt{a+}}{3465a^4e^4}$$

```
[Out] 2/3465*(233*a^3*d^3+48*b^3*e^3+a*b*e^2*(67*b*d-157*c*e)+4*a^2*d*e*(18*b*d-3
7*c*e))*x*(e*x+d)^(3/2)*(a+c/x^2+b/x)^(1/2)/a^3/e^4-2/693*(29*a^2*d^2+8*b^2
*e^2+a*e*(19*b*d-18*c*e))*x*(e*x+d)^(5/2)*(a+c/x^2+b/x)^(1/2)/a^2/e^4+2/99*
(a*d+b*e)*x*(e*x+d)^(7/2)*(a+c/x^2+b/x)^(1/2)/a/e^4-2/3465*(187*a^4*d^4+64*
b^4*e^4+4*a*b^2*e^3*(7*b*d-69*c*e)-4*a^3*d^2*e*(2*b*d+3*c*e)+3*a^2*e^2*(3*b
^2*d^2-29*b*c*d*e+50*c^2*e^2))*x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/a^4/e^4+
2/11*x^5*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+1/3465*(128*a^5*d^5+128*b^5*e^5-
4*a^4*d^3*e*(14*b*d-27*c*e)-8*a*b^3*e^4*(7*b*d+87*c*e)-a^2*b*e^3*(37*b^2*d^
2-258*b*c*d*e-771*c^2*e^2)-a^3*d*e^2*(37*b^2*d^2-135*b*c*d*e+156*c^2*e^2))*
x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/
(-4*a*c+b^2))^(1/2)/a^5/e^5/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^
2)^(1/2))))^(1/2)-2/3465*(a*d^2-e*(b*d-c*e))*(128*a^4*d^4-64*b^4*e^4-4*a*b^
2*e^3*(7*b*d-69*c*e)+4*a^3*d^2*e*(2*b*d+3*c*e)-3*a^2*e^2*(3*b^2*d^2-29*b*c*
d*e+50*c^2*e^2))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/
2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+
c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a
^5/e^5/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

Rubi [A]
time = 3.92, antiderivative size = 981, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1587, 932, 1667, 857, 732, 435, 430}

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
```

```
[Out] (-2*(187*a^4*d^4 + 64*b^4*e^4 + 4*a*b^2*e^3*(7*b*d - 69*c*e) - 4*a^3*d^2*e*
(2*b*d + 3*c*e) + 3*a^2*e^2*(3*b^2*d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a +
 c/x^2 + b/x]*x*Sqrt[d + e*x])/(3465*a^4*e^4) + (2*Sqrt[a + c/x^2 + b/x]*x^
5*Sqrt[d + e*x])/11 + (2*(233*a^3*d^3 + 48*b^3*e^3 + a*b*e^2*(67*b*d - 157*
c*e) + 4*a^2*d*e*(18*b*d - 37*c*e))*Sqrt[a + c/x^2 + b/x]*x*(d + e*x)^(3/2)
)/(3465*a^3*e^4) - (2*(29*a^2*d^2 + 8*b^2*e^2 + a*e*(19*b*d - 18*c*e))*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(693*a^2*e^4) + (2*(a*d + b*e)*Sqrt[a
+ c/x^2 + b/x]*x*(d + e*x)^(7/2))/(99*a*e^4) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
128*a^5*d^5 + 128*b^5*e^5 - 4*a^4*d^3*e*(14*b*d - 27*c*e) - 8*a*b^3*e^4*(7*
b*d + 87*c*e) - a^2*b*e^3*(37*b^2*d^2 - 258*b*c*d*e - 771*c^2*e^2) - a^3*d*
e^2*(37*b^2*d^2 - 135*b*c*d*e + 156*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[
d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt
[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
- 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3465*a^5*e^5*Sqrt[(a*(d
+ e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) - (2*Sqrt[2
]*Sqrt[b^2 - 4*a*c]*(a*d^2 - e*(b*d - c*e))*(128*a^4*d^4 - 64*b^4*e^4 - 4*a
*b^2*e^3*(7*b*d - 69*c*e) + 4*a^3*d^2*e*(2*b*d + 3*c*e) - 3*a^2*e^2*(3*b^2*
d^2 - 29*b*c*d*e + 50*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/
(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*
a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a
*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
])/(3465*a^5*e^5*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^4\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int x^3\sqrt{d+ex}\,\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^5\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{x^3(-3cd-2(bd+ce)x-(}{\sqrt{d+ex}\,\sqrt{c+bx}}}{11\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^5\sqrt{d+ex} + \frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{7/2}}{99ae^4} -$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^5\sqrt{d+ex} - \frac{2(29a^2d^2+8b^2e^2+ae(19bd-18ce))\sqrt{}}{693a^2e^4}$$

$$= \frac{2}{11}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^5\sqrt{d+ex} + \frac{2(233a^3d^3+48b^3e^3+abe^2(67bd-157ce}{}$$

$$= -\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2}{3465a^4e^4}$$

$$= -\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2}{3465a^4e^4}$$

$$= -\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2}{3465a^4e^4}$$

$$= -\frac{2(187a^4d^4+64b^4e^4+4ab^2e^3(7bd-69ce)-4a^3d^2e(2bd+3ce)+3a^2e^2}{3465a^4e^4}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.37, size = 10904, normalized size = 11.12

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^4*Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 11937 vs. $2(899) = 1798$.
time = 0.28, size = 11938, normalized size = 12.17

method	result	size
risch	Expression too large to display	5004
default	Expression too large to display	11938

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.16, size = 894, normalized size = 0.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/10395*((128*a^6*d^6 - 120*a^5*b*d^5*e - 3*(11*a^4*b^2 - 68*a^5*c)*d^4*e^
2 - (20*a^3*b^3 - 87*a^4*b*c)*d^3*e^3 - 3*(11*a^2*b^4 - 53*a^3*b^2*c + 34*a
^4*c^2)*d^2*e^4 - 3*(40*a*b^5 - 246*a^2*b^3*c + 329*a^3*b*c^2)*d*e^5 + (128
*b^6 - 888*a*b^4*c + 1599*a^2*b^2*c^2 - 450*a^3*c^3)*e^6)*sqrt(a)*e^(1/2)*w
```

```
eierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/a^2,
-4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a
*b*c)*e^3)*e^(-3)/a^3, 1/3*(a*d + (3*a*x + b)*e)*e^(-1)/a) + 3*(128*a^6*d^5
*e - 56*a^5*b*d^4*e^2 - (37*a^4*b^2 - 108*a^5*c)*d^3*e^3 - (37*a^3*b^3 - 13
5*a^4*b*c)*d^2*e^4 - 2*(28*a^2*b^4 - 129*a^3*b^2*c + 78*a^4*c^2)*d*e^5 + (1
28*a*b^5 - 696*a^2*b^3*c + 771*a^3*b*c^2)*e^6)*sqrt(a)*e^(1/2)*weierstrassZ
eta(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/a^2, -4/27*(2*a^3*d^
3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-
3)/a^3, weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)*e^(
-2)/a^2, -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*
b^3 - 9*a*b*c)*e^3)*e^(-3)/a^3, 1/3*(a*d + (3*a*x + b)*e)*e^(-1)/a)) + 3*(6
4*a^6*d^4*x*e^2 - (315*a^6*x^5 + 35*a^5*b*x^4 - 10*(4*a^4*b^2 - 9*a^5*c)*x^
3 + (48*a^3*b^3 - 157*a^4*b*c)*x^2 - 2*(32*a^2*b^4 - 138*a^3*b^2*c + 75*a^4
*c^2)*x)*e^6 - (35*a^6*d*x^4 + 10*a^5*b*d*x^3 - (13*a^4*b^2 - 32*a^5*c)*d*x
^2 + 10*(2*a^3*b^3 - 7*a^4*b*c)*d*x)*e^5 + (40*a^6*d^2*x^3 + 13*a^5*b*d^2*x
^2 - 2*(9*a^4*b^2 - 23*a^5*c)*d^2*x)*e^4 - 4*(12*a^6*d^3*x^2 + 5*a^5*b*d^3*
x)*e^3)*sqrt(x*e + d)*sqrt((a*x^2 + b*x + c)/x^2))*e^(-6)/a^6
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)*x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
[Out] int(x^4*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.80 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^3 \sqrt{d+ex}\, dx$

Optimal. Leaf size=778

$$\frac{2(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{315a^3e^3}+\frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x^4\sqrt{d+ex}-\cdots$$

```
[Out] -4/315*(8*a^2*d^2+3*b^2*e^2+a*e*(4*b*d-7*c*e))*x*(e*x+d)^(3/2)*(a+c/x^2+b/x
)^(1/2)/a^2/e^3+2/63*(a*d+b*e)*x*(e*x+d)^(5/2)*(a+c/x^2+b/x)^(1/2)/a/e^3+2/
315*(19*a^3*d^3-6*a^2*c*d*e^2+8*b^3*e^3+3*a*b*e^2*(b*d-9*c*e))*x*(a+c/x^2+b
/x)^(1/2)*(e*x+d)^(1/2)/a^3/e^3+2/9*x^4*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)-2
/315*(8*a^4*d^4+8*b^4*e^4-a^3*d^2*e*(4*b*d-9*c*e)-4*a*b^2*e^3*(b*d+9*c*e)-3
*a^2*e^2*(b^2*d^2-5*b*c*d*e-7*c^2*e^2))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b
^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x
)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a^4/e^4/(a*x^2+
b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+2/315*(16*a^3*d^3
+6*a^2*c*d*e^2-8*b^3*e^3-3*a*b*e^2*(b*d-9*c*e))*(a*d^2-e*(b*d-c*e))*x*Ellip
ticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-
2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4
*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(
a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a^4/e^4/(a*x^2+b*x+c)/(e*
x+d)^(1/2)
```

Rubi [A]
time = 1.56, antiderivative size = 778, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1587, 932, 1667, 857, 732, 435, 430}

[illegible]

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

```
[Out] (2*(19*a^3*d^3 - 6*a^2*c*d*e^2 + 8*b^3*e^3 + 3*a*b*e^2*(b*d - 9*c*e))*Sqrt[
a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(315*a^3*e^3) + (2*Sqrt[a + c/x^2 + b/x]*
x^4*Sqrt[d + e*x])/9 - (4*(8*a^2*d^2 + 3*b^2*e^2 + a*e*(4*b*d - 7*c*e))*Sqr
t[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(315*a^2*e^3) + (2*(a*d + b*e)*Sqrt[a
```

```
 + c/x^2 + b/x]*x*(d + e*x)^(5/2))/(63*a*e^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c
]*(8*a^4*d^4 + 8*b^4*e^4 - a^3*d^2*e*(4*b*d - 9*c*e) - 4*a*b^2*e^3*(b*d + 9
*c*e) - 3*a^2*e^2*(b^2*d^2 - 5*b*c*d*e - 7*c^2*e^2))*Sqrt[a + c/x^2 + b/x]*
x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcS
in[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sq
rt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(315*a^4*e^4*Sqrt[
(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*
Sqrt[2]*Sqrt[b^2 - 4*a*c]*(16*a^3*d^3 + 6*a^2*c*d*e^2 - 8*b^3*e^3 - 3*a*b*e
^2*(b*d - 9*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(
d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt
[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
4*a*c])*e)])/(315*a^4*e^4*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

<u>Rule 430</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

<u>Rule 435</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

<u>Rule 732</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

<u>Rule 857</u>

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

<u>Rule 932</u>

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] &&  !LtQ[m,
 -1]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^3\sqrt{d+ex}\, dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\right)\int x^2\sqrt{d+ex}\,\sqrt{c+bx+ax^2}\, dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^4\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\right)\int \frac{x^2(-3cd-2(bd+ce)x-(a}{\sqrt{d+ex}\,\sqrt{c+bx}}}{9\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^4\sqrt{d+ex} + \frac{2(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x(d+ex)^{5/2}}{63ae^3} -$$

$$= \frac{2}{9}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^4\sqrt{d+ex} - \frac{4(8a^2d^2+3b^2e^2+ae(4bd-7ce))\sqrt{a+}}{315a^2e^3}$$

$$= \frac{2(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}}{315a^3e^3}$$

$$= \frac{2(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}}{315a^3e^3}$$

$$= \frac{2(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}}{315a^3e^3}$$

$$= \frac{2(19a^3d^3-6a^2cde^2+8b^3e^3+3abe^2(bd-9ce))\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}}{315a^3e^3}$$

Mathematica [C] Result contains complex when optimal does not.

time = 32.99, size = 7531, normalized size = 9.68

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^3*Sqrt[d + e*x],x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 9181 vs. 2(702) = 1404.
time = 0.19, size = 9182, normalized size = 11.80

method	result	size
risch	Expression too large to display	3661
default	Expression too large to display	9182

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 710, normalized size = 0.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/945*((16*a^5*d^5 - 16*a^4*b*d^4*e - 5*(a^3*b^2 - 6*a^4*c)*d^3*e^2 - (5*a^
2*b^3 - 21*a^3*b*c)*d^2*e^3 - 2*(8*a*b^4 - 42*a^2*b^2*c + 33*a^3*c^2)*d*e^4
 + (16*b^5 - 96*a*b^3*c + 123*a^2*b*c^2)*e^5)*sqrt(a)*e^(1/2)*weierstrassPI
nverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/a^2, -4/27*(2*a^3
*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e
```

```
^(-3)/a^3, 1/3*(a*d + (3*a*x + b)*e)*e^(-1)/a) + 6*(8*a^5*d^4*e - 4*a^4*b*d
^3*e^2 - 3*(a^3*b^2 - 3*a^4*c)*d^2*e^3 - (4*a^2*b^3 - 15*a^3*b*c)*d*e^4 + (
8*a*b^4 - 36*a^2*b^2*c + 21*a^3*c^2)*e^5)*sqrt(a)*e^(1/2)*weierstrassZeta(4
/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/a^2, -4/27*(2*a^3*d^3 - 3
*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/a^
3, weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/a
^2, -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 -
 9*a*b*c)*e^3)*e^(-3)/a^3, 1/3*(a*d + (3*a*x + b)*e)*e^(-1)/a)) + 3*(8*a^5*
d^3*x*e^2 + (35*a^5*x^4 + 5*a^4*b*x^3 - 2*(3*a^3*b^2 - 7*a^4*c)*x^2 + (8*a^
2*b^3 - 27*a^3*b*c)*x)*e^5 + (5*a^5*d*x^3 + 2*a^4*b*d*x^2 - (3*a^3*b^2 - 8*
a^4*c)*d*x)*e^4 - 3*(2*a^5*d^2*x^2 + a^4*b*d^2*x)*e^3)*sqrt(x*e + d)*sqrt((
a*x^2 + b*x + c)/x^2))*e^(-5)/a^5
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

```
[Out] int(x^3*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.81 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^2\sqrt{d+ex}\, dx$

Optimal. Leaf size=636

$$-\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}\,(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-4be)x)}{105a^2e^2}+\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+e}}{7a}$$

```
[Out] -2/105*x*(4*a^2*d^2+4*b^2*e^2-a*e*(2*b*d-5*c*e)-3*a*e*(a*d-4*b*e)*x)*(a+c/x
^2+b/x)^(1/2)*(e*x+d)^(1/2)/a^2/e^2+2/7*x*(a*x^2+b*x+c)*(a+c/x^2+b/x)^(1/2)
*(e*x+d)^(1/2)/a+1/105*(8*a^3*d^3+8*b^3*e^3-a^2*d*e*(5*b*d-16*c*e)-a*b*e^2*
(5*b*d+29*c*e))*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^
(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2
))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-
a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a^3/e^3/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/105*(8*a^2*d^2-4*b^2*e^2-a*e*(b*d-10*c*
e))*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a
*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b
^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x
^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)/a^3/e^3/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 636, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1587, 846, 828, 857, 732, 435, 430}

$$-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,(4a^2d^2-ae(2bd-5ce)-3aex(ad-4be)+4b^2e^2)}{105a^2e^2}-\frac{2\sqrt{2}\,x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,(8a^2d^2-ae(bd-10ce)-4b^2e^2)(ad^2-e(bd-ce))\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{105a^3e^3\sqrt{d+ex}\,(ax^2+bx+c)}+\frac{\sqrt{2}\,x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a(ax^2+bx+c)}{b^2-4ac}}\,(8a^3d^3-a^2de(5bd-16ce)-abe^2(5bd+29ce)+8b^3e^3)\,E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{105a^3e^3(ax^2+bx+c)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,(ax^2+bx+c)}{7a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

```
[Out] (-2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*(4*a^2*d^2 + 4*b^2*e^2 - a*e*(2*b
*d - 5*c*e) - 3*a*e*(a*d - 4*b*e)*x))/(105*a^2*e^2) + (2*Sqrt[a + c/x^2 + b
/x]*x*Sqrt[d + e*x]*(c + b*x + a*x^2))/(7*a) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
8*a^3*d^3 + 8*b^3*e^3 - a^2*d*e*(5*b*d - 16*c*e) - a*b*e^2*(5*b*d + 29*c*e)
)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 -
 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 -
```

```
4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])
*e)])/(105*a^3*e^3*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*a^2*d^2 - 4*b^2*e^2 -
a*e*(b*d - 10*c*e))*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a
*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2
))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(105*a^3*e^3*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
```

```
|| IntegersQ[2*m, 2*p])
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x^2\sqrt{d+ex}\, dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\right)\int x\sqrt{d+ex}\,\sqrt{c+bx+ax^2}\, dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}\,(c+bx+ax^2)}{7a} + \frac{\left(2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\right)\int \frac{(\frac{1}{2}(-\cdots}{\cdots}}{7a\sqrt{\cdots}}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}\,(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-\cdots}{105a^2e^2}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}\,(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-\cdots}{105a^2e^2}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}\,(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-\cdots}{105a^2e^2}$$

$$= -\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}\,(4a^2d^2+4b^2e^2-ae(2bd-5ce)-3ae(ad-\cdots}{105a^2e^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.36, size = 851, normalized size = 1.34

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x^2*Sqrt[d + e*x],x]
```

```
[Out] (x*Sqrt[a + (c + b*x)/x^2]*((-4*e^2*(-8*a^3*d^3 - 8*b^3*e^3 + a^2*d*e*(5*b*
d - 16*c*e) + a*b*e^2*(5*b*d + 29*c*e)))/Sqrt[d + e*x] + 4*a*e^2*Sqrt[d + e
*x]*(-4*b^2*e^2 + a*e*(2*b*d + 10*c*e + 3*b*e*x) + a^2*(-4*d^2 + 3*d*e*x +
15*e^2*x^2)) - (I*(d + e*x)*Sqrt[1 - (2*(a*d^2 + e*(-(b*d) + c*e)))/((2*a*d
 - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[2 + (4*(a*d^2 + e*(-(b*d
) + c*e)))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*((2*a*d -
b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(8*a^3*d^3 + 8*b^3*e^3 + a^2*d*e*(-5*b*d + 1
6*c*e) - a*b*e^2*(5*b*d + 29*c*e))*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2
 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]]
, -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a
*c)*e^2]))] + (8*b^3*e^3*(b*e - Sqrt[(b^2 - 4*a*c)*e^2]) + 4*a^3*d^2*(c*e^2
 - 2*d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*b*e^2*(-13*b^2*d*e - 37*b*c*e^2 + 5*b*d
*Sqrt[(b^2 - 4*a*c)*e^2] + 29*c*e*Sqrt[(b^2 - 4*a*c)*e^2]) + a^2*e*(-(b^2*d
^2*e) + 4*c*e*(5*c*e^2 - 4*d*Sqrt[(b^2 - 4*a*c)*e^2]) + b*d*(52*c*e^2 + 5*d
*Sqrt[(b^2 - 4*a*c)*e^2])))*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*
e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2
*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2
]))]))/(Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*
e^2])]*(c + x*(b + a*x)))))/(210*a^3*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6301 vs. $2(572) = 1144$.
time = 0.18, size = 6302, normalized size = 9.91

method	result	size
risch	Expression too large to display	2662
default	Expression too large to display	6302

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.14, size = 581, normalized size = 0.91

$$-\frac{2\left(\left(8a^4d^4-9a^3bd^3e-2\left(2a^2b^2-11a^3c\right)d^2e^2-\left(9ab^3-41a^2bc\right)de^3+\left(8b^4-41ab^2c+30a^2c^2\right)e^4\right)\sqrt{a}\,e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(\frac{4\left(a^2d^2-abde+\left(b^2-3ac\right)e^2\right)e^{(-2)}}{3a^2},-\frac{4\left(2a^3d^3-3a^2bd^2e-3\left(ab^2-6a^2c\right)de^2+\left(2b^3-9abc\right)e^3\right)e^{(-3)}}{27a^3},\frac{\left(ad+\left(3ax+b\right)e\right)e^{(-1)}}{3a}\right)+3\left(8a^4d^3e-5a^3bd^2e^2-\left(5a^2b^2-16a^3c\right)de^3+\left(8ab^3-29a^2bc\right)e^4\right)\sqrt{a}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(\frac{4\left(a^2d^2-abde+\left(b^2-3ac\right)e^2\right)e^{(-2)}}{3a^2},-\frac{4\left(2a^3d^3-3a^2bd^2e-3\left(ab^2-6a^2c\right)de^2+\left(2b^3-9abc\right)e^3\right)e^{(-3)}}{27a^3},\mathrm{weierstrassPInverse}\left(\frac{4\left(a^2d^2-abde+\left(b^2-3ac\right)e^2\right)e^{(-2)}}{3a^2},-\frac{4\left(2a^3d^3-3a^2bd^2e-3\left(ab^2-6a^2c\right)de^2+\left(2b^3-9abc\right)e^3\right)e^{(-3)}}{27a^3},\frac{\left(ad+\left(3ax+b\right)e\right)e^{(-1)}}{3a}\right)\right)+3\left(4a^4d^2xe^2-\left(15a^4x^3+3a^3bx^2-2\left(2a^2b^2-5a^3c\right)x\right)e^4-\left(3a^4dx^2+2a^3bdx\right)e^3\right)\sqrt{xe+d}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)e^{(-4)}}{315a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*((8*a^4*d^4 - 9*a^3*b*d^3*e - 2*(2*a^2*b^2 - 11*a^3*c)*d^2*e^2 - (9*
a*b^3 - 41*a^2*b*c)*d*e^3 + (8*b^4 - 41*a*b^2*c + 30*a^2*c^2)*e^4)*sqrt(a)*
e^(1/2)*weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)*e^(
-2)/a^2, -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*
b^3 - 9*a*b*c)*e^3)*e^(-3)/a^3, 1/3*(a*d + (3*a*x + b)*e)*e^(-1)/a) + 3*(8*
a^4*d^3*e - 5*a^3*b*d^2*e^2 - (5*a^2*b^2 - 16*a^3*c)*d*e^3 + (8*a*b^3 - 29*
a^2*b*c)*e^4)*sqrt(a)*e^(1/2)*weierstrassZeta(4/3*(a^2*d^2 - a*b*d*e + (b^2
 - 3*a*c)*e^2)*e^(-2)/a^2, -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*
a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/a^3, weierstrassPInverse(4/3*(
a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/a^2, -4/27*(2*a^3*d^3 - 3*a^2
*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/a^3, 1
/3*(a*d + (3*a*x + b)*e)*e^(-1)/a)) + 3*(4*a^4*d^2*x*e^2 - (15*a^4*x^3 + 3*
a^3*b*x^2 - 2*(2*a^2*b^2 - 5*a^3*c)*x)*e^4 - (3*a^4*d*x^2 + 2*a^3*b*d*x)*e^
3)*sqrt(x*e + d)*sqrt((a*x^2 + b*x + c)/x^2))*e^(-4)/a^4
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

```
[Out] int(x^2*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.82 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}\, dx$

Optimal. Leaf size=550

$$-\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x\sqrt{d+ex}}{15ae}+\frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\, x(d+ex)^{3/2}}{5e}-\frac{2\sqrt{2}\sqrt{b^2-4ac}\,(a^2d^2+b^2e^2-ae}{}$$

```
[Out] 2/5*x*(e*x+d)^(3/2)*(a+c/x^2+b/x)^(1/2)/e-2/15*(2*a*d-b*e)*x*(a+c/x^2+b/x)^
(1/2)*(e*x+d)^(1/2)/a/e-2/15*(a^2*d^2+b^2*e^2-a*e*(b*d+3*c*e))*x*EllipticE(
1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(
-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+
b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2)
)^(1/2)/a^2/e^2/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^
(1/2)+2/15*(2*a*d-b*e)*(a*d^2-e*(b*d-c*e))*x*EllipticF(1/2*((b+2*a*x+(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2
*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+
b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4
*a*c+b^2)^(1/2))))^(1/2)/a^2/e^2/(a*x^2+b*x+c)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.47, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {1587, 748, 846, 857, 732, 435, 430}

$$\frac{2\sqrt{2}\,x\sqrt{b^2-4ac}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\,(2ad-be)\left(ad^2-e(bd-ce)\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{15a^2e^2\sqrt{d+ex}\left(ax^2+bx+c\right)}-\frac{2\sqrt{2}\,x\sqrt{b^2-4ac}\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\sqrt{-\frac{a\left(ax^2+bx+c\right)}{b^2-4ac}}\left(a^2d^2-ae(bd+3ce)+b^2e^2\right)E\left(\text{ArcSin}\left(\frac{\sqrt{\frac{b+2ax+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)|-\frac{2\sqrt{b^2-4ac}\,e}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{15a^2e^2\left(ax^2+bx+c\right)\sqrt{\frac{a(d+ex)}{2ad-e\left(\sqrt{b^2-4ac}+b\right)}}}+\frac{2x(d+ex)^{3/2}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{5e}-\frac{2x\sqrt{d+ex}\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,(2ad-be)}{15ae}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

```
[Out] (-2*(2*a*d - b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/(15*a*e) + (2*Sqrt
[a + c/x^2 + b/x]*x*(d + e*x)^(3/2))/(5*e) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(
a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x
]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c
]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[(a*(d + e*x))/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^
2 - 4*a*c]*(2*a*d - b*e)*(a*d^2 - e*(b*d - c*e))*Sqrt[a + c/x^2 + b/x]*x*Sq
```

```
rt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x +
a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*
x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt
[b^2 - 4*a*c])*e)])/(15*a^2*e^2*Sqrt[d + e*x]*(c + b*x + a*x^2))
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
```

```
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \sqrt{d+ex}\,\sqrt{c+bx+ax^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{\sqrt{d+ex}\,(bd-2ce+(2a}{\sqrt{c+bx+ax}}}{5e\sqrt{c+bx+ax^2}}$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} -$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} +$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} -$$

$$= -\frac{2(2ad-be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}}{15ae} + \frac{2\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x(d+ex)^{3/2}}{5e} -$$

<u>Mathematica</u> [C] Result contains complex when optimal does not.
time = 28.29, size = 1046, normalized size = 1.90

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x],x]
```

```
[Out] (x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]*((2*b)/a + (2*d)/e + 6*x - (4*e^2*
Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(
```

```
a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*(c + x*(b + a*x)) - I*Sqrt[2]*(2*a*d
 - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(a^2*d^2 + b^2*e^2 - a*e*(b*d + 3*c*e))*(
d + e*x)^(3/2)*Sqrt[(-2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*a*d*e*x + e*S
qrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c
)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*a*d*e*x +
 e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 -
 4*a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e
+ c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a
*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])
)] + I*Sqrt[2]*(b^2*e^2*(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2]) + a^2*d*(-8*c*e^
2 + d*Sqrt[(b^2 - 4*a*c)*e^2]) + a*e*(2*b^2*d*e + 4*b*c*e^2 - b*d*Sqrt[(b^2
 - 4*a*c)*e^2] - 3*c*e*Sqrt[(b^2 - 4*a*c)*e^2]))*(d + e*x)^(3/2)*Sqrt[(-2*c
*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x
+ b*e*(d - e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[
(2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2
]*x + b*e*(-d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]
*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e +
Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/(a^2*e^3*Sqrt[(a*d^2
+ e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(d + e*x)*(c
+ x*(b + a*x)))))/15
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4360 vs. $2(486) = 972$.
time = 0.23, size = 4361, normalized size = 7.93

method	result	size
risch	Expression too large to display	1711
default	Expression too large to display	4361

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(-10*a^2*b*d*e^3*x^2-3*2^(1
/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^
2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2
)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x
+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d
+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*d^2*e
^2+2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(
-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-
4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/
2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1
/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*
a*c*d*e^3+12*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((
```

-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e^3-8*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d*e^3-6*a^2*c*e^4*x^2-2*a*b^2*e^4*x^2-8*a^3*d*e^3*x^3-8*a^2*b*e^4*x^3-2*a^3*d^2*e^2*x^2-2*a^2*b*d^2*e^2*x-8*a^2*c*d*e^3*x-2*a*b^2*d*e^3*x-2*a*b*c*e^4*x-6*a^3*e^4*x^4+3*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2*d^2*e^2+8*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*b*d^3*e+2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b^2*d*e^3+2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a^2*d^3*e-3*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^3*d*e^3+3*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*e^4+4*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*

```
a*c+b^2)^(1/2)))^(1/2))*b^3*d*e^3-4*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/
2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+
b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*
a*d+e*b))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e
*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/
2)))^(1/2))*b^2*c*e^4-2*a^2*c*d^2*e^2-2*a*b*c*d*e^3-12*2^(1/2)*(-(e*x+d)*a/
(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2
*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)*x, x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.11, size = 475, normalized size = 0.86

$$\frac{2\left((2a^3d^3-3a^2bd^2e-3(ab^2-6a^2c)de^2+(2b^3-9abc)e^3)\sqrt{a}\,e^{\frac{1}{2}}\mathrm{weierstrassPInverse}\left(\frac{4(a^2d^2-abde+(b^2-3ac)e^2)e^{(-2)}}{3a^2},-\frac{4(2a^3d^3-3a^2bd^2e-3(ab^2-6a^2c)de^2+(2b^3-9abc)e^3)e^{(-3)}}{27a^3},\frac{(ad+(3ax+b)e)e^{(-1)}}{3a}\right)+6(a^3d^2e-a^2bde^2+(ab^2-3a^2c)e^3)\sqrt{a}\,e^{\frac{1}{2}}\mathrm{weierstrassZeta}\left(\frac{4(a^2d^2-abde+(b^2-3ac)e^2)e^{(-2)}}{3a^2},-\frac{4(2a^3d^3-3a^2bd^2e-3(ab^2-6a^2c)de^2+(2b^3-9abc)e^3)e^{(-3)}}{27a^3},\mathrm{weierstrassPInverse}\left(\frac{4(a^2d^2-abde+(b^2-3ac)e^2)e^{(-2)}}{3a^2},-\frac{4(2a^3d^3-3a^2bd^2e-3(ab^2-6a^2c)de^2+(2b^3-9abc)e^3)e^{(-3)}}{27a^3},\frac{(ad+(3ax+b)e)e^{(-1)}}{3a}\right)\right)+3(a^3dxe^2+(3a^3x^2+a^2bx)e^3)\sqrt{xe+d}\sqrt{\frac{ax^2+bx+c}{x^2}}\right)e^{(-3)}}{45a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/45*((2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 - 9*a
*b*c)*e^3)*sqrt(a)*e^(1/2)*weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^
2 - 3*a*c)*e^2)*e^(-2)/a^2, -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6
*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/a^3, 1/3*(a*d + (3*a*x + b)*e
)*e^(-1)/a) + 6*(a^3*d^2*e - a^2*b*d*e^2 + (a*b^2 - 3*a^2*c)*e^3)*sqrt(a)*e
^(1/2)*weierstrassZeta(4/3*(a^2*d^2 - a*b*d*e + (b^2 - 3*a*c)*e^2)*e^(-2)/a
^2, -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6*a^2*c)*d*e^2 + (2*b^3 -
 9*a*b*c)*e^3)*e^(-3)/a^3, weierstrassPInverse(4/3*(a^2*d^2 - a*b*d*e + (b^
2 - 3*a*c)*e^2)*e^(-2)/a^2, -4/27*(2*a^3*d^3 - 3*a^2*b*d^2*e - 3*(a*b^2 - 6
*a^2*c)*d*e^2 + (2*b^3 - 9*a*b*c)*e^3)*e^(-3)/a^3, 1/3*(a*d + (3*a*x + b)*e
)*e^(-1)/a)) + 3*(a^3*d*x*e^2 + (3*a^3*x^2 + a^2*b*x)*e^3)*sqrt(x*e + d)*sq
rt((a*x^2 + b*x + c)/x^2))*e^(-3)/a^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{d+ex}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

```
[Out] int(x*(d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.83 $\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}\,dx$

Optimal. Leaf size=955

$$\frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}+\frac{\sqrt{2}\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\sqrt{-\frac{a\,(c+bx+ax^2)}{b^2-4ac}}\,E\Bigg(\mathrm{s}\ldots}{3ae\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e\ldots}}\ldots}$$

[Out] 2/3*x*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+1/3*(a*d+b*e)*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/a/e/(a*x^2+b*x+c)/(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-2/3*d*(a*d+b*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a/e/(a*x^2+b*x+c)/(e*x+d)^(1/2)+4/3*(b*d+c*e)*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a/(a*x^2+b*x+c)/(e*x+d)^(1/2)-c*x*EllipticPi(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(2*a*d-b*e+e*(-4*a*c+b^2)^(1/2))/a/d,((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*2^(1/2)*(a+c/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)/a^(1/2)

Rubi [A]
time = 2.13, antiderivative size = 955, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {1463, 932, 6874, 732, 430, 948, 175, 552, 551, 857, 435}

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]

[Out] (2*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x])/3 + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d + b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2)

```
)/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqr
t[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 -
 4*a*c])*e)])/(3*a*e*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)
]*(c + b*x + a*x^2)) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*(a*d + b*e)*Sqrt[a +
c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt
[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^
2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(
2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(3*a*e*Sqrt[d + e*x]*(c + b*x + a*x^2)
) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(
a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^
2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/S
qrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2
 - 4*a*c])*e)])/(3*a*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[2]*c*Sqrt[2*a
*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d +
e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d
- (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e
)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^
2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)])/(Sqrt[a]*(c + b*x + a*x^2))
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
```

```
f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 932

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2*(d + e*x)^(m + 1)*Sqrt[f + g*x]*(
Sqrt[a + b*x + c*x^2]/(e*(2*m + 5))), x] - Dist[1/(e*(2*m + 5)), Int[((d +
e*x)^m/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*d*f - 3*a*e*f + a*d*g
+ 2*(c*d*f - b*e*f + b*d*g - a*e*g)*x - (c*e*f - 3*c*d*g + b*e*g)*x^2, x],
x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[e*f - d*g, 0] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[2*m] && !LtQ[m,
 -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
```

```
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1463

```
Int[((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_
)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^n + c/x^(2*
n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[((d + e*x^n)^q*(c
 + b*x^n + a*x^(2*n))^p)/x^(2*n*p), x], x] /; FreeQ[{a, b, c, d, e, n, p, q
}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] &&
 PosQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{\sqrt{d+ex}\,\sqrt{c+bx+ax^2}}{x}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{-3cd-2(bd+ce)x-(ad+\cdots}{x\sqrt{d+ex}\,\sqrt{c+bx-\cdots}}}{3\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} - \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \left(-\frac{2(bd+ce)}{\sqrt{d+ex}\,\sqrt{c+b\cdots}}\right.}{\cdots}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{\left(cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{1}{x\sqrt{d+ex}\,\sqrt{c+b\cdots}}}{\sqrt{c+bx+ax^2}}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{\left(cd\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{b-\sqrt{b^2-4ac}}+2\cdots\right.}{\cdots}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{4\sqrt{2}\,\sqrt{b^2-4ac}\,(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\cdots}{\cdots}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{\cdots}}{\cdots}$$

$$= \frac{2}{3}\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex} + \frac{\sqrt{2}\,\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 27.55, size = 1258, normalized size = 1.32

$$\frac{2}{3}x\sqrt{d+ex}\sqrt{a+\frac{c+bx}{x^2}}+\frac{x(d+ex)^{3/2}\sqrt{a+\frac{c+bx}{x^2}}\left(\frac{4e^2(ad+be)\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(c+x(b+ax))}{(d+ex)^2}-\frac{i\sqrt{2}(ad+be)\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)\sqrt{\frac{-2ce^2+2adex+be(d-ex)+\sqrt{(b^2-4ac)e^2}(d+ex)}{\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{\frac{2ce^2-2adex+be(-d+ex)+\sqrt{(b^2-4ac)e^2}(d+ex)}{\left(-2ad+be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}E\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{ad^2-bde+ce^2}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}}{\sqrt{d+ex}}\right)\middle|-\frac{-2ad+be+\sqrt{(b^2-4ac)e^2}}{2ad-be+\sqrt{(b^2-4ac)e^2}}\right)}{\sqrt{d+ex}}+\frac{i\sqrt{2}\left(be\left(-be+\sqrt{(b^2-4ac)e^2}\right)+a\left(3bde-2ce^2+d\sqrt{(b^2-4ac)e^2}\right)\right)\sqrt{\frac{-2ce^2+2adex+be(d-ex)+\sqrt{(b^2-4ac)e^2}(d+ex)}{\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{\frac{2ce^2-2adex+be(-d+ex)+\sqrt{(b^2-4ac)e^2}(d+ex)}{\left(-2ad+be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{ad^2-bde+ce^2}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}}{\sqrt{d+ex}}\right)\middle|-\frac{-2ad+be+\sqrt{(b^2-4ac)e^2}}{2ad-be+\sqrt{(b^2-4ac)e^2}}\right)}{\sqrt{d+ex}}+6i\sqrt{2}ace^2\sqrt{\frac{-2ce^2+2adex+be(d-ex)+\sqrt{(b^2-4ac)e^2}(d+ex)}{\left(2ad-be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\sqrt{\frac{2ce^2-2adex+be(-d+ex)+\sqrt{(b^2-4ac)e^2}(d+ex)}{\left(-2ad+be+\sqrt{(b^2-4ac)e^2}\right)(d+ex)}}\Pi\left(\frac{d\left(2ad-be-\sqrt{(b^2-4ac)e^2}\right)}{2(ad^2+e(-bd+ce))};i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{ad^2-bde+ce^2}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}}{\sqrt{d+ex}}\right)\middle|-\frac{-2ad+be+\sqrt{(b^2-4ac)e^2}}{2ad-be+\sqrt{(b^2-4ac)e^2}}\right)\right)}{6ae^2\sqrt{\frac{ad^2+e(-bd+ce)}{-2ad+be+\sqrt{(b^2-4ac)e^2}}}(c+x(b+ax))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x],x]
```

```
[Out] (2*x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2])/3 + (x*(d + e*x)^(3/2)*Sqrt[a +
 (c + b*x)/x^2]*((4*e^2*(a*d + b*e)*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d
 + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/(d + e*x)^2 - (I*Sqrt
[2]*(a*d + b*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*c*e^2 + 2*
a*d*e*x + b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e
+ Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d
+ e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*
a*c)*e^2])*(d + e*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c
*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d
+ b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])
/Sqrt[d + e*x] + (I*Sqrt[2]*(b*e*(-(b*e) + Sqrt[(b^2 - 4*a*c)*e^2]) + a*(3*
b*d*e - 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(-2*c*e^2 + 2*a*d*e*x +
b*e*(d - e*x) + Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^
2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + S
qrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])
*(d + e*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*
a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d +
e*x] + ((6*I)*Sqrt[2]*a*c*e^2*Sqrt[(-2*c*e^2 + 2*a*d*e*x + b*e*(d - e*x) +
Sqrt[(b^2 - 4*a*c)*e^2]*(d + e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])
*(d + e*x))]*Sqrt[(2*c*e^2 - 2*a*d*e*x + b*e*(-d + e*x) + Sqrt[(b^2 - 4*a*c
)*e^2]*(d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Ell
ipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*(a*d^2 + e*(-(b*d) +
 c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*
c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(6*a*e^
2*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
*(c + x*(b + a*x)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3022 vs. $2(836) = 1672$.
time = 0.23, size = 3023, normalized size = 3.17

method	result	size
default	Expression too large to display	3023

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/3*((a*x^2+b*x+c)/x^2)^(1/2)*x*(e*x+d)^(1/2)*(2^(1/2)*(-(e*x+d)*a/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b
+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^
2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1
/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a
*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*d^2*e-2^(1/2)*(-(e*x+d)*a/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-
e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-
4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2-2*2^(1/2)*(-(e*x+d)*a/
(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2
*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b
+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^3+3*2^(1/2)*(-(e*x+d)
*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e
/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e
*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*
a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-
e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*d^2*e+6*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*
c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+
e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2
)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/
2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*
c+b^2)^(1/2)))^(1/2))*a*c*d*e^2-3*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)
-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^
2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*
d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b
))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)
))^(1/2))*b^2*d*e^2-2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))
^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(
1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2
)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(
e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a^
2*d^3-2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*
x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x
+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^
(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)
^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e^2+2*2^(1
/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^
2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2
)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^(1/2)*(-(e*x
```

```
+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d
+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*d*e^2-2*2^(1/2)*(-(e*x+d
)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*
e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(
e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4
*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d
-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e^3+3*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e
*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+e*b))^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/
2)-2*a*d+e*b))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a*c
+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2
)^(1/2)*c*e^3-6*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)
*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*(
(b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*Elli
pticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),-1/2*(e*
(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d
-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e^2+3*2^(1/2)*(-(e*x+d)*a/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b
+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^
2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(
1/2)-2*a*d+e*b))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a
*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b*c*e^3+2
*a^2*e^3*x^3+2*a^2*d*e^2*x^2+2*a*b*e^3*x^2+2*a*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2),x)
```

```
[Out] Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2),x)
```

```
[Out] int((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2), x)
```

3.84 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{x}\,dx$

Optimal. Leaf size=929

$$-\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}+\frac{3\sqrt{b^2-4ac}\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\sqrt{-\frac{a\,(c+bx+ax^2)}{b^2-4ac}}\,E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+\sqrt{}}{}}}{}\right.\right.}{\sqrt{2}\,\sqrt{\frac{a(d+ex)}{2ad-\left(b+\sqrt{b^2-4ac}\right)e}}\,(c+bx}$$

```
[Out] -(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)+3/2*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d
-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(
e*x+d)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/(a*x^2+b*x+c)*2^(1/2)/(a
*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)-3*d*x*EllipticF(1/2*((b+2*
a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2
)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1/2)*(-4*a*c+b^2)^(1/2)
*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*
d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)/(e*x+d)^(1/2)+2*(a*d+b*e)*
x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(
1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*2^(1
/2)*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^
(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/a/(a*x^2+b*x+c)/(e
*x+d)^(1/2)-1/2*(b*d+c*e)*x*EllipticPi(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*d
-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(2*a*d-b*e+e*(-4*a*c+b^2)^(1/2))/a/d,(
(b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*(a+c/
x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(
1/2))))^(1/2)/d/(a*x^2+b*x+c)*2^(1/2)/a^(1/2)
```

Rubi [A]

time = 1.87, antiderivative size = 929, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$, Rules used = {1587, 930, 6874, 732, 430, 948, 175, 552, 551, 857, 435}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] -(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x]) + (3*Sqrt[b^2 - 4*a*c]*Sqrt[a + c/x^
2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*Ellip
ticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]
], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[2]*
Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x + a*x^2))
- (3*Sqrt[2]*Sqrt[b^2 - 4*a*c]*d*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))
/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4
*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*
a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e
)])/(Sqrt[d + e*x]*(c + b*x + a*x^2)) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(a*d +
 b*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4
*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^
2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*Sqrt[d + e*x]*(c + b
*x + a*x^2)) - ((b*d + c*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[a
+ c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 - 4*a*c])*
e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*EllipticP
i[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqrt[a]*Sqrt
[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2 - 4*a*c]
 - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[a]*d*(c +
 b*x + a*x^2))
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
```

```
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f + b
*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
```

```
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{x}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{\sqrt{d+ex}\,\sqrt{c+bx+ax^2}}{x^2}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{bd+ce+2(ad+be)x+3aex^2 \cdots}{x\sqrt{d+ex}\,\sqrt{c+bx+a \cdots}}}{2\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \left(\frac{2(ad+be)}{\sqrt{d+ex}\,\sqrt{c+bx+ \cdots}} \cdots\right.}{\cdots}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex} + \frac{\left(3ae\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{x}{\sqrt{d+ex}\,\sqrt{c+bx \cdots}}}{2\sqrt{c+bx+ax^2}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex} + \frac{\left((bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{b-\sqrt{b^2-4ac}} \cdots\right.}{\cdots}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex} + \frac{2\sqrt{2}\,\sqrt{b^2-4ac}\,(ad+be)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{\cdots}}{\cdots}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex} + \frac{3\sqrt{b^2-4ac}\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\sqrt{-\frac{a(\cdots}{\cdots}}}{\sqrt{2}\,\sqrt{\cdots}}$$

$$= -\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex} + \frac{3\sqrt{b^2-4ac}\,\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+ex}\,\sqrt{-\frac{a(\cdots}{\cdots}}}{\sqrt{2}\,\sqrt{\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 29.59, size = 1372, normalized size = 1.48

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x,x]
```

```
[Out] -(Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]) + (x*(d + e*x)^(3/2)*Sqrt[a + (c +
 b*x)/x^2]*(12*d*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b^2
- 4*a*c)*e^2])]*(a*(-1 + d/(d + e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(
d + e*x)))/(d + e*x)) - ((3*I)*Sqrt[2]*d*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*
e^2])*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(
d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(
d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e
^2])]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b
*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] +
(I*Sqrt[2]*(4*a*d^2 - b*d*e - 2*c*e^2 + 3*d*Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(
Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*d*(-1 + d/(d + e*x)) +
b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Sqrt[(
Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*d*(-1 + d/(d + e*x)) +
b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*Ellipt
icF[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^
2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + ((2*I)*Sqrt[
2]*e*(b*d + c*e)*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] - (2*c*e^2)/(d + e*x) - 2*a*
d*(-1 + d/(d + e*x)) + b*e*(-1 + (2*d)/(d + e*x)))/(2*a*d - b*e + Sqrt[(b^2
 - 4*a*c)*e^2])]*Sqrt[(Sqrt[(b^2 - 4*a*c)*e^2] + (2*c*e^2)/(d + e*x) + 2*a*
d*(-1 + d/(d + e*x)) + b*(e - (2*d*e)/(d + e*x)))/(-2*a*d + b*e + Sqrt[(b^2
 - 4*a*c)*e^2])]*EllipticPi[(d*(2*a*d - b*e - Sqrt[(b^2 - 4*a*c)*e^2]))/(2*
(a*d^2 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2
)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*
e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqr
t[d + e*x]))/(4*d*e*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-2*a*d + b*e + Sqrt[(b
^2 - 4*a*c)*e^2])]*Sqrt[c + b*x + a*x^2]*Sqrt[((d + e*x)^2*(a*(-1 + d/(d +
e*x))^2 + (e*(b - (b*d)/(d + e*x) + (c*e)/(d + e*x)))/(d + e*x)))/e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3552 vs. $2(818) = 1636$.
time = 0.23, size = 3553, normalized size = 3.82

method	result

risch	$-\sqrt{\frac{a\,x^2+bx+c}{x^2}}\,\sqrt{ex+d}+\frac{\left(3ae\left(\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2a}\right)\sqrt{\frac{x+\frac{d}{e}}{\frac{d}{e}-\frac{b+\sqrt{-4ac+b^2}}{2a}}}\,\sqrt{\frac{x-\frac{-b+\sqrt{-4ac+b^2}}{2a}}{-\frac{d}{e}-\frac{-b+\sqrt{-4ac+b^2}}{2a}}}\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)*(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e
*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c
+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*d^2*e*x-2*2^(1/2)*(-(e*x+d)*a/(e*
(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*
d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^
2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*
(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e^2*x+4*2^(1/2)*(-(e*x+d
)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*
e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(
e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4
*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d
-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*d^3*x-2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e
*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c
+b^2)^(1/2)))^(1/2))*a*b*d^2*e*x+6*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b
^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*
```

```
b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2
)))^(1/2))*a*c*d*e^2*x-2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*
b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2))
)^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(
1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),
(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))
*b^2*d*e^2*x-6*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*
((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((
b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*Ellip
ticE(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a
*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*d^3*x
+6*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4
*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*
a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^(1/2)
*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2
)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b*d^2*e*x-6*2^(1/2)
*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^
(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(
1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)
*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*
b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c*d*e^2*x+2^(1/2)*(-(e*x+d)*a
/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(
2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(
-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a
*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/a/d,(
-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*
(-4*a*c+b^2)^(1/2)*b*d*e^2*x+2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*
d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1
/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b
))^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(
1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*
d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*c*e^3*x-
2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*
a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a
*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticPi(2^(1/2)
*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),-1/2*(e*(-4*a*c+b^2)^(
1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c
+b^2)^(1/2)))^(1/2))*a*b*d^2*e*x-2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2
)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b
^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a
*d+e*b))^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e
*b))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a*c+b^2)^(1/2
)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)/x, x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x,x)
[Out] Integral(sqrt(d + e*x)*sqrt(a + b/x + c/x**2)/x, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)/x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x,x)
[Out] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x, x)
```

3.85 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{x^2}\,dx$

Optimal. Leaf size=1287

$$-\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd}-\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x}+\frac{\sqrt{b^2-4ac}\,(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\sqrt{d+}}{4}$$

```
[Out] -1/4*(b*d+c*e)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/c/d-1/2*(a+c/x^2+b/x)^(1/2
)*(e*x+d)^(1/2)/x+1/8*(b*d+c*e)*x*EllipticE(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2
))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(
-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(e*x+d)^
(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)/c/d/(a*x^2+b*x+c)*2^(1/2)/(a*(e
*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)+3/2*e*x*EllipticF(1/2*((b+2*a
*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2),(-2*e*(-4*a*c+b^2)
^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4*a*c+b^2)^(1/2)*(a+c/x^2
+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(a*(e*x+d)/(2*a*d-e*(b+(-
4*a*c+b^2)^(1/2))))^(1/2)/(a*x^2+b*x+c)*2^(1/2)/(e*x+d)^(1/2)-1/4*(b*d+c*e)
*x*EllipticF(1/2*((b+2*a*x+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2))^(1/2)*2^
(1/2),(-2*e*(-4*a*c+b^2)^(1/2)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2))*(-4
*a*c+b^2)^(1/2)*(a+c/x^2+b/x)^(1/2)*(-a*(a*x^2+b*x+c)/(-4*a*c+b^2))^(1/2)*(
a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/(a*x^2+b*x+c)*2^(1/2)/(
e*x+d)^(1/2)-1/2*(a*d+b*e)*x*EllipticPi(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(2*a*d-b*e+e*(-4*a*c+b^2)^(1/2))/a/d,
((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*(a+c
/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^
(1/2))))^(1/2)/d/(a*x^2+b*x+c)*2^(1/2)/a^(1/2)+1/8*(b*d+c*e)^2*x*EllipticPi
(2^(1/2)*a^(1/2)*(e*x+d)^(1/2)/(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2),1/2*(
2*a*d-b*e+e*(-4*a*c+b^2)^(1/2))/a/d,((b-2*a*d/e-(-4*a*c+b^2)^(1/2))/(b-2*a*
d/e+(-4*a*c+b^2)^(1/2)))^(1/2))*(a+c/x^2+b/x)^(1/2)*(1-2*a*(e*x+d)/(2*a*d-e
*(b-(-4*a*c+b^2)^(1/2))))^(1/2)*(2*a*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1-2
*a*(e*x+d)/(2*a*d-e*(b+(-4*a*c+b^2)^(1/2))))^(1/2)/c/d^2/(a*x^2+b*x+c)*2^(1
/2)/a^(1/2)
```

Rubi [A]
time = 3.57, antiderivative size = 1287, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$,

Rules used = {1587, 930, 6874, 732, 430, 953, 948, 175, 552, 551, 857, 435}

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] -1/4*((b*d + c*e)*Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/(c*d) - (Sqrt[a + c/
x^2 + b/x]*Sqrt[d + e*x])/(2*x) + (Sqrt[b^2 - 4*a*c]*(b*d + c*e)*Sqrt[a + c
/x^2 + b/x]*x*Sqrt[d + e*x]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*El
lipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt
[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(4*Sqr
t[2]*c*d*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*(c + b*x +
 a*x^2)) + (3*Sqrt[b^2 - 4*a*c]*e*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x)
)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 -
4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4
*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*
e)])/(Sqrt[2]*Sqrt[d + e*x]*(c + b*x + a*x^2)) - (Sqrt[b^2 - 4*a*c]*(b*d +
c*e)*Sqrt[a + c/x^2 + b/x]*x*Sqrt[(a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*
a*c])*e)]*Sqrt[-((a*(c + b*x + a*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqr
t[(b + Sqrt[b^2 - 4*a*c] + 2*a*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2
 - 4*a*c]*e)/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*Sqrt[2]*c*Sqrt[d + e*
x]*(c + b*x + a*x^2)) - ((a*d + b*e)*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e
]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b^2 -
 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*e)]*
EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]*Sqr
t[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt[b^2
 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(Sqrt[2]*Sqrt[
a]*d*(c + b*x + a*x^2)) + ((b*d + c*e)^2*Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c
])*e]*Sqrt[a + c/x^2 + b/x]*x*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b - Sqrt[b
^2 - 4*a*c])*e)]*Sqrt[1 - (2*a*(d + e*x))/(2*a*d - (b + Sqrt[b^2 - 4*a*c])*
e)]*EllipticPi[(2*a*d - b*e + Sqrt[b^2 - 4*a*c]*e)/(2*a*d), ArcSin[(Sqrt[2]
*Sqrt[a]*Sqrt[d + e*x])/Sqrt[2*a*d - (b - Sqrt[b^2 - 4*a*c])*e]], (b - Sqrt
[b^2 - 4*a*c] - (2*a*d)/e)/(b + Sqrt[b^2 - 4*a*c] - (2*a*d)/e)])/(4*Sqrt[2]
*Sqrt[a]*c*d^2*(c + b*x + a*x^2))
```

Rule 175

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] &&  !SimplerQ[e + f*x, c + d*x] &&  !SimplerQ[g + h*x, c + d
*x]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Sy
mbol] :> Dist[2*Rt[b^2 - 4*a*c, 2]*(d + e*x)^m*(Sqrt[(-c)*((a + b*x + c*x^2
)/(b^2 - 4*a*c))]/(c*Sqrt[a + b*x + c*x^2]*(2*c*((d + e*x)/(2*c*d - b*e - e
*Rt[b^2 - 4*a*c, 2])))^m)), Subst[Int[(1 + 2*e*Rt[b^2 - 4*a*c, 2]*(x^2/(2*c
*d - b*e - e*Rt[b^2 - 4*a*c, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(b + Rt[b^2
 - 4*a*c, 2] + 2*c*x)/(2*Rt[b^2 - 4*a*c, 2])]], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d -
 b*e, 0] && EqQ[m^2, 1/4]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 930

```
Int[((d_.) + (e_.)*(x_))^(m_.)*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[(d + e*x)^(m + 1)*Sqrt[f + g*x]*(Sq
rt[a + b*x + c*x^2]/(e*(m + 1))), x] - Dist[1/(2*e*(m + 1)), Int[((d + e*x)
^(m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[b*f + a*g + 2*(c*f + b
*g)*x + 3*c*g*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f
 - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Integ
erQ[2*m] && LtQ[m, -1]
```

Rule 948

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_
) + (c_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[Sqrt[b
 - q + 2*c*x]*(Sqrt[b + q + 2*c*x]/Sqrt[a + b*x + c*x^2]), Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[b - q + 2*c*x]*Sqrt[b + q + 2*c*x]), x], x]] /; FreeQ[{
a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ
[c*d^2 - b*d*e + a*e^2, 0]
```

Rule 953

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)*
(x_) + (c_.)*(x_)^2]), x_Symbol] :> Simp[e^2*(d + e*x)^(m + 1)*Sqrt[f + g*x
]*(Sqrt[a + b*x + c*x^2]/((m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/(2*(m + 1)*(e*f - d*g)*(c*d^2 - b*d*e + a*e^2)), Int[((d + e*x)^(
m + 1)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]))*Simp[2*d*(c*e*f - c*d*g + b*e
*g)*(m + 1) - e^2*(b*f + a*g)*(2*m + 3) + 2*e*(c*d*g*(m + 1) - e*(c*f + b*g
)*(m + 2))*x - c*e^2*g*(2*m + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
 a*e^2, 0] && IntegerQ[2*m] && LeQ[m, -2]
```

Rule 1587

```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(mn_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_)
+ (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + b/x^
n + c/x^(2*n))^FracPart[p]/(c + b*x^n + a*x^(2*n))^FracPart[p]), Int[x^(m -
 2*n*p)*(d + e*x^n)^q*(c + b*x^n + a*x^(2*n))^p, x], x] /; FreeQ[{a, b, c,
d, e, m, n, p, q}, x] && EqQ[mn, -n] && EqQ[mn2, 2*mn] &&  !IntegerQ[p] &&
 !IntegerQ[q] && PosQ[n]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{x^2}\,dx = \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{\sqrt{d+ex}\,\sqrt{c+bx+ax^2}}{x^3}\,dx}{\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{bd+ce+2(ad+be)x+3a}{x^2\sqrt{d+ex}\,\sqrt{c+bx}}}{4\sqrt{c+bx+ax^2}}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\left(\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \left(\frac{3ae}{\sqrt{d+ex}\,\sqrt{c+bx}}\right.}{}$$

$$= -\frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\left(3ae\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,x\right)\int \frac{1}{\sqrt{d+ex}\,\sqrt{c+b}}}{4\sqrt{c+bx+ax^2}}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{\left((ad\right.}{}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2}}{}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2}}{}$$

$$= -\frac{(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{4cd} - \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}}{2x} + \frac{3\sqrt{b^2}}{}$$

$$3\sqrt{b^2}$$

$$(bd+ce)\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex} \qquad \sqrt{a+\frac{c}{x^2}+\frac{b}{x}}\,\sqrt{d+ex}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.66, size = 1392, normalized size = 1.08

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + c/x^2 + b/x]*Sqrt[d + e*x])/x^2,x]
```

```
[Out] (x*Sqrt[d + e*x]*Sqrt[a + (c + b*x)/x^2]*((-4*(2*c*d + b*d*x + c*e*x))/(c*d
*x^2) + ((d + e*x)*((4*d*e^2*(b*d + c*e)*Sqrt[(a*d^2 + e*(-(b*d) + c*e))/(-
2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))/(d + e*x)^2 - (I
*Sqrt[2]*d*(b*d + c*e)*(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*Sqrt[(-2*c*e
^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] + 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x +
b*e*(d - e*x))/((2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2
*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*
x + b*e*(-d + e*x))/((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*E
llipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sq
rt[(b^2 - 4*a*c)*e^2])])/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*
c)*e^2])/(2*a*d - b*e + Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] + (I*Sqrt
[2]*(b^2*d^2*e + b*d*(-5*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]) + c*e*(4*a*d^2
+ 2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]))*Sqrt[(-2*c*e^2 + d*Sqrt[(b^2 - 4*a*
c)*e^2] + 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*a*d
- b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 + d*Sqrt[(b^2 -
4*a*c)*e^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-
2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticF[I*ArcSinh[(Sqr
t[2]*Sqrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]
)/Sqrt[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e +
 Sqrt[(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x] - ((2*I)*Sqrt[2]*e*(b^2*d^2 - 2*
b*c*d*e + c*(-4*a*d^2 + c*e^2))*Sqrt[(-2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e^2]
+ 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(d - e*x))/((2*a*d - b*e +
Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*Sqrt[(2*c*e^2 + d*Sqrt[(b^2 - 4*a*c)*e
^2] - 2*a*d*e*x + e*Sqrt[(b^2 - 4*a*c)*e^2]*x + b*e*(-d + e*x))/((-2*a*d +
b*e + Sqrt[(b^2 - 4*a*c)*e^2])*(d + e*x))]*EllipticPi[(d*(2*a*d - b*e - Sqr
t[(b^2 - 4*a*c)*e^2]))/(2*(a*d^2 + e*(-(b*d) + c*e))), I*ArcSinh[(Sqrt[2]*S
qrt[(a*d^2 - b*d*e + c*e^2)/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])])/Sqrt
[d + e*x]], -((-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])/(2*a*d - b*e + Sqrt[
(b^2 - 4*a*c)*e^2]))])/Sqrt[d + e*x]))/(c*d^2*e*Sqrt[(a*d^2 + e*(-(b*d) + c
*e))/(-2*a*d + b*e + Sqrt[(b^2 - 4*a*c)*e^2])]*(c + x*(b + a*x)))))/16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4956 vs. $2(1127) = 2254$.
time = 0.24, size = 4957, normalized size = 3.85

method	result	size

risch	Expression too large to display	1597
default	Expression too large to display	4957

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x,method=_RETURNVERBOSE)

[Out] 1/8*((a*x^2+b*x+c)/x^2)^(1/2)*(e*x+d)^(1/2)*(-4*a*c^2*d^3*e+2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*b*d^3*e*x^2+2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2*d^3*e*x^2-2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticE(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*c*d^3*e*x^2+12*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a^2*c*d^3*e*x^2-2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*b^2*d^3*e*x^2+2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d*e^3*x^2-2*a*b*c*d*e^3*x^3-8*a*b*c*d^2*e^2*x^2-2*a^2*b*d^2*e^2*x^4-2*a^2*c*d*e^3*x^4-2*a^2*b*d^3*e*x^3-6*a^2*c*d^2*e^2*x^3-2*a*b^2*d^2*e^2*x^3-4*a^2*c*d^3*e*x^2-2*a*b^2*d^3*e*x^2-2*a*c^2*d*e^3*x^2-6*a*c^2*d^2*e^2*x-6*a*b*c*d^3*e*x-5*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+

```
e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1
/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*a*c*d^2*e^2*x^2-5*2^(1/2)*(-(e*x+d)*a/(e*(-
4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-
e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c
+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticF(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)
^(1/2)-2*a*d+e*b))^(1/2),(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-
4*a*c+b^2)^(1/2)))^(1/2))*a*b*c*d^2*e^2*x^2+4*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*
c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+
e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2
)^(1/2)-2*a*d+e*b))^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1
/2)-2*a*d+e*b))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a*
c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^
2)^(1/2)*a*c*d^2*e^2*x^2+2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+
e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2
)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))
^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/
2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+
e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*c*d*e^3*
x^2+2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+
(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(
-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2)*EllipticPi(2^(
1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b))^(1/2),-1/2*(e*(-4*a*c+b^
2)^(1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4
*a*c+b^2)^(1/2)))^(1/2))*a*c^2*d*e^3*x^2+2*2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b
^2)^(1/2)-2*a*d+e*b))^(1/2)*((-2*a*x+(-4*a*c+b^2)^(1/2)-b)*e/(2*a*d-e*b+e*(
-4*a*c+b^2)^(1/2)))^(1/2)*((b+2*a*x+(-4*a*c+b^2)^(1/2))*e/(e*(-4*a*c+b^2)^(
1/2)-2*a*d+e*b))^(1/2)*EllipticPi(2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)
-2*a*d+e*b))^(1/2),-1/2*(e*(-4*a*c+b^2)^(1/2)-2*a*d+e*b)/a/d,(-(e*(-4*a*c+b
^2)^(1/2)-2*a*d+e*b)/(2*a*d-e*b+e*(-4*a*c+b^2)^(1/2)))^(1/2))*b^2*c*d*e^3*x
^2-2^(1/2)*(-(e*x+d)*a/(e*(-4*a*c+b^2)^(1/2)-2*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x**2+b/x)**(1/2)*(e*x+d)**(1/2)/x**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2+b/x)^(1/2)*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x*e + d)*sqrt(a + b/x + c/x^2)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{d+e\,x}\,\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2,x)
```

```
[Out] int(((d + e*x)^(1/2)*(a + b/x + c/x^2)^(1/2))/x^2, x)
```

3.86 $\int (fx)^m (d+ex^n)^q (a+cx^{2n})^p \, dx$

Optimal. Leaf size=29

$$\text{Int}\left((fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p, x\right)$$

```
[Out] Unintegrable((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
[Out] Defer[Int][(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Rubi steps

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx = \int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Mathematica [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int (fx)^m (d+ex^n)^q \left(a+cx^{2n}\right)^p \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p,x]
[Out] Integrate[(f*x)^m*(d + e*x^n)^q*(a + c*x^(2*n))^p, x]
```

Maple [A]
time = 0.18, size = 0, normalized size = 0.00

$$\int (fx)^m (d+e\,x^n)^q \left(a+c\,x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m*(x^n*e + d)^q, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m*(x^n*e + d)^q, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**q*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q*(a+c*x^(2*n))^p,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m*(x^n*e + d)^q, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int (a+c\,x^{2\,n})^p\,(f\,x)^m\,(d+e\,x^n)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q,x)
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^q, x)
```

3.87 $\int (fx)^m (d+ex^n)^3 (a+cx^{2n})^p \, dx$

Optimal. Leaf size=358

$$\frac{d^3(fx)^{1+m}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)}+\frac{3d^2ex^{1+n}(fx)^m(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)}{1+m}$$

```
[Out] d^3*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n]
,-c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+3*d^2*e*x^(1+n)*(f*x)^m*(a+c*x^(
2*n))^p*hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+
n)/((1+c*x^(2*n)/a)^p)+3*d*e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom(
[-p, 1/2*(1+m+2*n)/n],[1/2*(1+m+4*n)/n],-c*x^(2*n)/a)/(1+m+2*n)/((1+c*x^(2*
n)/a)^p)+e^3*x^(1+3*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m+3*n)
/n],[1/2*(1+m+5*n)/n],-c*x^(2*n)/a)/(1+m+3*n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A]
time = 0.16, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1575, 372, 371, 20}

$$\frac{d^3(fx)^{m+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{3d^2ex^{n+1}(fx)^m(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{3de^2x^{2n+1}(fx)^m(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+2n+1}{2n},-p;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+2n+1}+\frac{e^3x^{3n+1}(fx)^m(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+3n+1}{2n},-p;\frac{m+5n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+3n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (d^3*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (3
*d^2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2
*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*
n))/a)^p) + (3*d*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F
1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m
+ 2*n)*(1 + (c*x^(2*n))/a)^p) + (e^3*x^(1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*
Hypergeometric2F1[(1 + m + 3*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n
))/a)])/((1 + m + 3*n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1575

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int (fx)^m (d+ex^n)^3 \left(a+cx^{2n}\right)^p \, dx &= \int \left(d^3(fx)^m \left(a+cx^{2n}\right)^p + 3d^2ex^n(fx)^m \left(a+cx^{2n}\right)^p + 3de^2x^{2n}(fx)^m\right.\\ &= d^3 \int (fx)^m \left(a+cx^{2n}\right)^p \, dx + \left(3d^2e\right) \int x^n(fx)^m \left(a+cx^{2n}\right)^p \, dx + (3d\\ &= \left(3d^2ex^{-m}(fx)^m\right) \int x^{m+n}\left(a+cx^{2n}\right)^p \, dx + \left(3de^2x^{-m}(fx)^m\right) \int x^{m+2n}\\ &= \frac{d^3(fx)^{1+m}\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} -\\ &= \frac{d^3(fx)^{1+m}\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} -\end{aligned}$$

Mathematica [A]

time = 0.39, size = 249, normalized size = 0.70

$$x(fx)^m \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(\frac{d^3 \, {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{1+m} + ex^n \left(\frac{3d^2 \, {}_2F_1\left(\frac{1+m+n}{2n},-p;\frac{1+m+3n}{2n};-\frac{cx^{2n}}{a}\right)}{1+m+n} + ex^n \left(\frac{3d \, {}_2F_1\left(\frac{1+m+2n}{2n},-p;\frac{1+m+4n}{2n};-\frac{cx^{2n}}{a}\right)}{1+m+2n} + \frac{ex^n \, {}_2F_1\left(\frac{1+m+3n}{2n},-p;\frac{1+m+5n}{2n};-\frac{cx^{2n}}{a}\right)}{1+m+3n}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^3*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^3*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((3*d^2*Hypergeometric2F
```

```
1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + n
) + e*x^n*((3*d*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2
*n), -((c*x^(2*n))/a)])/(1 + m + 2*n) + (e*x^n*Hypergeometric2F1[(1 + m + 3
*n)/(2*n), -p, (1 + m + 5*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + 3*n)))))/(1
 + (c*x^(2*n))/a)^p
```

Maple [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{m} \left(d+e\,x^{n}\right)^{3} \left(a+c\,x^{2 n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((x^n*e + d)^3*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x^(3*n)*e^3)*(c*x^(2*n) + a
)^p*(f*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**3*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^3*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARx near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+c\,x^{2\,n})^p\,(f\,x)^m\,(d+e\,x^n)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3,x)
```

```
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^3, x)
```

3.88 $\int (fx)^m (d+ex^n)^2 (a+cx^{2n})^p \, dx$

Optimal. Leaf size=262

$$\frac{d^2(fx)^{1+m}(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)}+\frac{2dex^{1+n}(fx)^m(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-}}{1+m+}$$

```
[Out] d^2*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n]
,-c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+2*d*e*x^(1+n)*(f*x)^m*(a+c*x^(2*
n))^p*hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+n)
/((1+c*x^(2*n)/a)^p)+e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*n))^p*hypergeom([-p, 1
/2*(1+m+2*n)/n],[1/2*(1+m+4*n)/n],-c*x^(2*n)/a)/(1+m+2*n)/((1+c*x^(2*n)/a)^
p)
```

Rubi [A]
time = 0.10, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1575, 372, 371, 20}

$$\frac{d^2(fx)^{m+1}(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{2dex^{n+1}(fx)^m(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}+\frac{e^2x^{2n+1}(fx)^m(a+cx^{2n})^p\left(\frac{cx^{2n}}{a}+1\right)^{-p} {}_2F_1\left(\frac{m+2n+1}{2n},-p;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1
 + (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (2
*d*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n
), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n)
)/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 +
 m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + 2*n)
*(1 + (c*x^(2*n))/a)^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1575

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\int (fx)^m (d+ex^n)^2 \left(a+cx^{2n}\right)^p \, dx = \int \left(d^2(fx)^m \left(a+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+cx^{2n}\right)^p + e^2x^{2n}(fx)^m \left(a\right.\right.$$

$$= d^2 \int (fx)^m \left(a+cx^{2n}\right)^p \, dx + (2de) \int x^n(fx)^m \left(a+cx^{2n}\right)^p \, dx + e^2 \int$$

$$= \left(2dex^{-m}(fx)^m\right) \int x^{m+n}\left(a+cx^{2n}\right)^p \, dx + \left(e^2x^{-m}(fx)^m\right) \int x^{m+2n}(a$$

$$= \frac{d^2(fx)^{1+m} \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n}, -p; 1+\frac{1+m}{2n}; -\frac{cx^{2n}}{a}\right)}{f(1+m)}$$

$$= \frac{d^2(fx)^{1+m} \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n}, -p; 1+\frac{1+m}{2n}; -\frac{cx^{2n}}{a}\right)}{f(1+m)}$$

Mathematica [A]
time = 0.21, size = 189, normalized size = 0.72

$$x(fx)^m \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(\frac{d^2\,{}_2F_1\left(\frac{1+m}{2n}, -p; 1+\frac{1+m}{2n}; -\frac{cx^{2n}}{a}\right)}{1+m} + ex^n \left(\frac{2d\,{}_2F_1\left(\frac{1+m+n}{2n}, -p; \frac{1+m+3n}{2n}; -\frac{cx^{2n}}{a}\right)}{1+m+n} + \frac{ex^n\,{}_2F_1\left(\frac{1+m+2n}{2n}, -p; \frac{1+m+4n}{2n}; -\frac{cx^{2n}}{a}\right)}{1+m+2n}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*((d^2*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(1 + m) + e*x^n*((2*d*Hypergeometric2F1[
(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/(1 + m + n)
+ (e*x^n*Hypergeometric2F1[(1 + m + 2*n)/(2*n), -p, (1 + m + 4*n)/(2*n), -(
(c*x^(2*n))/a)])/(1 + m + 2*n))))/(1 + (c*x^(2*n))/a)^p
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^m (d+e\,x^n)^2 \left(a+c\,x^{2n}\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
[Out] int((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="maxima")
[Out] integrate((x^n*e + d)^2*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="fricas")
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**2*(a+c*x**(2*n))**p,x)
[Out] Timed out
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARx near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+c\,x^{2\,n})^p\,(f\,x)^m\,(d+e\,x^n)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2,x)
```

```
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n)^2, x)
```

3.89 $\int (fx)^m (d+ex^n) (a+cx^{2n})^p \, dx$

Optimal. Leaf size=166

$$\frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)}+\frac{ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}{}_2F \ldots}{1+m+\ldots}$$

```
[Out] d*(f*x)^(1+m)*(a+c*x^(2*n))^p*hypergeom([-p, 1/2*(1+m)/n],[1+1/2*(1+m)/n],-
c*x^(2*n)/a)/f/(1+m)/((1+c*x^(2*n)/a)^p)+e*x^(1+n)*(f*x)^m*(a+c*x^(2*n))^p*
hypergeom([-p, 1/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],-c*x^(2*n)/a)/(1+m+n)/((1+c
*x^(2*n)/a)^p)
```

Rubi [A]
time = 0.06, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1575, 372, 371, 20}

$$\frac{d(fx)^{m+1}\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+1}{2n},-p;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a}\right)}{f(m+1)}+\frac{ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}{}_2F_1\left(\frac{m+n+1}{2n},-p;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a}\right)}{m+n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (d*(f*x)^(1 + m)*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m)/(2*n), -p, 1 +
 (1 + m)/(2*n), -((c*x^(2*n))/a)])/(f*(1 + m)*(1 + (c*x^(2*n))/a)^p) + (e*x
^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*Hypergeometric2F1[(1 + m + n)/(2*n), -p,
 (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)])/((1 + m + n)*(1 + (c*x^(2*n))/a)^p
)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
```

```
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1575

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^
(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^n)^q*(a + c*
x^(2*n))^p, x], x] /; FreeQ[{a, c, d, e, f, m, n, p, q}, x] && EqQ[n2, 2*n]
 && (IGtQ[p, 0] || IGtQ[q, 0])
```

Rubi steps

$$\begin{aligned}\int (fx)^m (d+ex^n) \left(a+cx^{2n}\right)^p \, dx &= \int \left(d(fx)^m \left(a+cx^{2n}\right)^p + ex^n(fx)^m \left(a+cx^{2n}\right)^p\right) \, dx \\ &= d\int (fx)^m \left(a+cx^{2n}\right)^p \, dx + e\int x^n(fx)^m \left(a+cx^{2n}\right)^p \, dx \\ &= \left(ex^{-m}(fx)^m\right)\int x^{m+n}\left(a+cx^{2n}\right)^p \, dx + \left(d\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p}\right. \\ &= \frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} + \\ &= \frac{d(fx)^{1+m}\left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)}{f(1+m)} +\end{aligned}$$

Mathematica [A]
time = 0.11, size = 136, normalized size = 0.82

$$\frac{x(fx)^m \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} \left(d(1+m+n)\, {}_2F_1\left(\frac{1+m}{2n},-p;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a}\right)+e(1+m)x^n\, {}_2F_1\left(\frac{1+m+n}{2n},-p;\frac{1+m+3n}{2n};-\frac{cx^{2n}}{a}\right)\right)}{(1+m)(1+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)*(a + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*(d*(1 + m + n)*Hypergeometric2F1[(1 + m)/(2*n)
, -p, 1 + (1 + m)/(2*n), -((c*x^(2*n))/a)] + e*(1 + m)*x^n*Hypergeometric2F
1[(1 + m + n)/(2*n), -p, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a)]))/((1 + m)*
(1 + m + n)*(1 + (c*x^(2*n))/a)^p)
```

Maple [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+e\,x^n\right)\left(a+c\,x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)*(c*x^(2*n) + a)^p*(f*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+c\,x^{2\,n}\right)^p\,{\left(f\,x\right)}^m\,\left(d+e\,x^n\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n),x)
[Out] int((a + c*x^(2*n))^p*(f*x)^m*(d + e*x^n), x)
```

3.90 $\int \frac{(fx)^m (a+cx^{2n})^p}{d+ex^n} \, dx$

Optimal. Leaf size=194

$$\frac{x(fx)^m \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1+m}{2n};-p,1;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(1+m)}-\frac{ex^{1+n}(fx)^m \left(a+cx^{2n}\right)^p \left(1+\frac{cx^{2n}}{a}\right)}{d^2(1}$$

```
[Out] x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,1,-p,1+1/2*(1+m)/n,e^2*x^(2*
n)/d^2,-c*x^(2*n)/a)/d/(1+m)/((1+c*x^(2*n)/a)^p)-e*x^(1+n)*(f*x)^m*(a+c*x^(
2*n))^p*AppellF1(1/2*(1+m+n)/n,1,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*x^(2
*n)/a)/d^2/(1+m+n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A]
time = 0.14, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1576, 525, 524}

$$\frac{x(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} F_1\left(\frac{m+1}{2n};-p,1;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(m+1)}-\frac{ex^{n+1}(fx)^m \left(a+cx^{2n}\right)^p \left(\frac{cx^{2n}}{a}+1\right)^{-p} F_1\left(\frac{m+n+1}{2n};-p,1;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 1, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d*(1 + m)*(1 + (c*x^(2*n))/a)^p)
 - (e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -p, 1
, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m +
n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1576

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/(d^2 - e^2*x^(2*n))))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{d+ex^n}\,dx = \left(x^{-m}(fx)^m\right)\int\left(\frac{dx^m(a+cx^{2n})^p}{d^2-e^2x^{2n}}+\frac{ex^{m+n}(a+cx^{2n})^p}{-d^2+e^2x^{2n}}\right)dx$$

$$= \left(dx^{-m}(fx)^m\right)\int\frac{x^m(a+cx^{2n})^p}{d^2-e^2x^{2n}}\,dx + \left(ex^{-m}(fx)^m\right)\int\frac{x^{m+n}(a+cx^{2n})^p}{-d^2+e^2x^{2n}}\,dx$$

$$= \left(dx^{-m}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int\frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{d^2-e^2x^{2n}}\,dx + \left(ex^{-m}(fx)^m\right.$$

$$= \frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,1;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d(1+m)} - \frac{ex^{1+}}{}$$

Mathematica [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{d+ex^n}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n),x]
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n), x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+c\,x^{2n}\right)^p}{d+e\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(x^n*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(x^n*e + d), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(x^n*e + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + c\,x^{2\,n})^p\,(f\,x)^m}{d + e\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n),x)
[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n), x)
```

3.91 $\int \frac{(fx)^m(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=302

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,2;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m)}-\frac{2ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right.}{d^3}$$

```
[Out] x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,2,-p,1+1/2*(1+m)/n,e^2*x^(2*
n)/d^2,-c*x^(2*n)/a)/d^2/(1+m)/((1+c*x^(2*n)/a)^p)-2*e*x^(1+n)*(f*x)^m*(a+c
*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n,2,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*
x^(2*n)/a)/d^3/(1+m+n)/((1+c*x^(2*n)/a)^p)+e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(2*
n))^p*AppellF1(1/2*(1+m+2*n)/n,2,-p,1/2*(1+m+4*n)/n,e^2*x^(2*n)/d^2,-c*x^(2
*n)/a)/d^4/(1+m+2*n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A]
time = 0.21, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1576, 525, 524}

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,2;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(m+1)}+\frac{e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+2n+1}{2n};-p,2;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(m+2n+1)}-\frac{2ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};-p,2;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 2, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^2*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (2*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 2, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^p*
AppellF1[(1 + m + 2*n)/(2*n), -p, 2, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a),
 (e^2*x^(2*n))/d^2])/(d^4*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1576

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/(d^2 - e^2*x^(2*n))))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rubi steps

$$\int \frac{(fx)^m\,(a+cx^{2n})^p}{(d+ex^n)^2}\,dx = \left(x^{-m}(fx)^m\right)\int \left(\frac{d^2x^m(a+cx^{2n})^p}{(d^2-e^2x^{2n})^2} - \frac{2dex^{m+n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2} + \frac{e^2x^{m+2n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\right.$$

$$= \left(d^2x^{-m}(fx)^m\right)\int \frac{x^m(a+cx^{2n})^p}{(d^2-e^2x^{2n})^2}\,dx - \left(2dex^{-m}(fx)^m\right)\int \frac{x^{m+n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^2}\,dx + ($$

$$= \left(d^2x^{-m}(fx)^m\,(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}\right)\int \frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{(d^2-e^2x^{2n})^2}\,dx - \left(2dex^{-m}(fx\right.$$

$$= \frac{x(fx)^m\,(a+cx^{2n})^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,2;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^2(1+m)} - \frac{2ex^{1+n}}{}$$

Mathematica [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Maple [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+c\,x^{2n})^p}{(d+e\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(x^n*e + d)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(2*d*x^n*e + d^2 + x^(2*n)*e^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(x^n*e + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+c\,x^{2\,n})^p\,(f\,x)^m}{(d+e\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2,x)
[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^2, x)
```

3.92 $\int \frac{(fx)^m\left(a+cx^{2n}\right)^p}{\left(d+ex^n\right)^3}\,dx$

Optimal. Leaf size=412

$$\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right)^{-p}F_1\left(\frac{1+m}{2n};-p,3;1+\frac{1+m}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m)}-\frac{3ex^{1+n}(fx)^m\left(a+cx^{2n}\right)^p\left(1+\frac{cx^{2n}}{a}\right.}{d^4(}$$

```
[Out] x*(f*x)^m*(a+c*x^(2*n))^p*AppellF1(1/2*(1+m)/n,3,-p,1+1/2*(1+m)/n,e^2*x^(2*
n)/d^2,-c*x^(2*n)/a)/d^3/(1+m)/((1+c*x^(2*n)/a)^p)-3*e*x^(1+n)*(f*x)^m*(a+c
*x^(2*n))^p*AppellF1(1/2*(1+m+n)/n,3,-p,1/2*(1+m+3*n)/n,e^2*x^(2*n)/d^2,-c*
x^(2*n)/a)/d^4/(1+m+n)/((1+c*x^(2*n)/a)^p)+3*e^2*x^(1+2*n)*(f*x)^m*(a+c*x^(
2*n))^p*AppellF1(1/2*(1+m+2*n)/n,3,-p,1/2*(1+m+4*n)/n,e^2*x^(2*n)/d^2,-c*x^
(2*n)/a)/d^5/(1+m+2*n)/((1+c*x^(2*n)/a)^p)-e^3*x^(1+3*n)*(f*x)^m*(a+c*x^(2*
n))^p*AppellF1(1/2*(1+m+3*n)/n,3,-p,1/2*(1+m+5*n)/n,e^2*x^(2*n)/d^2,-c*x^(2
*n)/a)/d^6/(1+m+3*n)/((1+c*x^(2*n)/a)^p)
```

Rubi [A]
time = 0.29, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1576, 525, 524}

$$-\frac{e^3x^{3n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+3n+1}{2n};-p,3;\frac{m+5n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^6(m+3n+1)}+\frac{3e^2x^{2n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+2n+1}{2n};-p,3;\frac{m+4n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^5(m+2n+1)}-\frac{3ex^{n+1}(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+n+1}{2n};-p,3;\frac{m+3n+1}{2n};-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^4(m+n+1)}+\frac{x(fx)^m\left(a+cx^{2n}\right)^p\left(\frac{cx^{2n}}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2n};-p,3;\frac{m+1}{2n}+1;-\frac{cx^{2n}}{a},\frac{e^2x^{2n}}{d^2}\right)}{d^3(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
```

```
[Out] (x*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m)/(2*n), -p, 3, 1 + (1 + m)/(2*
n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^3*(1 + m)*(1 + (c*x^(2*n))/a)^
p) - (3*e*x^(1 + n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + n)/(2*n), -
p, 3, (1 + m + 3*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^4*(1 +
m + n)*(1 + (c*x^(2*n))/a)^p) + (3*e^2*x^(1 + 2*n)*(f*x)^m*(a + c*x^(2*n))^
p*AppellF1[(1 + m + 2*n)/(2*n), -p, 3, (1 + m + 4*n)/(2*n), -((c*x^(2*n))/a
), (e^2*x^(2*n))/d^2])/(d^5*(1 + m + 2*n)*(1 + (c*x^(2*n))/a)^p) - (e^3*x^(
1 + 3*n)*(f*x)^m*(a + c*x^(2*n))^p*AppellF1[(1 + m + 3*n)/(2*n), -p, 3, (1
+ m + 5*n)/(2*n), -((c*x^(2*n))/a), (e^2*x^(2*n))/d^2])/(d^6*(1 + m + 3*n)*
(1 + (c*x^(2*n))/a)^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1576

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_)*((a_) + (c_.)*(x_)^(n2
_))^(p_), x_Symbol] :> Dist[(f*x)^m/x^m, Int[ExpandIntegrand[x^m*(a + c*x^(
2*n))^p, (d/(d^2 - e^2*x^(2*n)) - e*(x^n/(d^2 - e^2*x^(2*n))))^(-q), x], x]
, x] /; FreeQ[{a, c, d, e, f, m, n, p}, x] && EqQ[n2, 2*n] &&  !IntegerQ[p]
 && ILtQ[q, 0]
```

Rubi steps

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3} dx = (x^{-m}(fx)^m) \int \left(\frac{d^3x^m(a+cx^{2n})^p}{(d^2-e^2x^{2n})^3} + \frac{3d^2ex^{m+n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3} - \frac{3de^2x^{m+2n}(a+cx}{(-d^2+e^2x^{2n})^3}\right.$$

$$= (d^3x^{-m}(fx)^m) \int \frac{x^m(a+cx^{2n})^p}{(d^2-e^2x^{2n})^3} dx + (3d^2ex^{-m}(fx)^m) \int \frac{x^{m+n}(a+cx^{2n})^p}{(-d^2+e^2x^{2n})^3} dx$$

$$= \left(d^3x^{-m}(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p}\right) \int \frac{x^m\left(1+\frac{cx^{2n}}{a}\right)^p}{(d^2-e^2x^{2n})^3} dx + \left(3d^2ex^{-m}(\right.$$

$$= \frac{x(fx)^m (a+cx^{2n})^p \left(1+\frac{cx^{2n}}{a}\right)^{-p} F_1\left(\frac{1+m}{2n}; -p, 3; 1+\frac{1+m}{2n}; -\frac{cx^{2n}}{a}, \frac{e^2x^{2n}}{d^2}\right)}{d^3(1+m)} - \frac{3ex^{1+}}{}$$

Mathematica [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a+cx^{2n})^p}{(d+ex^n)^3} dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3,x]
[Out] Integrate[((f*x)^m*(a + c*x^(2*n))^p)/(d + e*x^n)^3, x]
```

Maple [F]
time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a+c\,x^{2n})^p}{(d+e\,x^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

```
[Out] int((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(x^n*e + d)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + a)^p*(f*x)^m/(3*d^2*x^n*e + d^3 + 3*d*x^(2*n)*e^2 + x
^(3*n)*e^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+c*x**(2*n))**p/(d+e*x**n)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+c*x^(2*n))^p/(d+e*x^n)^3,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + a)^p*(f*x)^m/(x^n*e + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+c\,x^{2\,n})^p\,(f\,x)^m}{(d+e\,x^n)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3,x)
[Out] int(((a + c*x^(2*n))^p*(f*x)^m)/(d + e*x^n)^3, x)
```

3.93 $\int (b+2cx) \left(a+bx+cx^2\right)^{13} \, dx$

Optimal. Leaf size=16

$$\frac{1}{14} \left(a+bx+cx^2\right)^{14}$$

```
[Out] 1/14*(c*x^2+b*x+a)^14
```

Rubi [A]

time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {643}

$$\frac{1}{14} \left(a+bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (a + b*x + c*x^2)^14/14
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx) \left(a+bx+cx^2\right)^{13} \, dx = \frac{1}{14} \left(a+bx+cx^2\right)^{14}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(16) = 32.

time = 0.12, size = 201, normalized size = 12.56

$$\frac{1}{14}x(b+cx) \left(14a^{13}+91a^{12}x(b+cx)+364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3+2002a^9x^4(b+cx)^4+3003a^8x^5(b+cx)^5+3432a^7x^6(b+cx)^6+3003a^6x^7(b+cx)^7+2002a^5x^8(b+cx)^8+1001a^4x^9(b+cx)^9+364a^3x^{10}(b+cx)^{10}+91a^2x^{11}(b+cx)^{11}+14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(14*a^13 + 91*a^12*x*(b + c*x) + 364*a^11*x^2*(b + c*x)^2 + 10
01*a^10*x^3*(b + c*x)^3 + 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x)
^5 + 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 + 2002*a^5*x^8*(b
```

```
+ c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 + 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x^
11*(b + c*x)^11 + 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

Maple [A]
time = 0.31, size = 15, normalized size = 0.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*(c*x^2+b*x+a)^14
```

Maxima [A]
time = 0.28, size = 14, normalized size = 0.88

$$\frac{1}{14}\left(cx^2+bx+a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x + a)^14
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1234 vs. $2(14) = 28$.
time = 0.36, size = 1234, normalized size = 77.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 1/2*(13*b^2*c^12 + 2*a*c^13)*x^26 + 13*(2*b^
3*c^11 + a*b*c^12)*x^25 + 13/2*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^12)*x^2
4 + 13*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^23 + 13/2*(33*b^6*c^8
+ 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^22 + 143/7*(12*b^7*c^7 +
63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^21 + 143/2*(3*b^8*c^6 + 24
*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^20 + 143*(b^9*c^
5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^19 + 14
3/2*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 45*a^4*b^
2*c^8 + 2*a^5*c^9)*x^18 + 13*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 +
 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^17 + 13/2*(b^12*c^2 +
44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792
*a^5*b^2*c^7 + 33*a^6*c^8)*x^16 + (b^13*c + 78*a*b^11*c^2 + 1430*a^2*b^9*c^
3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 + 1716*a^6*b*c
^7)*x^15 + a^13*b*x + 1/14*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 + 60060
*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6
+ 3432*a^7*c^7)*x^14 + (a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2 + 8580*a^
```

```
4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^13 +
13/2*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*
a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^12 + 13*(2*a^3*b^11 + 55*a^4*
b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 99*a^8*b*c^5)
*x^11 + 143/2*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a^7*b^4*c^3 +
 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^10 + 143*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*b^
5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^9 + 143/2*(3*a^6*b^8 + 24*a^7*b^6*c
 + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^8 + 143/7*(12*a^7*b^7 + 63
*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^7 + 13/2*(33*a^8*b^6 + 110*a
^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^6 + 13*(11*a^9*b^5 + 22*a^10*b^3
*c + 6*a^11*b*c^2)*x^5 + 13/2*(11*a^10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^4
+ 13*(2*a^11*b^3 + a^12*b*c)*x^3 + 1/2*(13*a^12*b^2 + 2*a^13*c)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. $2(12) = 24$.

time = 0.14, size = 1326, normalized size = 82.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**13,x)
```

```
[Out] a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(a*c**13 + 13*b**2*c**12
/2) + x**25*(13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 + 78*a*
b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 + 286*a*b**3*c**10
+ 143*b**5*c**9) + x**22*(26*a**3*c**11 + 429*a**2*b**2*c**10 + 715*a*b**4*
c**9 + 429*b**6*c**8/2) + x**21*(286*a**3*b*c**10 + 1430*a**2*b**3*c**9 + 1
287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 + 1430*a**3*b
**2*c**9 + 6435*a**2*b**4*c**8/2 + 1716*a*b**6*c**7 + 429*b**8*c**6/2) + x*
*19*(715*a**4*b*c**9 + 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 + 1716*a*b
**7*c**6 + 143*b**9*c**5) + x**18*(143*a**5*c**9 + 6435*a**4*b**2*c**8/2 +
8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 + 1287*a*b**8*c**5 + 143*b**10*c*
*4/2) + x**17*(1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 + 12012*a**3*b**5*c**
6 + 5148*a**2*b**7*c**5 + 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(429*a**
6*c**8/2 + 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 + 12012*a**3*b**6*c**
5 + 6435*a**2*b**8*c**4/2 + 286*a*b**10*c**3 + 13*b**12*c**2/2) + x**15*(17
16*a**6*b*c**7 + 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 + 8580*a**3*b*
*7*c**4 + 1430*a**2*b**9*c**3 + 78*a*b**11*c**2 + b**13*c) + x**14*(1716*a*
*7*c**7/7 + 6006*a**6*b**2*c**6 + 18018*a**5*b**4*c**5 + 15015*a**4*b**6*c*
*4 + 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 + 13*a*b**12*c + b**14/14) +
 x**13*(1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 + 18018*a**5*b**5*c**4 + 85
80*a**4*b**7*c**3 + 1430*a**3*b**9*c**2 + 78*a**2*b**11*c + a*b**13) + x**1
2*(429*a**8*c**6/2 + 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 + 12012*a**
5*b**6*c**3 + 6435*a**4*b**8*c**2/2 + 286*a**3*b**10*c + 13*a**2*b**12/2) +
 x**11*(1287*a**8*b*c**5 + 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c**3 + 514
```

```
8*a**5*b**7*c**2 + 715*a**4*b**9*c + 26*a**3*b**11) + x**10*(143*a**9*c**5
+ 6435*a**8*b**2*c**4/2 + 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c**2 + 1287*
a**5*b**8*c + 143*a**4*b**10/2) + x**9*(715*a**9*b*c**4 + 4290*a**8*b**3*c*
*3 + 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c + 143*a**5*b**9) + x**8*(143*a*
*10*c**4/2 + 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 + 1716*a**7*b**6*c
 + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 + 1430*a**9*b**3*c**2 + 1287*a
**8*b**5*c + 1716*a**7*b**7/7) + x**6*(26*a**11*c**3 + 429*a**10*b**2*c**2
+ 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(78*a**11*b*c**2 + 286*a**10*b*
*3*c + 143*a**9*b**5) + x**4*(13*a**12*c**2/2 + 78*a**11*b**2*c + 143*a**10
*b**4/2) + x**3*(13*a**12*b*c + 26*a**11*b**3) + x**2*(a**13*c + 13*a**12*b
**2/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(14) = 28$.
time = 3.20, size = 216, normalized size = 13.50

$$\frac{1}{14}\left(cx^2+bx\right)^{14}+\left(cx^2+bx\right)^{13}a+\frac{13}{2}\left(cx^2+bx\right)^{12}a^2+26\left(cx^2+bx\right)^{11}a^3+\frac{143}{2}\left(cx^2+bx\right)^{10}a^4+143\left(cx^2+bx\right)^9a^5+\frac{429}{2}\left(cx^2+bx\right)^8a^6+\frac{1716}{7}\left(cx^2+bx\right)^7a^7+\frac{429}{2}\left(cx^2+bx\right)^6a^8+143\left(cx^2+bx\right)^5a^9+\frac{143}{2}\left(cx^2+bx\right)^4a^{10}+26\left(cx^2+bx\right)^3a^{11}+\frac{13}{2}\left(cx^2+bx\right)^2a^{12}+\left(cx^2+bx\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14 + (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 + 26
*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 + 143*(c*x^2 + b*x)^9*a^
5 + 429/2*(c*x^2 + b*x)^8*a^6 + 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 +
 b*x)^6*a^8 + 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 + 26*(c*
x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 + (c*x^2 + b*x)*a^13
```

Mupad [B]
time = 3.34, size = 1203, normalized size = 75.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(a + b*x + c*x^2)^13,x)
```

```
[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*c + (6435*a^4*b^8*c^
2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7*b^2*c^5) + x^16*((4
29*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 + 1
2012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148*a^5*b^2*c^7) + x^13*(a*b^13 + 7
8*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018
*a^5*b^5*c^4 + 12012*a^6*b^3*c^5) + x^15*(b^13*c + 78*a*b^11*c^2 + 1716*a^6
*b*c^7 + 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^
5*b^3*c^6) + x^6*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 429*a^10*
b^2*c^2) + x^22*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 429*a^2*b^
2*c^10) + x^10*((143*a^4*b^10)/2 + 143*a^9*c^5 + 1287*a^5*b^8*c + 6006*a^6*
b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) + x^18*(143*a^5*c^9 + (1
```

```
43*b^10*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 + (64
35*a^4*b^2*c^8)/2) + x^14*(b^14/14 + (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 +
4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6
 + 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 + 1716*a^7*b^6*c
+ (6435*a^8*b^4*c^2)/2 + 1430*a^9*b^2*c^3) + x^20*((143*a^4*c^10)/2 + (429*
b^8*c^6)/2 + 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9) + (c
^14*x^28)/14 + x^2*(a^13*c + (13*a^12*b^2)/2) + (13*a^10*x^4*(11*b^4 + a^2*
c^2 + 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/2 + b
*c^13*x^27 + (c^12*x^26*(2*a*c + 13*b^2))/2 + a^13*b*x + (143*a^7*b*x^7*(12
*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + (143*b*c^7*x^21*(12*b
^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/7 + 143*a^5*b*x^9*(b^8 + 5*
a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 143*b*c^5*x^19*(b
^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c) + 13*a^3*b*x
^11*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*
c^4 + 55*a*b^8*c) + 13*b*c^3*x^17*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 +
924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c) + 13*a^9*b*x^5*(11*b^4 + 6*
a^2*c^2 + 22*a*b^2*c) + 13*b*c^9*x^23*(11*b^4 + 6*a^2*c^2 + 22*a*b^2*c) + 1
3*a^11*b*x^3*(a*c + 2*b^2) + 13*b*c^11*x^25*(a*c + 2*b^2)
```

3.94 $\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

```
[Out] 1/28*(c*x^4+b*x^2+a)^14
```

Rubi [A]
time = 0.21, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1261, 643}

$$\frac{1}{28}\left(a+bx^2+cx^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a + b*x^2 + c*x^4)^14/28
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^{13}\,dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^2\right)\\ &= \frac{1}{28}\left(a+bx^2+cx^4\right)^{14}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(18) = 36.
time = 0.12, size = 233, normalized size = 12.94

$$\frac{1}{28}x^2\left(b+cx^2\right)\left(14a^{13}+91a^{12}x^2\left(b+cx^2\right)+364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3+2002a^9x^8\left(b+cx^2\right)^4+3003a^8x^{10}\left(b+cx^2\right)^5+3432a^7x^{12}\left(b+cx^2\right)^6+3003a^6x^{14}\left(b+cx^2\right)^7+2002a^5x^{16}\left(b+cx^2\right)^8+1001a^4x^{18}\left(b+cx^2\right)^9+364a^3x^{20}\left(b+cx^2\right)^{10}+91a^2x^{22}\left(b+cx^2\right)^{11}+14ax^{24}\left(b+cx^2\right)^{12}+x^{26}\left(b+cx^2\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(14*a^13 + 91*a^12*x^2*(b + c*x^2) + 364*a^11*x^4*(b + c*x
^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 + 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^8
*x^10*(b + c*x^2)^5 + 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x^
2)^7 + 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 + 364*a^3*
x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 + 14*a*x^24*(b + c*x^2)^12
 + x^26*(b + c*x^2)^13))/28
```

Maple [A]
time = 0.09, size = 17, normalized size = 0.94

method	result	size
default	$\frac{\left(c\,x^{4}+b\,x^{2}+a\right)^{14}}{28}$	17
gosper	Expression too large to display	1455
risch	Expression too large to display	1460

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*(c*x^4+b*x^2+a)^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(16) = 32$.
time = 0.30, size = 1240, normalized size = 68.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78*a*b^11*c^2
```

```
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 + 13/2*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^20 + 143/2*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^16 + 1/2*a^
13*b*x^2 + 143/14*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)
*x^12 + 13/2*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^8 + 13/2*(2*a^11*b^3 + a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 + 2*a^13*c)*x^4
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(16) = 32$.

time = 0.35, size = 1240, normalized size = 68.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 + 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 + a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^28 + 1/2*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
```

```
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 + 13/2*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^20 + 143/2*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^16 + 1/2*a^
13*b*x^2 + 143/14*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)
*x^12 + 13/2*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^8 + 13/2*(2*a^11*b^3 + a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 + 2*a^13*c)*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. $2(14) = 28$.
time = 0.17, size = 1384, normalized size = 76.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**13,x)
```

```
[Out] a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(a*c**13/2 + 13*b
**2*c**12/4) + x**50*(13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c**1
2/4 + 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 + 143*a*
b**3*c**10 + 143*b**5*c**9/2) + x**44*(13*a**3*c**11 + 429*a**2*b**2*c**10/
2 + 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(143*a**3*b*c**10 + 715*a*
*2*b**3*c**9 + 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4*c**1
0/4 + 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 + 858*a*b**6*c**7 + 429*b*
*8*c**6/4) + x**38*(715*a**4*b*c**9/2 + 2145*a**3*b**3*c**8 + 2574*a**2*b**
5*c**7 + 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(143*a**5*c**9/2 + 6435
*a**4*b**2*c**8/4 + 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 + 1287*a*b**8
*c**5/2 + 143*b**10*c**4/4) + x**34*(1287*a**5*b*c**8/2 + 4290*a**4*b**3*c*
*7 + 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 + 715*a*b**9*c**4/2 + 13*b**
11*c**3) + x**32*(429*a**6*c**8/4 + 2574*a**5*b**2*c**7 + 15015*a**4*b**4*c
**6/2 + 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 + 143*a*b**10*c**3 + 13
*b**12*c**2/4) + x**30*(858*a**6*b*c**7 + 6006*a**5*b**3*c**6 + 9009*a**4*b
**5*c**5 + 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 + 39*a*b**11*c**2 + b**
13*c/2) + x**28*(858*a**7*c**7/7 + 3003*a**6*b**2*c**6 + 9009*a**5*b**4*c**
5 + 15015*a**4*b**6*c**4/2 + 2145*a**3*b**8*c**3 + 429*a**2*b**10*c**2/2 +
13*a*b**12*c/2 + b**14/28) + x**26*(858*a**7*b*c**6 + 6006*a**6*b**3*c**5 +
 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 + 715*a**3*b**9*c**2 + 39*a**2*b
**11*c + a*b**13/2) + x**24*(429*a**8*c**6/4 + 2574*a**7*b**2*c**5 + 15015*
a**6*b**4*c**4/2 + 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 + 143*a**3*b
**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 + 4290*a**7*b**3*c**4
 + 6006*a**6*b**5*c**3 + 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 + 13*a**3*
```

```
b**11) + x**20*(143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 + 4290*a**7*b**4*c*
*3 + 3003*a**6*b**6*c**2 + 1287*a**5*b**8*c/2 + 143*a**4*b**10/4) + x**18*(
715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 + 2574*a**7*b**5*c**2 + 858*a**6*b*
*7*c + 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 + 715*a**9*b**2*c**3 + 64
35*a**8*b**4*c**2/4 + 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14*(143*a**10
*b*c**3 + 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 + 858*a**7*b**7/7) + x**1
2*(13*a**11*c**3 + 429*a**10*b**2*c**2/2 + 715*a**9*b**4*c/2 + 429*a**8*b**
6/4) + x**10*(39*a**11*b*c**2 + 143*a**10*b**3*c + 143*a**9*b**5/2) + x**8*
(13*a**12*c**2/4 + 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*(13*a**12*b*c
/2 + 13*a**11*b**3) + x**4*(a**13*c/2 + 13*a**12*b**2/4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(16) = 32$.
time = 3.60, size = 246, normalized size = 13.67

$$\frac{1}{28}\left(cx^4+bx^2\right)^{14}+\frac{1}{2}\left(cx^4+bx^2\right)^{13}a+\frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2+13\left(cx^4+bx^2\right)^{11}a^3+\frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4+\frac{143}{2}\left(cx^4+bx^2\right)^{9}a^5+\frac{429}{4}\left(cx^4+bx^2\right)^{8}a^6+\frac{858}{7}\left(cx^4+bx^2\right)^{7}a^7+\frac{429}{4}\left(cx^4+bx^2\right)^{6}a^8+\frac{143}{2}\left(cx^4+bx^2\right)^{5}a^9+\frac{143}{4}\left(cx^4+bx^2\right)^{4}a^{10}+13\left(cx^4+bx^2\right)^{3}a^{11}+\frac{13}{4}\left(cx^4+bx^2\right)^{2}a^{12}+\frac{1}{2}\left(cx^4+bx^2\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14 + 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^1
2*a^2 + 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 + 143/2*(c
*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 + 858/7*(c*x^4 + b*x^2)^7
*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 + 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c
*x^4 + b*x^2)^4*a^10 + 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a
^12 + 1/2*(c*x^4 + b*x^2)*a^13
```

Mupad [B]
time = 3.23, size = 1210, normalized size = 67.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^13,x)
```

```
[Out] x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 + 143*a^3*b^10*c + (6435*a^4*b^8*c^
2)/4 + 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 + 2574*a^7*b^2*c^5) + x^32*
((429*a^6*c^8)/4 + (13*b^12*c^2)/4 + 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)/4
+ 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 + 2574*a^5*b^2*c^7) + x^26*((a*b
^13)/2 + 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 + 4290*a^4*b^7*c^3
 + 9009*a^5*b^5*c^4 + 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 + 39*a*b^11*c^2
+ 858*a^6*b*c^7 + 715*a^2*b^9*c^3 + 4290*a^3*b^7*c^4 + 9009*a^4*b^5*c^5 + 6
006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 + 13*a^11*c^3 + (715*a^9*b^4*c)/2
+ (429*a^10*b^2*c^2)/2) + x^44*(13*a^3*c^11 + (429*b^6*c^8)/4 + (715*a*b^4*
c^9)/2 + (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 + (143*a^9*c^5)/2 +
 (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 + 4290*a^7*b^4*c^3 + (6435*a^8*b^2*c
```

```
^4)/4) + x^36*((143*a^5*c^9)/2 + (143*b^10*c^4)/4 + (1287*a*b^8*c^5)/2 + 30
03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/4) + x^28*(b^14/28 +
 (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 + 2145*a^3*b^8*c^3 + (15015*a^4*b^6
*c^4)/2 + 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 + (13*a*b^12*c)/2) + x^16*((4
29*a^6*b^8)/4 + (143*a^10*c^4)/4 + 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 + 7
15*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4 + 858*a*b^6*c^7
+ (6435*a^2*b^4*c^8)/4 + 715*a^3*b^2*c^9) + (c^14*x^56)/28 + x^4*((a^13*c)/
2 + (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (1
3*c^10*x^48*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/4 + (a^13*b*x^2)/2 + (b*c^13*x
^54)/2 + (c^12*x^52*(2*a*c + 13*b^2))/4 + (143*a^7*b*x^14*(12*b^6 + 14*a^3*
c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*b*c^7*x^42*(12*b^6 + 14*a^3*c
^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/14 + (143*a^5*b*x^18*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (143*b*c^5*x^38*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/2 + (13*a^3*b*x^2
2*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 + 55*a*b^8*c))/2 + (13*b*c^3*x^34*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2
+ 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/2 + (13*a^9*b*x^10*(11*b
^4 + 6*a^2*c^2 + 22*a*b^2*c))/2 + (13*b*c^9*x^46*(11*b^4 + 6*a^2*c^2 + 22*a
*b^2*c))/2 + (13*a^11*b*x^6*(a*c + 2*b^2))/2 + (13*b*c^11*x^50*(a*c + 2*b^2
))/2
```

3.95 $\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{42}\left(a+bx^3+cx^6\right)^{14}$$

```
[Out] 1/42*(c*x^6+b*x^3+a)^14
```

Rubi [A]
time = 0.19, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1482, 643}

$$\frac{1}{42}\left(a+bx^3+cx^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a + b*x^3 + c*x^6)^14/42
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^{13}\,dx &= \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^3\right)\\ &= \frac{1}{42}\left(a+bx^3+cx^6\right)^{14}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(18) = 36.
time = 0.12, size = 233, normalized size = 12.94

$$\frac{1}{42}x^3\left(b+cx^3\right)\left(14a^{13}+91a^{12}x^3\left(b+cx^3\right)+364a^{11}x^6\left(b+cx^3\right)^2+1001a^{10}x^9\left(b+cx^3\right)^3+2002a^9x^{12}\left(b+cx^3\right)^4+3003a^8x^{15}\left(b+cx^3\right)^5+3432a^7x^{18}\left(b+cx^3\right)^6+3003a^6x^{21}\left(b+cx^3\right)^7+2002a^5x^{24}\left(b+cx^3\right)^8+1001a^4x^{27}\left(b+cx^3\right)^9+364a^3x^{30}\left(b+cx^3\right)^{10}+91a^2x^{33}\left(b+cx^3\right)^{11}+14ax^{36}\left(b+cx^3\right)^{12}+x^{39}\left(b+cx^3\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(14*a^13 + 91*a^12*x^3*(b + c*x^3) + 364*a^11*x^6*(b + c*x
^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 + 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a^
8*x^15*(b + c*x^3)^5 + 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*x
^3)^7 + 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 + 364*a^3
*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 + 14*a*x^36*(b + c*x^3)^1
2 + x^39*(b + c*x^3)^13))/42
```

Maple [A]
time = 0.11, size = 17, normalized size = 0.94

method	result	size
default	$\frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{14}}{42}$	17
gosper	Expression too large to display	1455
risch	Expression too large to display	1460

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*(c*x^6+b*x^3+a)^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. 2(16) = 32.
time = 0.29, size = 1240, normalized size = 68.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78*a*b^11*c^2
```

```
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 + 13/3*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^30 + 143/3*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^24 + 143/21
*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^18 + 1/3*a^13*
b*x^3 + 13/3*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^12 + 13/3*(2*a^11*b^3 + a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 + 2*a^13*c)*x^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1240 vs. $2(16) = 32$.

time = 0.36, size = 1240, normalized size = 68.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="fricas")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 + 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 + a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 + 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 + 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 + 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 + 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 + 18*a*b^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 + 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 + 44*a*b^10*c^3 + 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 + 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c + 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 +
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 + 3432*a^7*c^7)*x^42 + 1/3*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c
```

```
^3 + 2310*a^6*b^4*c^4 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 + 13/3*(2*a^3*b^
11 + 55*a^4*b^9*c + 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 + 2*a^9*c^5)*x^30 + 143/3*(a^5*b^9 + 12*a^6*b^7
*c + 36*a^7*b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 + 24*a^7*b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^24 + 143/21
*(12*a^7*b^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^18 + 1/3*a^13*
b*x^3 + 13/3*(11*a^9*b^5 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 + 12*a^11*b^2*c + a^12*c^2)*x^12 + 13/3*(2*a^11*b^3 + a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 + 2*a^13*c)*x^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(14) = 28$.
time = 0.18, size = 1394, normalized size = 77.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**13,x)
```

```
[Out] a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(a*c**13/3 + 13*b
**2*c**12/6) + x**75*(13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2*c*
*12/6 + 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 + 286*
a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(26*a**3*c**11/3 + 143*a**2*b**2*
c**10 + 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(286*a**3*b*c**10/3 +
1430*a**2*b**3*c**9/3 + 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(143*a**
4*c**10/6 + 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 + 572*a*b**6*c**7
 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 + 1430*a**3*b**3*c**8 + 1716
*a**2*b**5*c**7 + 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(143*a**5*c**9
/3 + 2145*a**4*b**2*c**8/2 + 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**6 + 42
9*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(429*a**5*b*c**8 + 2860*a**4*b**3
*c**7 + 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 + 715*a*b**9*c**4/3 + 26*
b**11*c**3/3) + x**48*(143*a**6*c**8/2 + 1716*a**5*b**2*c**7 + 5005*a**4*b*
*4*c**6 + 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 + 286*a*b**10*c**3/3
+ 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 + 4004*a**5*b**3*c**6 + 6006*a*
*4*b**5*c**5 + 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 + 26*a*b**11*c**
2 + b**13*c/3) + x**42*(572*a**7*c**7/7 + 2002*a**6*b**2*c**6 + 6006*a**5*b
**4*c**5 + 5005*a**4*b**6*c**4 + 1430*a**3*b**8*c**3 + 143*a**2*b**10*c**2
+ 13*a*b**12*c/3 + b**14/42) + x**39*(572*a**7*b*c**6 + 4004*a**6*b**3*c**5
 + 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 + 1430*a**3*b**9*c**2/3 + 26*a
**2*b**11*c + a*b**13/3) + x**36*(143*a**8*c**6/2 + 1716*a**7*b**2*c**5 + 5
005*a**6*b**4*c**4 + 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 + 286*a**3
*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 + 2860*a**7*b**3*c**
4 + 4004*a**6*b**5*c**3 + 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3 + 26*a**3
```

```
*b**11/3) + x**30*(143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 + 2860*a**7*b**4
*c**3 + 2002*a**6*b**6*c**2 + 429*a**5*b**8*c + 143*a**4*b**10/6) + x**27*(
715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 + 1716*a**7*b**5*c**2 + 572*a**6*b*
*7*c + 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 + 1430*a**9*b**2*c**3/3 +
 2145*a**8*b**4*c**2/2 + 572*a**7*b**6*c + 143*a**6*b**8/2) + x**21*(286*a*
*10*b*c**3/3 + 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c + 572*a**7*b**7/7) +
 x**18*(26*a**11*c**3/3 + 143*a**10*b**2*c**2 + 715*a**9*b**4*c/3 + 143*a**
8*b**6/2) + x**15*(26*a**11*b*c**2 + 286*a**10*b**3*c/3 + 143*a**9*b**5/3)
+ x**12*(13*a**12*c**2/6 + 26*a**11*b**2*c + 143*a**10*b**4/6) + x**9*(13*a
**12*b*c/3 + 26*a**11*b**3/3) + x**6*(a**13*c/3 + 13*a**12*b**2/6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(16) = 32$.

time = 3.74, size = 246, normalized size = 13.67

$$\frac{1}{42}\left(cx^{6}+bx^{3}\right)^{14}+\frac{1}{3}\left(cx^{6}+bx^{3}\right)^{13}a+\frac{13}{6}\left(cx^{6}+bx^{3}\right)^{12}a^{2}+\frac{26}{3}\left(cx^{6}+bx^{3}\right)^{11}a^{3}+\frac{143}{6}\left(cx^{6}+bx^{3}\right)^{10}a^{4}+\frac{143}{3}\left(cx^{6}+bx^{3}\right)^{9}a^{5}+\frac{143}{2}\left(cx^{6}+bx^{3}\right)^{8}a^{6}+\frac{572}{7}\left(cx^{6}+bx^{3}\right)^{7}a^{7}+\frac{143}{2}\left(cx^{6}+bx^{3}\right)^{6}a^{8}+\frac{143}{3}\left(cx^{6}+bx^{3}\right)^{5}a^{9}+\frac{143}{6}\left(cx^{6}+bx^{3}\right)^{4}a^{10}+\frac{26}{3}\left(cx^{6}+bx^{3}\right)^{3}a^{11}+\frac{13}{6}\left(cx^{6}+bx^{3}\right)^{2}a^{12}+\frac{1}{3}\left(cx^{6}+bx^{3}\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14 + 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^1
2*a^2 + 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 + 143/3*
(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 + 572/7*(c*x^6 + b*x^3)
^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 + 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*
(c*x^6 + b*x^3)^4*a^10 + 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)
^2*a^12 + 1/3*(c*x^6 + b*x^3)*a^13
```

Mupad [B]

time = 3.18, size = 1210, normalized size = 67.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^13,x)
```

```
[Out] x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 + (286*a^3*b^10*c)/3 + (2145*a^4*b^
8*c^2)/2 + 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 + 1716*a^7*b^2*c^5) + x^48*(
(143*a^6*c^8)/2 + (13*b^12*c^2)/6 + (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c^4)
/2 + 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 + 1716*a^5*b^2*c^7) + x^39*((a*b^1
3)/3 + 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 + 2860*a^4*b^7*
c^3 + 6006*a^5*b^5*c^4 + 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 + 26*a*b^11*c
^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 + 2860*a^3*b^7*c^4 + 6006*a^4*b^5
*c^5 + 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 + (26*a^11*c^3)/3 + (715*a
^9*b^4*c)/3 + 143*a^10*b^2*c^2) + x^66*((26*a^3*c^11)/3 + (143*b^6*c^8)/2 +
 (715*a*b^4*c^9)/3 + 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 + (143*a^9*
c^5)/3 + 429*a^5*b^8*c + 2002*a^6*b^6*c^2 + 2860*a^7*b^4*c^3 + (2145*a^8*b^
```

```
2*c^4)/2) + x^54*((143*a^5*c^9)/3 + (143*b^10*c^4)/6 + 429*a*b^8*c^5 + 2002
*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 + (2145*a^4*b^2*c^8)/2) + x^42*(b^14/42 + (
572*a^7*c^7)/7 + 143*a^2*b^10*c^2 + 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 + 6
006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 + (13*a*b^12*c)/3) + x^24*((143*a^6*b^8)
/2 + (143*a^10*c^4)/6 + 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 + (1430*a^9*b^
2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 + 572*a*b^6*c^7 + (214
5*a^2*b^4*c^8)/2 + (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42 + x^6*((a^13*c)/3
 + (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (1
3*c^10*x^72*(11*b^4 + a^2*c^2 + 12*a*b^2*c))/6 + (a^13*b*x^3)/3 + (b*c^13*x
^81)/3 + (c^12*x^78*(2*a*c + 13*b^2))/6 + (143*a^7*b*x^21*(12*b^6 + 14*a^3*
c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*b*c^7*x^63*(12*b^6 + 14*a^3*c
^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/21 + (143*a^5*b*x^27*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (143*b*c^5*x^57*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12*a*b^6*c))/3 + (13*a^3*b*x^3
3*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 + 55*a*b^8*c))/3 + (13*b*c^3*x^51*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2
+ 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/3 + (13*a^9*b*x^15*(11*b
^4 + 6*a^2*c^2 + 22*a*b^2*c))/3 + (13*b*c^9*x^69*(11*b^4 + 6*a^2*c^2 + 22*a
*b^2*c))/3 + (13*a^11*b*x^9*(a*c + 2*b^2))/3 + (13*b*c^11*x^75*(a*c + 2*b^2
))/3
```

3.96 $\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

```
[Out] 1/14*(a+b*x^n+c*x^(2*n))^14/n
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1482, 643}

$$\frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a + b*x^n + c*x^(2*n))^14/(14*n)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^{13}\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^{13}\,dx,x,x^n\right)}{n}\\ &= \frac{\left(a+bx^n+cx^{2n}\right)^{14}}{14n}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 260 vs. 2(23) = 46.
time = 0.39, size = 260, normalized size = 11.30

$$\frac{x^n(b+cx^n)\left(14a^{13}+91a^{12}x^n(b+cx^n)+364a^{11}x^{2n}(b+cx^n)^2+1001a^{10}x^{3n}(b+cx^n)^3+2002a^9x^{4n}(b+cx^n)^4+3003a^8x^{5n}(b+cx^n)^5+3432a^7x^{6n}(b+cx^n)^6+3003a^6x^{7n}(b+cx^n)^7+2002a^5x^{8n}(b+cx^n)^8+1001a^4x^{9n}(b+cx^n)^9+364a^3x^{10n}(b+cx^n)^{10}+91a^2x^{11n}(b+cx^n)^{11}+14ax^{12n}(b+cx^n)^{12}+x^{13n}(b+cx^n)^{13}\right)}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^n*(b + c*x^n)*(14*a^13 + 91*a^12*x^n*(b + c*x^n) + 364*a^11*x^(2*n)*(b +
 c*x^n)^2 + 1001*a^10*x^(3*n)*(b + c*x^n)^3 + 2002*a^9*x^(4*n)*(b + c*x^n)^
4 + 3003*a^8*x^(5*n)*(b + c*x^n)^5 + 3432*a^7*x^(6*n)*(b + c*x^n)^6 + 3003*
a^6*x^(7*n)*(b + c*x^n)^7 + 2002*a^5*x^(8*n)*(b + c*x^n)^8 + 1001*a^4*x^(9*
n)*(b + c*x^n)^9 + 364*a^3*x^(10*n)*(b + c*x^n)^10 + 91*a^2*x^(11*n)*(b + c
*x^n)^11 + 14*a*x^(12*n)*(b + c*x^n)^12 + x^(13*n)*(b + c*x^n)^13))/(14*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2041 vs. $2(21) = 42$.
time = 0.07, size = 2042, normalized size = 88.78

method	result	size
risch	Expression too large to display	2042

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x,method=_RETURNVERBOSE)
```

```
[Out] 429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12*b^12+26*b^3*c^11/n*(x^n)^25+17
16/7*a^7*b^7/n*(x^n)^7+143*c^9*b^5/n*(x^n)^23+143*a^9*b^5/n*(x^n)^5+b*c^13/
n*(x^n)^27+b*a^13/n*x^n+26*c^11/n*(x^n)^22*a^3+429/2*c^8/n*(x^n)^22*b^6+a^1
3/n*(x^n)^2*c+13/2*a^12/n*(x^n)^2*b^2+143/2*c^10/n*(x^n)^20*a^4+429/2*c^6/n
*(x^n)^20*b^8+143*c^9/n*(x^n)^18*a^5+6435/2*a^8/n*(x^n)^10*b^2*c^4+8580*a^7
/n*(x^n)^10*b^4*c^3+6006*a^6/n*(x^n)^10*b^6*c^2+1287*a^5/n*(x^n)^10*b^8*c+1
430*a^9/n*(x^n)^8*b^2*c^3+6435/2*a^8/n*(x^n)^8*b^4*c^2+1716*a^7/n*(x^n)^8*b
^6*c+78*c^11/n*(x^n)^24*a*b^2+c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b^2+42
9/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(x^n)^16*b^12+26*a^11/n*(x^n)^6*c^3+429/2
*a^8/n*(x^n)^6*b^6+b^13*c/n*(x^n)^15+26*a^11*b^3/n*(x^n)^3+a*b^13/n*(x^n)^1
3+1716/7*b^7*c^7/n*(x^n)^21+26*b^11*c^3/n*(x^n)^17+429*c^10/n*(x^n)^22*a^2*
b^2+715*c^9/n*(x^n)^22*a*b^4+1430*c^9/n*(x^n)^20*a^3*b^2+6435/2*c^8/n*(x^n)
^20*a^2*b^4+1716*c^7/n*(x^n)^20*a*b^6+6435/2*c^8/n*(x^n)^18*a^4*b^2+8580*c^
7/n*(x^n)^18*a^3*b^4+6006*c^6/n*(x^n)^18*a^2*b^6+1287*c^5/n*(x^n)^18*a*b^8+
143/2*c^4/n*(x^n)^18*b^10+78*c^11*b/n*(x^n)^23*a^2+286*c^10*b^3/n*(x^n)^23*
a+78*a^11*b/n*(x^n)^5*c^2+286*a^10*b^3/n*(x^n)^5*c+5148*c^7/n*(x^n)^16*a^5*
b^2+15015*c^6/n*(x^n)^16*a^4*b^4+12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c^4/n*
(x^n)^16*a^2*b^8+286*c^3/n*(x^n)^16*a*b^10+429*a^10/n*(x^n)^6*b^2*c^2+715*a
^9/n*(x^n)^6*b^4*c+78*a^11/n*(x^n)^4*b^2*c+1/14*c^14/n*(x^n)^28+13/2*a^12/n
*(x^n)^4*c^2+143/2*a^10/n*(x^n)^4*b^4+143*a^9/n*(x^n)^10*c^5+143/2*a^4/n*(x
^n)^10*b^10+143/2*a^10/n*(x^n)^8*c^4+429/2*a^6/n*(x^n)^8*b^8+13/2*c^12/n*(x
^n)^24*a^2+143/2*c^10/n*(x^n)^24*b^4+143*b^9*c^5/n*(x^n)^19+1716/7/n*(x^n)^
14*a^7*c^7+5148*a^7/n*(x^n)^12*b^2*c^5+15015*a^6/n*(x^n)^12*b^4*c^4+1/14/n*
(x^n)^14*b^14+143*a^5*b^9/n*(x^n)^9+26*a^3*b^11/n*(x^n)^11+18018/n*(x^n)^14
*a^5*b^4*c^5+15015/n*(x^n)^14*a^4*b^6*c^4+4290/n*(x^n)^14*a^3*b^8*c^3+429/n
```

```
*(x^n)^14*a^2*b^10*c^2+13/n*(x^n)^14*a*b^12*c+715*a^9*b/n*(x^n)^9*c^4+4290*
a^8*b^3/n*(x^n)^9*c^3+5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)^9*c+1
287*a^8*b/n*(x^n)^11*c^5+8580*a^7*b^3/n*(x^n)^11*c^4+12012*a^6*b^5/n*(x^n)^
11*c^3+5148*a^5*b^7/n*(x^n)^11*c^2+715*a^4*b^9/n*(x^n)^11*c+715*b*c^9/n*(x^
n)^19*a^4+4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x^n)^19*a^2+1716*b^7*
c^6/n*(x^n)^19*a+6006/n*(x^n)^14*a^6*b^2*c^6+12012*a^5/n*(x^n)^12*b^6*c^3+6
435/2*a^4/n*(x^n)^12*b^8*c^2+286*a^3/n*(x^n)^12*b^10*c+13*b*c^12/n*(x^n)^25
*a+286*a^10*b/n*(x^n)^7*c^3+1430*a^9*b^3/n*(x^n)^7*c^2+1287*a^8*b^5/n*(x^n)
^7*c+286*b*c^10/n*(x^n)^21*a^3+1430*b^3*c^9/n*(x^n)^21*a^2+1287*b^5*c^8/n*(
x^n)^21*a+1287*b*c^8/n*(x^n)^17*a^5+8580*b^3*c^7/n*(x^n)^17*a^4+12012*b^5*c
^6/n*(x^n)^17*a^3+5148*b^7*c^5/n*(x^n)^17*a^2+715*b^9*c^4/n*(x^n)^17*a+1716
*b*c^7/n*(x^n)^15*a^6+12012*b^3*c^6/n*(x^n)^15*a^5+18018*b^5*c^5/n*(x^n)^15
*a^4+8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n)^15*a^2+78*b^11*c^2/n*
(x^n)^15*a+13*a^12*b/n*(x^n)^3*c+1716*a^7*b/n*(x^n)^13*c^6+12012*a^6*b^3/n*
(x^n)^13*c^5+18018*a^5*b^5/n*(x^n)^13*c^4+8580*a^4*b^7/n*(x^n)^13*c^3+1430*
a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^n)^13*c
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2041 vs. $2(21) = 42$.
time = 0.33, size = 2041, normalized size = 88.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima"
)
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + a*c^1
3*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n + 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*
c^10*x^(24*n)/n + 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143
*b^5*c^9*x^(23*n)/n + 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n
+ 429/2*b^6*c^8*x^(22*n)/n + 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^
(22*n)/n + 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 1287*a*b^5*
c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n + 286*a^3*b*c^10*x^(21*n)/n +
429/2*b^8*c^6*x^(20*n)/n + 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x
^(20*n)/n + 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b
^9*c^5*x^(19*n)/n + 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n
 + 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*
x^(18*n)/n + 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n + 8580
*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n + 143*a^5*c^9*x^(18
*n)/n + 26*b^11*c^3*x^(17*n)/n + 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^
5*x^(17*n)/n + 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n +
 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n + 286*a*b^10*c^3*x^(1
6*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n + 12012*a^3*b^6*c^5*x^(16*n)/n + 150
15*a^4*b^4*c^6*x^(16*n)/n + 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(
```

```
16*n)/n + b^13*c*x^(15*n)/n + 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x
^(15*n)/n + 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n + 12
012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)
/n + 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n + 4290*a^3*b^8*c^
3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n + 18018*a^5*b^4*c^5*x^(14*n)/n
+ 6006*a^6*b^2*c^6*x^(14*n)/n + 1716/7*a^7*c^7*x^(14*n)/n + a*b^13*x^(13*n)
/n + 78*a^2*b^11*c*x^(13*n)/n + 1430*a^3*b^9*c^2*x^(13*n)/n + 8580*a^4*b^7*
c^3*x^(13*n)/n + 18018*a^5*b^5*c^4*x^(13*n)/n + 12012*a^6*b^3*c^5*x^(13*n)/
n + 1716*a^7*b*c^6*x^(13*n)/n + 13/2*a^2*b^12*x^(12*n)/n + 286*a^3*b^10*c*x
^(12*n)/n + 6435/2*a^4*b^8*c^2*x^(12*n)/n + 12012*a^5*b^6*c^3*x^(12*n)/n +
15015*a^6*b^4*c^4*x^(12*n)/n + 5148*a^7*b^2*c^5*x^(12*n)/n + 429/2*a^8*c^6*
x^(12*n)/n + 26*a^3*b^11*x^(11*n)/n + 715*a^4*b^9*c*x^(11*n)/n + 5148*a^5*b
^7*c^2*x^(11*n)/n + 12012*a^6*b^5*c^3*x^(11*n)/n + 8580*a^7*b^3*c^4*x^(11*n
)/n + 1287*a^8*b*c^5*x^(11*n)/n + 143/2*a^4*b^10*x^(10*n)/n + 1287*a^5*b^8*
c*x^(10*n)/n + 6006*a^6*b^6*c^2*x^(10*n)/n + 8580*a^7*b^4*c^3*x^(10*n)/n +
6435/2*a^8*b^2*c^4*x^(10*n)/n + 143*a^9*c^5*x^(10*n)/n + 143*a^5*b^9*x^(9*n
)/n + 1716*a^6*b^7*c*x^(9*n)/n + 5148*a^7*b^5*c^2*x^(9*n)/n + 4290*a^8*b^3*
c^3*x^(9*n)/n + 715*a^9*b*c^4*x^(9*n)/n + 429/2*a^6*b^8*x^(8*n)/n + 1716*a^
7*b^6*c*x^(8*n)/n + 6435/2*a^8*b^4*c^2*x^(8*n)/n + 1430*a^9*b^2*c^3*x^(8*n)
/n + 143/2*a^10*c^4*x^(8*n)/n + 1716/7*a^7*b^7*x^(7*n)/n + 1287*a^8*b^5*c*x
^(7*n)/n + 1430*a^9*b^3*c^2*x^(7*n)/n + 286*a^10*b*c^3*x^(7*n)/n + 429/2*a^
8*b^6*x^(6*n)/n + 715*a^9*b^4*c*x^(6*n)/n + 429*a^10*b^2*c^2*x^(6*n)/n + 26
*a^11*c^3*x^(6*n)/n + 143*a^9*b^5*x^(5*n)/n + 286*a^10*b^3*c*x^(5*n)/n + 78
*a^11*b*c^2*x^(5*n)/n + 143/2*a^10*b^4*x^(4*n)/n + 78*a^11*b^2*c*x^(4*n)/n
+ 13/2*a^12*c^2*x^(4*n)/n + 26*a^11*b^3*x^(3*n)/n + 13*a^12*b*c*x^(3*n)/n +
 13/2*a^12*b^2*x^(2*n)/n + a^13*c*x^(2*n)/n + a^13*b*x^n/n
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1297 vs. $2(21) = 42$.
time = 0.40, size = 1297, normalized size = 56.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas"
)
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 14*a^13*b*x^n + 7*(13*b^2*c^12 +
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 + a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 + 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 + 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 + 110*a*b^4*c^9 + 66*a^2*b^2*c^10 +
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 + 63*a*b^5*c^8 + 70*a^2*b^3*c^9 + 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 + 24*a*b^6*c^7 + 45*a^2*b^4*c^8 +
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 + 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 + 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 + 18*a*b
```

```
^8*c^5 + 84*a^2*b^6*c^6 + 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 + 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 + 55*a*b^9*c^4 + 396*a^2*b^7*c^5 + 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 + 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 + 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 + 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 + 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c + 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 + 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 + 182*a*b^12*c + 6006*a^2*b^10*c^2 + 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 + 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 + 3432*a^7*c^7)*x^(1
4*n) + 14*(a*b^13 + 78*a^2*b^11*c + 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 + 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 + 44*a^3*b^10*c + 495*a^4*b^8*c^2 + 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 + 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) + 182*(2*a^3*b^11 + 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 + 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 + 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 + 18*a^5*b^8*c + 84*a^6*b^6*c^2 + 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 + 2*a^9*c^5)*x^(10*n) + 2002*(a^5*b^9 + 12*a^6*b^7*c + 36*a^7*
b^5*c^2 + 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 + 24*a^7*
b^6*c + 45*a^8*b^4*c^2 + 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) + 286*(12*a^7*b
^7 + 63*a^8*b^5*c + 70*a^9*b^3*c^2 + 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 + 110*a^9*b^4*c + 66*a^10*b^2*c^2 + 4*a^11*c^3)*x^(6*n) + 182*(11*a^9*b^5
 + 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 + 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) + 182*(2*a^11*b^3 + a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
+ 2*a^13*c)*x^(2*n))/n
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**13,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1693 vs. $2(21) = 42$.
time = 4.38, size = 1693, normalized size = 73.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) + 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) + 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) + 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
```

*x^(22*n) + 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) + 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) + 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) + 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) + 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) + 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) + 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) + 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) + 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) + 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) + 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) + 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) + 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) + 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) + 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) + 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) + 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
) + 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
+ 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) + 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) + 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) + 3432*a^7*c^7*x^(14*n) + 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) + 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) + 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) + 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) + 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) + 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) + 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) + 364*a^3*b^11*x^(1
1*n) + 10010*a^4*b^9*c*x^(11*n) + 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) + 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) + 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) + 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) + 2002*a^9*c
^5*x^(10*n) + 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) + 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) + 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) + 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) + 200
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) + 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) + 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) + 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) + 364*a^11*c^3*x^(6*n) + 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) +
1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) + 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) + 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) + 14*a^13*c*x^(2*n) + 14*a^13*b*x^n)/n

Mupad [B]
time = 5.78, size = 1395, normalized size = 60.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^13,x)
```

```
[Out] x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 + 286*a^3*b^10*
c + (6435*a^4*b^8*c^2)/2 + 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 + 5148*a^7
*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 + 286*a*b^1
0*c^3 + (6435*a^2*b^8*c^4)/2 + 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 + 5148
*a^5*b^2*c^7))/n + (x^(12*n + 1)*(a*b^13 + 78*a^2*b^11*c + 1716*a^7*b*c^6 +
 1430*a^3*b^9*c^2 + 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 + 12012*a^6*b^3*c^
5))/n + (x^(14*n + 1)*(b^13*c + 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 1430*a^2*b
^9*c^3 + 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 + 12012*a^5*b^3*c^6))/n + (x^
(5*n + 1)*((429*a^8*b^6)/2 + 26*a^11*c^3 + 715*a^9*b^4*c + 429*a^10*b^2*c^2
))/n + (x^(21*n + 1)*(26*a^3*c^11 + (429*b^6*c^8)/2 + 715*a*b^4*c^9 + 429*a
^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 + 143*a^9*c^5 + 1287*a^5*b
^8*c + 6006*a^6*b^6*c^2 + 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2))/n + (x^
(17*n + 1)*(143*a^5*c^9 + (143*b^10*c^4)/2 + 1287*a*b^8*c^5 + 6006*a^2*b^6*
c^6 + 8580*a^3*b^4*c^7 + (6435*a^4*b^2*c^8)/2))/n + (x^(13*n + 1)*(b^14/14
+ (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 + 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^
4 + 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 + 13*a*b^12*c))/n + (x^(7*n + 1)*(
(429*a^6*b^8)/2 + (143*a^10*c^4)/2 + 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2
+ 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2 + (429*b^8*c^6)/2
+ 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 + 1430*a^3*b^2*c^9))/n + (c^14*x^(2
7*n + 1))/(14*n) + (a^12*x^(n + 1)*(a*c + (13*b^2)/2))/n + (a^10*x^(3*n + 1
)*((143*b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (c^10*x^(23*n + 1)*((143
*b^4)/2 + (13*a^2*c^2)/2 + 78*a*b^2*c))/n + (b*c^13*x^(26*n + 1))/n + (c^12
*x^(25*n + 1)*(a*c + (13*b^2)/2))/n + (a^13*b*x)/n + (143*a^7*b*x^(6*n + 1)
*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (143*b*c^7*x^
(20*n + 1)*(12*b^6 + 14*a^3*c^3 + 70*a^2*b^2*c^2 + 63*a*b^4*c))/(7*n) + (14
3*a^5*b*x^(8*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 + 30*a^3*b^2*c^3 + 12
*a*b^6*c))/n + (143*b*c^5*x^(18*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 +
30*a^3*b^2*c^3 + 12*a*b^6*c))/n + (13*a^3*b*x^(10*n + 1)*(2*b^10 + 99*a^5*c
^5 + 396*a^2*b^6*c^2 + 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n +
 (13*b*c^3*x^(16*n + 1)*(2*b^10 + 99*a^5*c^5 + 396*a^2*b^6*c^2 + 924*a^3*b^
4*c^3 + 660*a^4*b^2*c^4 + 55*a*b^8*c))/n + (13*a^9*b*x^(4*n + 1)*(11*b^4 +
6*a^2*c^2 + 22*a*b^2*c))/n + (13*b*c^9*x^(22*n + 1)*(11*b^4 + 6*a^2*c^2 + 2
2*a*b^2*c))/n + (13*a^11*b*x^(2*n + 1)*(a*c + 2*b^2))/n + (13*b*c^11*x^(24*
n + 1)*(a*c + 2*b^2))/n)
```

3.97 $\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=18

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

```
[Out] 1/14*(-c*x^2-b*x+a)^14
```

Rubi [A]
time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {643}

$$\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (a - b*x - c*x^2)^14/14
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx=\frac{1}{14}\left(a-bx-cx^2\right)^{14}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(18) = 36.
time = 0.12, size = 201, normalized size = 11.17

$$\frac{1}{14}x(b+cx)\left(-14a^{13}+91a^{12}x(b+cx)-364a^{11}x^2(b+cx)^2+1001a^{10}x^3(b+cx)^3-2002a^9x^4(b+cx)^4+3003a^8x^5(b+cx)^5-3432a^7x^6(b+cx)^6+3003a^6x^7(b+cx)^7-2002a^5x^8(b+cx)^8+1001a^4x^9(b+cx)^9-364a^3x^{10}(b+cx)^{10}+91a^2x^{11}(b+cx)^{11}-14ax^{12}(b+cx)^{12}+x^{13}(b+cx)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^13,x]
```

```
[Out] (x*(b + c*x)*(-14*a^13 + 91*a^12*x*(b + c*x) - 364*a^11*x^2*(b + c*x)^2 + 1
001*a^10*x^3*(b + c*x)^3 - 2002*a^9*x^4*(b + c*x)^4 + 3003*a^8*x^5*(b + c*x
)^5 - 3432*a^7*x^6*(b + c*x)^6 + 3003*a^6*x^7*(b + c*x)^7 - 2002*a^5*x^8*(b
```

```
 + c*x)^8 + 1001*a^4*x^9*(b + c*x)^9 - 364*a^3*x^10*(b + c*x)^10 + 91*a^2*x
^11*(b + c*x)^11 - 14*a*x^12*(b + c*x)^12 + x^13*(b + c*x)^13))/14
```

Maple [A]
time = 0.24, size = 17, normalized size = 0.94 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*(c*x^2+b*x-a)^14
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.89

$$\frac{1}{14}\left(cx^2+bx-a\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="maxima")
```

```
[Out] 1/14*(c*x^2 + b*x - a)^14
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1238 vs. 2(16) = 32.
time = 0.36, size = 1238, normalized size = 68.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="fricas")
```

```
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 1/2*(13*b^2*c^12 - 2*a*c^13)*x^26 + 13*(2*b^
3*c^11 - a*b*c^12)*x^25 + 13/2*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^12)*x^2
4 + 13*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^23 + 13/2*(33*b^6*c^8
- 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^22 + 143/7*(12*b^7*c^7 -
63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^21 + 143/2*(3*b^8*c^6 - 24
*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^20 + 143*(b^9*c^
5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^19 + 14
3/2*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4*b^
2*c^8 - 2*a^5*c^9)*x^18 + 13*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 -
 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^17 + 13/2*(b^12*c^2 -
44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792
*a^5*b^2*c^7 + 33*a^6*c^8)*x^16 + (b^13*c - 78*a*b^11*c^2 + 1430*a^2*b^9*c^
3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a^6*b*c
^7)*x^15 - a^13*b*x + 1/14*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060
*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6
- 3432*a^7*c^7)*x^14 - (a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^
```

```
4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^13 +
13/2*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*
a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^12 - 13*(2*a^3*b^11 - 55*a^4*
b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 99*a^8*b*c^5)
*x^11 + 143/2*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a^7*b^4*c^3 +
 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^10 - 143*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*b^
5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^9 + 143/2*(3*a^6*b^8 - 24*a^7*b^6*c
 + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^8 - 143/7*(12*a^7*b^7 - 63
*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^7 + 13/2*(33*a^8*b^6 - 110*a
^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^6 - 13*(11*a^9*b^5 - 22*a^10*b^3
*c + 6*a^11*b*c^2)*x^5 + 13/2*(11*a^10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^4
- 13*(2*a^11*b^3 - a^12*b*c)*x^3 + 1/2*(13*a^12*b^2 - 2*a^13*c)*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1326 vs. $2(12) = 24$.

time = 0.15, size = 1326, normalized size = 73.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**13,x)
```

```
[Out] -a**13*b*x + b*c**13*x**27 + c**14*x**28/14 + x**26*(-a*c**13 + 13*b**2*c**
12/2) + x**25*(-13*a*b*c**12 + 26*b**3*c**11) + x**24*(13*a**2*c**12/2 - 78
*a*b**2*c**11 + 143*b**4*c**10/2) + x**23*(78*a**2*b*c**11 - 286*a*b**3*c**
10 + 143*b**5*c**9) + x**22*(-26*a**3*c**11 + 429*a**2*b**2*c**10 - 715*a*b
**4*c**9 + 429*b**6*c**8/2) + x**21*(-286*a**3*b*c**10 + 1430*a**2*b**3*c**
9 - 1287*a*b**5*c**8 + 1716*b**7*c**7/7) + x**20*(143*a**4*c**10/2 - 1430*a
**3*b**2*c**9 + 6435*a**2*b**4*c**8/2 - 1716*a*b**6*c**7 + 429*b**8*c**6/2)
 + x**19*(715*a**4*b*c**9 - 4290*a**3*b**3*c**8 + 5148*a**2*b**5*c**7 - 171
6*a*b**7*c**6 + 143*b**9*c**5) + x**18*(-143*a**5*c**9 + 6435*a**4*b**2*c**
8/2 - 8580*a**3*b**4*c**7 + 6006*a**2*b**6*c**6 - 1287*a*b**8*c**5 + 143*b*
*10*c**4/2) + x**17*(-1287*a**5*b*c**8 + 8580*a**4*b**3*c**7 - 12012*a**3*b
**5*c**6 + 5148*a**2*b**7*c**5 - 715*a*b**9*c**4 + 26*b**11*c**3) + x**16*(
429*a**6*c**8/2 - 5148*a**5*b**2*c**7 + 15015*a**4*b**4*c**6 - 12012*a**3*b
**6*c**5 + 6435*a**2*b**8*c**4/2 - 286*a*b**10*c**3 + 13*b**12*c**2/2) + x*
*15*(1716*a**6*b*c**7 - 12012*a**5*b**3*c**6 + 18018*a**4*b**5*c**5 - 8580*
a**3*b**7*c**4 + 1430*a**2*b**9*c**3 - 78*a*b**11*c**2 + b**13*c) + x**14*(
-1716*a**7*c**7/7 + 6006*a**6*b**2*c**6 - 18018*a**5*b**4*c**5 + 15015*a**4
*b**6*c**4 - 4290*a**3*b**8*c**3 + 429*a**2*b**10*c**2 - 13*a*b**12*c + b**
14/14) + x**13*(-1716*a**7*b*c**6 + 12012*a**6*b**3*c**5 - 18018*a**5*b**5*
c**4 + 8580*a**4*b**7*c**3 - 1430*a**3*b**9*c**2 + 78*a**2*b**11*c - a*b**1
3) + x**12*(429*a**8*c**6/2 - 5148*a**7*b**2*c**5 + 15015*a**6*b**4*c**4 -
12012*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/2 - 286*a**3*b**10*c + 13*a**2*b
**12/2) + x**11*(1287*a**8*b*c**5 - 8580*a**7*b**3*c**4 + 12012*a**6*b**5*c
```

```
**3 - 5148*a**5*b**7*c**2 + 715*a**4*b**9*c - 26*a**3*b**11) + x**10*(-143*
a**9*c**5 + 6435*a**8*b**2*c**4/2 - 8580*a**7*b**4*c**3 + 6006*a**6*b**6*c*
*2 - 1287*a**5*b**8*c + 143*a**4*b**10/2) + x**9*(-715*a**9*b*c**4 + 4290*a
**8*b**3*c**3 - 5148*a**7*b**5*c**2 + 1716*a**6*b**7*c - 143*a**5*b**9) + x
**8*(143*a**10*c**4/2 - 1430*a**9*b**2*c**3 + 6435*a**8*b**4*c**2/2 - 1716*
a**7*b**6*c + 429*a**6*b**8/2) + x**7*(286*a**10*b*c**3 - 1430*a**9*b**3*c*
*2 + 1287*a**8*b**5*c - 1716*a**7*b**7/7) + x**6*(-26*a**11*c**3 + 429*a**1
0*b**2*c**2 - 715*a**9*b**4*c + 429*a**8*b**6/2) + x**5*(-78*a**11*b*c**2 +
 286*a**10*b**3*c - 143*a**9*b**5) + x**4*(13*a**12*c**2/2 - 78*a**11*b**2*
c + 143*a**10*b**4/2) + x**3*(13*a**12*b*c - 26*a**11*b**3) + x**2*(-a**13*
c + 13*a**12*b**2/2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(16) = 32$.
time = 2.09, size = 218, normalized size = 12.11

$$\frac{1}{14}\left(cx^2+bx\right)^{14}-\left(cx^2+bx\right)^{13}a+\frac{13}{2}\left(cx^2+bx\right)^{12}a^2-26\left(cx^2+bx\right)^{11}a^3+\frac{143}{2}\left(cx^2+bx\right)^{10}a^4-143\left(cx^2+bx\right)^9a^5+\frac{429}{2}\left(cx^2+bx\right)^8a^6-\frac{1716}{7}\left(cx^2+bx\right)^7a^7+\frac{429}{2}\left(cx^2+bx\right)^6a^8-143\left(cx^2+bx\right)^5a^9+\frac{143}{2}\left(cx^2+bx\right)^4a^{10}-26\left(cx^2+bx\right)^3a^{11}+\frac{13}{2}\left(cx^2+bx\right)^2a^{12}-\left(cx^2+bx\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14 - (c*x^2 + b*x)^13*a + 13/2*(c*x^2 + b*x)^12*a^2 - 26
*(c*x^2 + b*x)^11*a^3 + 143/2*(c*x^2 + b*x)^10*a^4 - 143*(c*x^2 + b*x)^9*a^
5 + 429/2*(c*x^2 + b*x)^8*a^6 - 1716/7*(c*x^2 + b*x)^7*a^7 + 429/2*(c*x^2 +
 b*x)^6*a^8 - 143*(c*x^2 + b*x)^5*a^9 + 143/2*(c*x^2 + b*x)^4*a^10 - 26*(c*
x^2 + b*x)^3*a^11 + 13/2*(c*x^2 + b*x)^2*a^12 - (c*x^2 + b*x)*a^13
```

Mupad [B]
time = 1.38, size = 1208, normalized size = 67.11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(b*x - a + c*x^2)^13,x)
```

```
[Out] x^12*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*c + (6435*a^4*b^8*c^
2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7*b^2*c^5) + x^16*((4
29*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*b^10*c^3 + (6435*a^2*b^8*c^4)/2 - 1
2012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 - 5148*a^5*b^2*c^7) - x^13*(a*b^13 - 7
8*a^2*b^11*c + 1716*a^7*b*c^6 + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018
*a^5*b^5*c^4 - 12012*a^6*b^3*c^5) + x^15*(b^13*c - 78*a*b^11*c^2 + 1716*a^6
*b*c^7 + 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^
5*b^3*c^6) + x^6*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 429*a^10*
b^2*c^2) - x^22*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c^9 - 429*a^2*b^
2*c^10) + x^10*((143*a^4*b^10)/2 - 143*a^9*c^5 - 1287*a^5*b^8*c + 6006*a^6*
b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2) - x^18*(143*a^5*c^9 - (1
```

```
43*b^10*c^4)/2 + 1287*a*b^8*c^5 - 6006*a^2*b^6*c^6 + 8580*a^3*b^4*c^7 - (64
35*a^4*b^2*c^8)/2) + x^14*(b^14/14 - (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 -
4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^4 - 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6
 - 13*a*b^12*c) + x^8*((429*a^6*b^8)/2 + (143*a^10*c^4)/2 - 1716*a^7*b^6*c
+ (6435*a^8*b^4*c^2)/2 - 1430*a^9*b^2*c^3) + x^20*((143*a^4*c^10)/2 + (429*
b^8*c^6)/2 - 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c^9) + (c
^14*x^28)/14 - x^2*(a^13*c - (13*a^12*b^2)/2) + (13*a^10*x^4*(11*b^4 + a^2*
c^2 - 12*a*b^2*c))/2 + (13*c^10*x^24*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/2 + b
*c^13*x^27 - (c^12*x^26*(2*a*c - 13*b^2))/2 - a^13*b*x - (143*a^7*b*x^7*(12
*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/7 + (143*b*c^7*x^21*(12*b
^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/7 - 143*a^5*b*x^9*(b^8 + 5*
a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c) + 143*b*c^5*x^19*(b
^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c) - 13*a^3*b*x
^11*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*
c^4 - 55*a*b^8*c) + 13*b*c^3*x^17*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 -
924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c) - 13*a^9*b*x^5*(11*b^4 + 6*
a^2*c^2 - 22*a*b^2*c) + 13*b*c^9*x^23*(11*b^4 + 6*a^2*c^2 - 22*a*b^2*c) + 1
3*a^11*b*x^3*(a*c - 2*b^2) - 13*b*c^11*x^25*(a*c - 2*b^2)
```

3.98 $\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=20

$$\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

```
[Out] 1/28*(-c*x^4-b*x^2+a)^14
```

Rubi [A]
time = 0.21, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1261, 643}

$$\frac{1}{28}\left(a-bx^2-cx^4\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (a - b*x^2 - c*x^4)^14/28
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^{13}\,dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx,x,x^2\right)\\ &= \frac{1}{28}\left(a-bx^2-cx^4\right)^{14}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(20) = 40.
time = 0.12, size = 233, normalized size = 11.65

$$\frac{1}{28}x^2\left(b+cx^2\right)\left(-14a^{13}+91a^{12}x^2\left(b+cx^2\right)-364a^{11}x^4\left(b+cx^2\right)^2+1001a^{10}x^6\left(b+cx^2\right)^3-2002a^9x^8\left(b+cx^2\right)^4+3003a^8x^{10}\left(b+cx^2\right)^5-3432a^7x^{12}\left(b+cx^2\right)^6+3003a^6x^{14}\left(b+cx^2\right)^7-2002a^5x^{16}\left(b+cx^2\right)^8+1001a^4x^{18}\left(b+cx^2\right)^9-364a^3x^{20}\left(b+cx^2\right)^{10}+91a^2x^{22}\left(b+cx^2\right)^{11}-14ax^{24}\left(b+cx^2\right)^{12}+x^{26}\left(b+cx^2\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^13,x]
```

```
[Out] (x^2*(b + c*x^2)*(-14*a^13 + 91*a^12*x^2*(b + c*x^2) - 364*a^11*x^4*(b + c*
x^2)^2 + 1001*a^10*x^6*(b + c*x^2)^3 - 2002*a^9*x^8*(b + c*x^2)^4 + 3003*a^
8*x^10*(b + c*x^2)^5 - 3432*a^7*x^12*(b + c*x^2)^6 + 3003*a^6*x^14*(b + c*x
^2)^7 - 2002*a^5*x^16*(b + c*x^2)^8 + 1001*a^4*x^18*(b + c*x^2)^9 - 364*a^3
*x^20*(b + c*x^2)^10 + 91*a^2*x^22*(b + c*x^2)^11 - 14*a*x^24*(b + c*x^2)^1
2 + x^26*(b + c*x^2)^13))/28
```

Maple [A]
time = 0.07, size = 19, normalized size = 0.95

method	result
default	$\frac{\left(c\,x^{4}+b\,x^{2}-a\right)^{14}}{28}$
gosper	$-\frac{x^{2}\left(-c^{14}x^{54}-14b\,c^{13}x^{52}+14x^{50}a\,c^{13}-91x^{50}b^{2}c^{12}+182x^{48}ab\,c^{12}-364x^{48}b^{3}c^{11}-91x^{46}a^{2}c^{12}+1092x^{46}a\,b^{2}c^{11}-1001x^{46}b^{4}c^{10}-1092\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*(c*x^4+b*x^2-a)^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. 2(18) = 36.
time = 0.29, size = 1242, normalized size = 62.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78*a*b^11*c^2
```

```
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 - 13/2*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^20 - 143/2*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^16 - 1/2*a^
13*b*x^2 - 143/14*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)
*x^12 - 13/2*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^8 - 13/2*(2*a^11*b^3 - a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 - 2*a^13*c)*x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(18) = 36$.

time = 0.40, size = 1242, normalized size = 62.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="fricas")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 1/4*(13*b^2*c^12 - 2*a*c^13)*x^52 + 13/2
*(2*b^3*c^11 - a*b*c^12)*x^50 + 13/4*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^48 + 13/2*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^46 + 13/4*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^44 + 143/14*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^42 + 143/4*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^40 + 14
3/2*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^38 + 143/4*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^36 + 13/2*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^34 + 13/4
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^32 + 1/2*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^30 + 1/28*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^28 - 1/2*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^26 + 13/4*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
```

```
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^24 - 13/2*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^22 + 143/4*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^20 - 143/2*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^18 + 143/4*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^16 - 1/2*a^
13*b*x^2 - 143/14*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c
^3)*x^14 + 13/4*(33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)
*x^12 - 13/2*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^10 + 13/4*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^8 - 13/2*(2*a^11*b^3 - a^12*b*c)*x^6 +
 1/4*(13*a^12*b^2 - 2*a^13*c)*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1384 vs. $2(14) = 28$.
time = 0.17, size = 1384, normalized size = 69.20

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**13,x)
```

```
[Out] -a**13*b*x**2/2 + b*c**13*x**54/2 + c**14*x**56/28 + x**52*(-a*c**13/2 + 13
*b**2*c**12/4) + x**50*(-13*a*b*c**12/2 + 13*b**3*c**11) + x**48*(13*a**2*c
**12/4 - 39*a*b**2*c**11 + 143*b**4*c**10/4) + x**46*(39*a**2*b*c**11 - 143
*a*b**3*c**10 + 143*b**5*c**9/2) + x**44*(-13*a**3*c**11 + 429*a**2*b**2*c*
*10/2 - 715*a*b**4*c**9/2 + 429*b**6*c**8/4) + x**42*(-143*a**3*b*c**10 + 7
15*a**2*b**3*c**9 - 1287*a*b**5*c**8/2 + 858*b**7*c**7/7) + x**40*(143*a**4
*c**10/4 - 715*a**3*b**2*c**9 + 6435*a**2*b**4*c**8/4 - 858*a*b**6*c**7 + 4
29*b**8*c**6/4) + x**38*(715*a**4*b*c**9/2 - 2145*a**3*b**3*c**8 + 2574*a**
2*b**5*c**7 - 858*a*b**7*c**6 + 143*b**9*c**5/2) + x**36*(-143*a**5*c**9/2
+ 6435*a**4*b**2*c**8/4 - 4290*a**3*b**4*c**7 + 3003*a**2*b**6*c**6 - 1287*
a*b**8*c**5/2 + 143*b**10*c**4/4) + x**34*(-1287*a**5*b*c**8/2 + 4290*a**4*
b**3*c**7 - 6006*a**3*b**5*c**6 + 2574*a**2*b**7*c**5 - 715*a*b**9*c**4/2 +
 13*b**11*c**3) + x**32*(429*a**6*c**8/4 - 2574*a**5*b**2*c**7 + 15015*a**4
*b**4*c**6/2 - 6006*a**3*b**6*c**5 + 6435*a**2*b**8*c**4/4 - 143*a*b**10*c*
*3 + 13*b**12*c**2/4) + x**30*(858*a**6*b*c**7 - 6006*a**5*b**3*c**6 + 9009
*a**4*b**5*c**5 - 4290*a**3*b**7*c**4 + 715*a**2*b**9*c**3 - 39*a*b**11*c**
2 + b**13*c/2) + x**28*(-858*a**7*c**7/7 + 3003*a**6*b**2*c**6 - 9009*a**5*
b**4*c**5 + 15015*a**4*b**6*c**4/2 - 2145*a**3*b**8*c**3 + 429*a**2*b**10*c
**2/2 - 13*a*b**12*c/2 + b**14/28) + x**26*(-858*a**7*b*c**6 + 6006*a**6*b*
*3*c**5 - 9009*a**5*b**5*c**4 + 4290*a**4*b**7*c**3 - 715*a**3*b**9*c**2 +
39*a**2*b**11*c - a*b**13/2) + x**24*(429*a**8*c**6/4 - 2574*a**7*b**2*c**5
 + 15015*a**6*b**4*c**4/2 - 6006*a**5*b**6*c**3 + 6435*a**4*b**8*c**2/4 - 1
43*a**3*b**10*c + 13*a**2*b**12/4) + x**22*(1287*a**8*b*c**5/2 - 4290*a**7*
b**3*c**4 + 6006*a**6*b**5*c**3 - 2574*a**5*b**7*c**2 + 715*a**4*b**9*c/2 -
```

```
 13*a**3*b**11) + x**20*(-143*a**9*c**5/2 + 6435*a**8*b**2*c**4/4 - 4290*a*
*7*b**4*c**3 + 3003*a**6*b**6*c**2 - 1287*a**5*b**8*c/2 + 143*a**4*b**10/4)
 + x**18*(-715*a**9*b*c**4/2 + 2145*a**8*b**3*c**3 - 2574*a**7*b**5*c**2 +
858*a**6*b**7*c - 143*a**5*b**9/2) + x**16*(143*a**10*c**4/4 - 715*a**9*b**
2*c**3 + 6435*a**8*b**4*c**2/4 - 858*a**7*b**6*c + 429*a**6*b**8/4) + x**14
*(143*a**10*b*c**3 - 715*a**9*b**3*c**2 + 1287*a**8*b**5*c/2 - 858*a**7*b**
7/7) + x**12*(-13*a**11*c**3 + 429*a**10*b**2*c**2/2 - 715*a**9*b**4*c/2 +
429*a**8*b**6/4) + x**10*(-39*a**11*b*c**2 + 143*a**10*b**3*c - 143*a**9*b*
*5/2) + x**8*(13*a**12*c**2/4 - 39*a**11*b**2*c + 143*a**10*b**4/4) + x**6*
(13*a**12*b*c/2 - 13*a**11*b**3) + x**4*(-a**13*c/2 + 13*a**12*b**2/4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(18) = 36$.
time = 4.80, size = 246, normalized size = 12.30

$$\frac{1}{28}\left(cx^4+bx^2\right)^{14}-\frac{1}{2}\left(cx^4+bx^2\right)^{13}a+\frac{13}{4}\left(cx^4+bx^2\right)^{12}a^2-13\left(cx^4+bx^2\right)^{11}a^3+\frac{143}{4}\left(cx^4+bx^2\right)^{10}a^4-\frac{143}{2}\left(cx^4+bx^2\right)^{9}a^5+\frac{429}{4}\left(cx^4+bx^2\right)^{8}a^6-\frac{858}{7}\left(cx^4+bx^2\right)^{7}a^7+\frac{429}{4}\left(cx^4+bx^2\right)^{6}a^8-\frac{143}{2}\left(cx^4+bx^2\right)^{5}a^9+\frac{143}{4}\left(cx^4+bx^2\right)^{4}a^{10}-13\left(cx^4+bx^2\right)^{3}a^{11}+\frac{13}{4}\left(cx^4+bx^2\right)^{2}a^{12}-\frac{1}{2}\left(cx^4+bx^2\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14 - 1/2*(c*x^4 + b*x^2)^13*a + 13/4*(c*x^4 + b*x^2)^1
2*a^2 - 13*(c*x^4 + b*x^2)^11*a^3 + 143/4*(c*x^4 + b*x^2)^10*a^4 - 143/2*(c
*x^4 + b*x^2)^9*a^5 + 429/4*(c*x^4 + b*x^2)^8*a^6 - 858/7*(c*x^4 + b*x^2)^7
*a^7 + 429/4*(c*x^4 + b*x^2)^6*a^8 - 143/2*(c*x^4 + b*x^2)^5*a^9 + 143/4*(c
*x^4 + b*x^2)^4*a^10 - 13*(c*x^4 + b*x^2)^3*a^11 + 13/4*(c*x^4 + b*x^2)^2*a
^12 - 1/2*(c*x^4 + b*x^2)*a^13
```

Mupad [B]
time = 3.25, size = 1214, normalized size = 60.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^13,x)
```

```
[Out] x^24*((13*a^2*b^12)/4 + (429*a^8*c^6)/4 - 143*a^3*b^10*c + (6435*a^4*b^8*c^
2)/4 - 6006*a^5*b^6*c^3 + (15015*a^6*b^4*c^4)/2 - 2574*a^7*b^2*c^5) + x^32*
((429*a^6*c^8)/4 + (13*b^12*c^2)/4 - 143*a*b^10*c^3 + (6435*a^2*b^8*c^4)/4
- 6006*a^3*b^6*c^5 + (15015*a^4*b^4*c^6)/2 - 2574*a^5*b^2*c^7) - x^26*((a*b
^13)/2 - 39*a^2*b^11*c + 858*a^7*b*c^6 + 715*a^3*b^9*c^2 - 4290*a^4*b^7*c^3
 + 9009*a^5*b^5*c^4 - 6006*a^6*b^3*c^5) + x^30*((b^13*c)/2 - 39*a*b^11*c^2
+ 858*a^6*b*c^7 + 715*a^2*b^9*c^3 - 4290*a^3*b^7*c^4 + 9009*a^4*b^5*c^5 - 6
006*a^5*b^3*c^6) + x^12*((429*a^8*b^6)/4 - 13*a^11*c^3 - (715*a^9*b^4*c)/2
+ (429*a^10*b^2*c^2)/2) - x^44*(13*a^3*c^11 - (429*b^6*c^8)/4 + (715*a*b^4*
c^9)/2 - (429*a^2*b^2*c^10)/2) + x^20*((143*a^4*b^10)/4 - (143*a^9*c^5)/2 -
 (1287*a^5*b^8*c)/2 + 3003*a^6*b^6*c^2 - 4290*a^7*b^4*c^3 + (6435*a^8*b^2*c
```

```
^4)/4) - x^36*((143*a^5*c^9)/2 - (143*b^10*c^4)/4 + (1287*a*b^8*c^5)/2 - 30
03*a^2*b^6*c^6 + 4290*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/4) + x^28*(b^14/28 -
 (858*a^7*c^7)/7 + (429*a^2*b^10*c^2)/2 - 2145*a^3*b^8*c^3 + (15015*a^4*b^6
*c^4)/2 - 9009*a^5*b^4*c^5 + 3003*a^6*b^2*c^6 - (13*a*b^12*c)/2) + x^16*((4
29*a^6*b^8)/4 + (143*a^10*c^4)/4 - 858*a^7*b^6*c + (6435*a^8*b^4*c^2)/4 - 7
15*a^9*b^2*c^3) + x^40*((143*a^4*c^10)/4 + (429*b^8*c^6)/4 - 858*a*b^6*c^7
+ (6435*a^2*b^4*c^8)/4 - 715*a^3*b^2*c^9) + (c^14*x^56)/28 - x^4*((a^13*c)/
2 - (13*a^12*b^2)/4) + (13*a^10*x^8*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 + (1
3*c^10*x^48*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/4 - (a^13*b*x^2)/2 + (b*c^13*x
^54)/2 - (c^12*x^52*(2*a*c - 13*b^2))/4 - (143*a^7*b*x^14*(12*b^6 - 14*a^3*
c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 + (143*b*c^7*x^42*(12*b^6 - 14*a^3*c
^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/14 - (143*a^5*b*x^18*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 + (143*b*c^5*x^38*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/2 - (13*a^3*b*x^2
2*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 - 55*a*b^8*c))/2 + (13*b*c^3*x^34*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2
- 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/2 - (13*a^9*b*x^10*(11*b
^4 + 6*a^2*c^2 - 22*a*b^2*c))/2 + (13*b*c^9*x^46*(11*b^4 + 6*a^2*c^2 - 22*a
*b^2*c))/2 + (13*a^11*b*x^6*(a*c - 2*b^2))/2 - (13*b*c^11*x^50*(a*c - 2*b^2
))/2
```

3.99 $\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^{13}\,dx$

Optimal. Leaf size=20

$$\frac{1}{42}\left(a-bx^3-cx^6\right)^{14}$$

```
[Out] 1/42*(-c*x^6-b*x^3+a)^14
```

Rubi [A]
time = 0.20, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1482, 643}

$$\frac{1}{42}\left(a-bx^3-cx^6\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (a - b*x^3 - c*x^6)^14/42
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^{13}\,dx &= \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx,x,x^3\right)\\ &= \frac{1}{42}\left(a-bx^3-cx^6\right)^{14}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 233 vs. 2(20) = 40.
time = 0.12, size = 233, normalized size = 11.65

$$\frac{1}{42}x^3\left(b+cx^3\right)\left(-14a^{13}+91a^{12}x^3\left(b+cx^3\right)-364a^{11}x^6\left(b+cx^3\right)^2+1001a^{10}x^9\left(b+cx^3\right)^3-2002a^9x^{12}\left(b+cx^3\right)^4+3003a^8x^{15}\left(b+cx^3\right)^5-3432a^7x^{18}\left(b+cx^3\right)^6+3003a^6x^{21}\left(b+cx^3\right)^7-2002a^5x^{24}\left(b+cx^3\right)^8+1001a^4x^{27}\left(b+cx^3\right)^9-364a^3x^{30}\left(b+cx^3\right)^{10}+91a^2x^{33}\left(b+cx^3\right)^{11}-14ax^{36}\left(b+cx^3\right)^{12}+x^{39}\left(b+cx^3\right)^{13}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^3*(b + c*x^3)*(-14*a^13 + 91*a^12*x^3*(b + c*x^3) - 364*a^11*x^6*(b + c*
x^3)^2 + 1001*a^10*x^9*(b + c*x^3)^3 - 2002*a^9*x^12*(b + c*x^3)^4 + 3003*a
^8*x^15*(b + c*x^3)^5 - 3432*a^7*x^18*(b + c*x^3)^6 + 3003*a^6*x^21*(b + c*
x^3)^7 - 2002*a^5*x^24*(b + c*x^3)^8 + 1001*a^4*x^27*(b + c*x^3)^9 - 364*a^
3*x^30*(b + c*x^3)^10 + 91*a^2*x^33*(b + c*x^3)^11 - 14*a*x^36*(b + c*x^3)^
12 + x^39*(b + c*x^3)^13))/42
```

Maple [A]
time = 0.09, size = 19, normalized size = 0.95

method	result
default	$\frac{\left(c\,x^{6}+b\,x^{3}-a\right)^{14}}{42}$
gosper	$-\frac{x^{3}\left(-c^{14}x^{81}-14b\,c^{13}x^{78}+14x^{75}a\,c^{13}-91x^{75}b^{2}c^{12}+182x^{72}ab\,c^{12}-364x^{72}b^{3}c^{11}-91x^{69}a^{2}c^{12}+1092x^{69}a\,b^{2}c^{11}-1001x^{69}b^{4}c^{10}-1092\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/42*(c*x^6+b*x^3-a)^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. 2(18) = 36.
time = 0.29, size = 1242, normalized size = 62.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78*a*b^11*c^2
```

```
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 - 13/3*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^30 - 143/3*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^24 - 143/21
*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^18 - 1/3*a^13*
b*x^3 - 13/3*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^12 - 13/3*(2*a^11*b^3 - a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 - 2*a^13*c)*x^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. $2(18) = 36$.
time = 0.33, size = 1242, normalized size = 62.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="fricas")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 1/6*(13*b^2*c^12 - 2*a*c^13)*x^78 + 13/3
*(2*b^3*c^11 - a*b*c^12)*x^75 + 13/6*(11*b^4*c^10 - 12*a*b^2*c^11 + a^2*c^1
2)*x^72 + 13/3*(11*b^5*c^9 - 22*a*b^3*c^10 + 6*a^2*b*c^11)*x^69 + 13/6*(33*
b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 - 4*a^3*c^11)*x^66 + 143/21*(12*b
^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 14*a^3*b*c^10)*x^63 + 143/6*(3*b^8
*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 - 20*a^3*b^2*c^9 + a^4*c^10)*x^60 + 14
3/3*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9
)*x^57 + 143/6*(b^10*c^4 - 18*a*b^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7
+ 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^54 + 13/3*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*
a^2*b^7*c^5 - 924*a^3*b^5*c^6 + 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^51 + 13/6
*(b^12*c^2 - 44*a*b^10*c^3 + 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*
b^4*c^6 - 792*a^5*b^2*c^7 + 33*a^6*c^8)*x^48 + 1/3*(b^13*c - 78*a*b^11*c^2
+ 1430*a^2*b^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c
^6 + 1716*a^6*b*c^7)*x^45 + 1/42*(b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 -
 60060*a^3*b^8*c^3 + 210210*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^
2*c^6 - 3432*a^7*c^7)*x^42 - 1/3*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2
 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^
6)*x^39 + 13/6*(a^2*b^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c
```

```
^3 + 2310*a^6*b^4*c^4 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^36 - 13/3*(2*a^3*b^
11 - 55*a^4*b^9*c + 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 9
9*a^8*b*c^5)*x^33 + 143/6*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a
^7*b^4*c^3 + 45*a^8*b^2*c^4 - 2*a^9*c^5)*x^30 - 143/3*(a^5*b^9 - 12*a^6*b^7
*c + 36*a^7*b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^27 + 143/6*(3*a^6*b^8
 - 24*a^7*b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^24 - 143/21
*(12*a^7*b^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^21 + 13/6*(
33*a^8*b^6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^18 - 1/3*a^13*
b*x^3 - 13/3*(11*a^9*b^5 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^15 + 13/6*(11*a^
10*b^4 - 12*a^11*b^2*c + a^12*c^2)*x^12 - 13/3*(2*a^11*b^3 - a^12*b*c)*x^9
+ 1/6*(13*a^12*b^2 - 2*a^13*c)*x^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1394 vs. $2(14) = 28$.
time = 0.15, size = 1394, normalized size = 69.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**13,x)
```

```
[Out] -a**13*b*x**3/3 + b*c**13*x**81/3 + c**14*x**84/42 + x**78*(-a*c**13/3 + 13
*b**2*c**12/6) + x**75*(-13*a*b*c**12/3 + 26*b**3*c**11/3) + x**72*(13*a**2
*c**12/6 - 26*a*b**2*c**11 + 143*b**4*c**10/6) + x**69*(26*a**2*b*c**11 - 2
86*a*b**3*c**10/3 + 143*b**5*c**9/3) + x**66*(-26*a**3*c**11/3 + 143*a**2*b
**2*c**10 - 715*a*b**4*c**9/3 + 143*b**6*c**8/2) + x**63*(-286*a**3*b*c**10
/3 + 1430*a**2*b**3*c**9/3 - 429*a*b**5*c**8 + 572*b**7*c**7/7) + x**60*(14
3*a**4*c**10/6 - 1430*a**3*b**2*c**9/3 + 2145*a**2*b**4*c**8/2 - 572*a*b**6
*c**7 + 143*b**8*c**6/2) + x**57*(715*a**4*b*c**9/3 - 1430*a**3*b**3*c**8 +
 1716*a**2*b**5*c**7 - 572*a*b**7*c**6 + 143*b**9*c**5/3) + x**54*(-143*a**
5*c**9/3 + 2145*a**4*b**2*c**8/2 - 2860*a**3*b**4*c**7 + 2002*a**2*b**6*c**
6 - 429*a*b**8*c**5 + 143*b**10*c**4/6) + x**51*(-429*a**5*b*c**8 + 2860*a*
*4*b**3*c**7 - 4004*a**3*b**5*c**6 + 1716*a**2*b**7*c**5 - 715*a*b**9*c**4/
3 + 26*b**11*c**3/3) + x**48*(143*a**6*c**8/2 - 1716*a**5*b**2*c**7 + 5005*
a**4*b**4*c**6 - 4004*a**3*b**6*c**5 + 2145*a**2*b**8*c**4/2 - 286*a*b**10*
c**3/3 + 13*b**12*c**2/6) + x**45*(572*a**6*b*c**7 - 4004*a**5*b**3*c**6 +
6006*a**4*b**5*c**5 - 2860*a**3*b**7*c**4 + 1430*a**2*b**9*c**3/3 - 26*a*b*
*11*c**2 + b**13*c/3) + x**42*(-572*a**7*c**7/7 + 2002*a**6*b**2*c**6 - 600
6*a**5*b**4*c**5 + 5005*a**4*b**6*c**4 - 1430*a**3*b**8*c**3 + 143*a**2*b**
10*c**2 - 13*a*b**12*c/3 + b**14/42) + x**39*(-572*a**7*b*c**6 + 4004*a**6*
b**3*c**5 - 6006*a**5*b**5*c**4 + 2860*a**4*b**7*c**3 - 1430*a**3*b**9*c**2
/3 + 26*a**2*b**11*c - a*b**13/3) + x**36*(143*a**8*c**6/2 - 1716*a**7*b**2
*c**5 + 5005*a**6*b**4*c**4 - 4004*a**5*b**6*c**3 + 2145*a**4*b**8*c**2/2 -
 286*a**3*b**10*c/3 + 13*a**2*b**12/6) + x**33*(429*a**8*b*c**5 - 2860*a**7
*b**3*c**4 + 4004*a**6*b**5*c**3 - 1716*a**5*b**7*c**2 + 715*a**4*b**9*c/3
```

```
- 26*a**3*b**11/3) + x**30*(-143*a**9*c**5/3 + 2145*a**8*b**2*c**4/2 - 2860
*a**7*b**4*c**3 + 2002*a**6*b**6*c**2 - 429*a**5*b**8*c + 143*a**4*b**10/6)
 + x**27*(-715*a**9*b*c**4/3 + 1430*a**8*b**3*c**3 - 1716*a**7*b**5*c**2 +
572*a**6*b**7*c - 143*a**5*b**9/3) + x**24*(143*a**10*c**4/6 - 1430*a**9*b*
*2*c**3/3 + 2145*a**8*b**4*c**2/2 - 572*a**7*b**6*c + 143*a**6*b**8/2) + x*
*21*(286*a**10*b*c**3/3 - 1430*a**9*b**3*c**2/3 + 429*a**8*b**5*c - 572*a**
7*b**7/7) + x**18*(-26*a**11*c**3/3 + 143*a**10*b**2*c**2 - 715*a**9*b**4*c
/3 + 143*a**8*b**6/2) + x**15*(-26*a**11*b*c**2 + 286*a**10*b**3*c/3 - 143*
a**9*b**5/3) + x**12*(13*a**12*c**2/6 - 26*a**11*b**2*c + 143*a**10*b**4/6)
 + x**9*(13*a**12*b*c/3 - 26*a**11*b**3/3) + x**6*(-a**13*c/3 + 13*a**12*b*
*2/6)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(18) = 36.
time = 5.50, size = 246, normalized size = 12.30

$$\frac{1}{42}\left(cx^{6}+bx^{3}\right)^{14}-\frac{1}{3}\left(cx^{6}+bx^{3}\right)^{13}a+\frac{13}{6}\left(cx^{6}+bx^{3}\right)^{12}a^{2}-\frac{26}{3}\left(cx^{6}+bx^{3}\right)^{11}a^{3}+\frac{143}{6}\left(cx^{6}+bx^{3}\right)^{10}a^{4}-\frac{143}{3}\left(cx^{6}+bx^{3}\right)^{9}a^{5}+\frac{143}{2}\left(cx^{6}+bx^{3}\right)^{8}a^{6}-\frac{572}{7}\left(cx^{6}+bx^{3}\right)^{7}a^{7}+\frac{143}{2}\left(cx^{6}+bx^{3}\right)^{6}a^{8}-\frac{143}{3}\left(cx^{6}+bx^{3}\right)^{5}a^{9}+\frac{143}{6}\left(cx^{6}+bx^{3}\right)^{4}a^{10}-\frac{26}{3}\left(cx^{6}+bx^{3}\right)^{3}a^{11}+\frac{13}{6}\left(cx^{6}+bx^{3}\right)^{2}a^{12}-\frac{1}{3}\left(cx^{6}+bx^{3}\right)a^{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14 - 1/3*(c*x^6 + b*x^3)^13*a + 13/6*(c*x^6 + b*x^3)^1
2*a^2 - 26/3*(c*x^6 + b*x^3)^11*a^3 + 143/6*(c*x^6 + b*x^3)^10*a^4 - 143/3*
(c*x^6 + b*x^3)^9*a^5 + 143/2*(c*x^6 + b*x^3)^8*a^6 - 572/7*(c*x^6 + b*x^3)
^7*a^7 + 143/2*(c*x^6 + b*x^3)^6*a^8 - 143/3*(c*x^6 + b*x^3)^5*a^9 + 143/6*
(c*x^6 + b*x^3)^4*a^10 - 26/3*(c*x^6 + b*x^3)^3*a^11 + 13/6*(c*x^6 + b*x^3)
^2*a^12 - 1/3*(c*x^6 + b*x^3)*a^13
```

Mupad [**B**]
time = 1.28, size = 1214, normalized size = 60.70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^13,x)
```

```
[Out] x^36*((13*a^2*b^12)/6 + (143*a^8*c^6)/2 - (286*a^3*b^10*c)/3 + (2145*a^4*b^
8*c^2)/2 - 4004*a^5*b^6*c^3 + 5005*a^6*b^4*c^4 - 1716*a^7*b^2*c^5) + x^48*(
(143*a^6*c^8)/2 + (13*b^12*c^2)/6 - (286*a*b^10*c^3)/3 + (2145*a^2*b^8*c^4)
/2 - 4004*a^3*b^6*c^5 + 5005*a^4*b^4*c^6 - 1716*a^5*b^2*c^7) - x^39*((a*b^1
3)/3 - 26*a^2*b^11*c + 572*a^7*b*c^6 + (1430*a^3*b^9*c^2)/3 - 2860*a^4*b^7*
c^3 + 6006*a^5*b^5*c^4 - 4004*a^6*b^3*c^5) + x^45*((b^13*c)/3 - 26*a*b^11*c
^2 + 572*a^6*b*c^7 + (1430*a^2*b^9*c^3)/3 - 2860*a^3*b^7*c^4 + 6006*a^4*b^5
*c^5 - 4004*a^5*b^3*c^6) + x^18*((143*a^8*b^6)/2 - (26*a^11*c^3)/3 - (715*a
^9*b^4*c)/3 + 143*a^10*b^2*c^2) - x^66*((26*a^3*c^11)/3 - (143*b^6*c^8)/2 +
 (715*a*b^4*c^9)/3 - 143*a^2*b^2*c^10) + x^30*((143*a^4*b^10)/6 - (143*a^9*
```

```
c^5)/3 - 429*a^5*b^8*c + 2002*a^6*b^6*c^2 - 2860*a^7*b^4*c^3 + (2145*a^8*b^
2*c^4)/2) - x^54*((143*a^5*c^9)/3 - (143*b^10*c^4)/6 + 429*a*b^8*c^5 - 2002
*a^2*b^6*c^6 + 2860*a^3*b^4*c^7 - (2145*a^4*b^2*c^8)/2) + x^42*(b^14/42 - (
572*a^7*c^7)/7 + 143*a^2*b^10*c^2 - 1430*a^3*b^8*c^3 + 5005*a^4*b^6*c^4 - 6
006*a^5*b^4*c^5 + 2002*a^6*b^2*c^6 - (13*a*b^12*c)/3) + x^24*((143*a^6*b^8)
/2 + (143*a^10*c^4)/6 - 572*a^7*b^6*c + (2145*a^8*b^4*c^2)/2 - (1430*a^9*b^
2*c^3)/3) + x^60*((143*a^4*c^10)/6 + (143*b^8*c^6)/2 - 572*a*b^6*c^7 + (214
5*a^2*b^4*c^8)/2 - (1430*a^3*b^2*c^9)/3) + (c^14*x^84)/42 - x^6*((a^13*c)/3
 - (13*a^12*b^2)/6) + (13*a^10*x^12*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 + (1
3*c^10*x^72*(11*b^4 + a^2*c^2 - 12*a*b^2*c))/6 - (a^13*b*x^3)/3 + (b*c^13*x
^81)/3 - (c^12*x^78*(2*a*c - 13*b^2))/6 - (143*a^7*b*x^21*(12*b^6 - 14*a^3*
c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 + (143*b*c^7*x^63*(12*b^6 - 14*a^3*c
^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/21 - (143*a^5*b*x^27*(b^8 + 5*a^4*c^4 +
36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 + (143*b*c^5*x^57*(b^8 + 5
*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12*a*b^6*c))/3 - (13*a^3*b*x^3
3*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^
4 - 55*a*b^8*c))/3 + (13*b*c^3*x^51*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2
- 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/3 - (13*a^9*b*x^15*(11*b
^4 + 6*a^2*c^2 - 22*a*b^2*c))/3 + (13*b*c^9*x^69*(11*b^4 + 6*a^2*c^2 - 22*a
*b^2*c))/3 + (13*a^11*b*x^9*(a*c - 2*b^2))/3 - (13*b*c^11*x^75*(a*c - 2*b^2
))/3
```

3.100 $\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=25

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

```
[Out] 1/14*(a-b*x^n-c*x^(2*n))^14/n
```

Rubi [A]
time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1482, 643}

$$\frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (a - b*x^n - c*x^(2*n))^14/(14*n)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^{13}\,dx &= \frac{\operatorname{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^{13}\,dx,x,x^n\right)}{n}\\ &= \frac{\left(a-bx^n-cx^{2n}\right)^{14}}{14n}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 260 vs. 2(25) = 50.
time = 0.40, size = 260, normalized size = 10.40

$$\frac{x^n(b+cx^n)\left(-14a^{13}+91a^{12}x^n(b+cx^n)-364a^{11}x^{2n}(b+cx^n)^2+1001a^{10}x^{3n}(b+cx^n)^3-2002a^9x^{4n}(b+cx^n)^4+3003a^8x^{5n}(b+cx^n)^5-3432a^7x^{6n}(b+cx^n)^6+3003a^6x^{7n}(b+cx^n)^7-2002a^5x^{8n}(b+cx^n)^8+1001a^4x^{9n}(b+cx^n)^9-364a^3x^{10n}(b+cx^n)^{10}+91a^2x^{11n}(b+cx^n)^{11}-14ax^{12n}(b+cx^n)^{12}+x^{13n}(b+cx^n)^{13}\right)}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^n*(b + c*x^n)*(-14*a^13 + 91*a^12*x^n*(b + c*x^n) - 364*a^11*x^(2*n)*(b
+ c*x^n)^2 + 1001*a^10*x^(3*n)*(b + c*x^n)^3 - 2002*a^9*x^(4*n)*(b + c*x^n)
^4 + 3003*a^8*x^(5*n)*(b + c*x^n)^5 - 3432*a^7*x^(6*n)*(b + c*x^n)^6 + 3003
*a^6*x^(7*n)*(b + c*x^n)^7 - 2002*a^5*x^(8*n)*(b + c*x^n)^8 + 1001*a^4*x^(9
*n)*(b + c*x^n)^9 - 364*a^3*x^(10*n)*(b + c*x^n)^10 + 91*a^2*x^(11*n)*(b +
c*x^n)^11 - 14*a*x^(12*n)*(b + c*x^n)^12 + x^(13*n)*(b + c*x^n)^13))/(14*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2045 vs. $2(23) = 46$.
time = 0.05, size = 2046, normalized size = 81.84

method	result	size
risch	Expression too large to display	2046

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x,method=_RETURNVERBOSE)
```

```
[Out] 26*b^3*c^11/n*(x^n)^25+b^13*c/n*(x^n)^15-1716/7*a^7*b^7/n*(x^n)^7+143/2*c^1
0/n*(x^n)^20*a^4+429/2*c^6/n*(x^n)^20*b^8-a^13/n*(x^n)^2*c+13/2*a^12/n*(x^n
)^2*b^2-26*c^11/n*(x^n)^22*a^3+429/2*c^8/n*(x^n)^22*b^6+b*c^13/n*(x^n)^27-b
*a^13/n*x^n+715*a^4*b^9/n*(x^n)^11*c+13*b*a^12/n*(x^n)^3*c-13*b*c^12/n*(x^n
)^25*a+1716*b*c^7/n*(x^n)^15*a^6-12012*b^3*c^6/n*(x^n)^15*a^5+18018*b^5*c^5
/n*(x^n)^15*a^4-8580*b^7*c^4/n*(x^n)^15*a^3+1430*b^9*c^3/n*(x^n)^15*a^2-78*
b^11*c^2/n*(x^n)^15*a+26*b^11*c^3/n*(x^n)^17-143*a^5*b^9/n*(x^n)^9+143*b^5*
c^9/n*(x^n)^23-26*a^3*b^11/n*(x^n)^11-26*b^3*a^11/n*(x^n)^3+286*a^10*b/n*(x
^n)^7*c^3-1430*a^9*b^3/n*(x^n)^7*c^2+1287*a^8*b^5/n*(x^n)^7*c+429*c^10/n*(x
^n)^22*a^2*b^2-715*c^9/n*(x^n)^22*a*b^4-1430*c^9/n*(x^n)^20*a^3*b^2+6435/2*
c^8/n*(x^n)^20*a^2*b^4-1716*c^7/n*(x^n)^20*a*b^6+143/2*a^10/n*(x^n)^8*c^4+4
29/2*a^6/n*(x^n)^8*b^8-26*a^11/n*(x^n)^6*c^3-1287*b*c^8/n*(x^n)^17*a^5+8580
*b^3*c^7/n*(x^n)^17*a^4-12012*b^5*c^6/n*(x^n)^17*a^3+5148*b^7*c^5/n*(x^n)^1
7*a^2-715*b^9*c^4/n*(x^n)^17*a-715*a^9*b/n*(x^n)^9*c^4+4290*a^8*b^3/n*(x^n)
^9*c^3-5148*a^7*b^5/n*(x^n)^9*c^2+1716*a^6*b^7/n*(x^n)^9*c+78*b*c^11/n*(x^n
)^23*a^2-286*b^3*c^10/n*(x^n)^23*a+1287*a^8*b/n*(x^n)^11*c^5-8580*a^7*b^3/n
*(x^n)^11*c^4+12012*a^6*b^5/n*(x^n)^11*c^3-5148*a^5*b^7/n*(x^n)^11*c^2+429/
2*a^8/n*(x^n)^6*b^6+429/2*a^8/n*(x^n)^12*c^6+13/2*a^2/n*(x^n)^12*b^12-1716/
7/n*(x^n)^14*a^7*c^7-c^13/n*(x^n)^26*a+13/2*c^12/n*(x^n)^26*b^2-143*c^9/n*(
x^n)^18*a^5+143/2*c^4/n*(x^n)^18*b^10+429/2*c^8/n*(x^n)^16*a^6+13/2*c^2/n*(
x^n)^16*b^12+1/14/n*(x^n)^14*b^14+13/2*a^12/n*(x^n)^4*c^2+143/2*a^10/n*(x^n
)^4*b^4+13/2*c^12/n*(x^n)^24*a^2+143/2*c^10/n*(x^n)^24*b^4+6435/2*c^8/n*(x^
n)^18*a^4*b^2-8580*c^7/n*(x^n)^18*a^3*b^4+6006*c^6/n*(x^n)^18*a^2*b^6-1287*
c^5/n*(x^n)^18*a*b^8-5148*c^7/n*(x^n)^16*a^5*b^2+15015*c^6/n*(x^n)^16*a^4*b
^4-12012*c^5/n*(x^n)^16*a^3*b^6+6435/2*c^4/n*(x^n)^16*a^2*b^8-286*c^3/n*(x^
```

```
n)^16*a*b^10-78*a^11/n*(x^n)^4*b^2*c-78*c^11/n*(x^n)^24*a*b^2-a*b^13/n*(x^n
)^13-143*a^9*b^5/n*(x^n)^5+1716/7*b^7*c^7/n*(x^n)^21+143*b^9*c^5/n*(x^n)^19
-143*a^9/n*(x^n)^10*c^5+143/2*a^4/n*(x^n)^10*b^10+1/14*c^14/n*(x^n)^28+1430
*b^3*c^9/n*(x^n)^21*a^2-1287*b^5*c^8/n*(x^n)^21*a+715*b*c^9/n*(x^n)^19*a^4-
4290*b^3*c^8/n*(x^n)^19*a^3+5148*b^5*c^7/n*(x^n)^19*a^2-1716*b^7*c^6/n*(x^n
)^19*a+6435/2*a^8/n*(x^n)^10*b^2*c^4-8580*a^7/n*(x^n)^10*b^4*c^3+6006*a^6/n
*(x^n)^10*b^6*c^2-1287*a^5/n*(x^n)^10*b^8*c-1716*a^7*b/n*(x^n)^13*c^6+12012
*a^6*b^3/n*(x^n)^13*c^5-18018*a^5*b^5/n*(x^n)^13*c^4+8580*a^4*b^7/n*(x^n)^1
3*c^3-1430*a^3*b^9/n*(x^n)^13*c^2+78*a^2*b^11/n*(x^n)^13*c-78*a^11*b/n*(x^n
)^5*c^2+286*a^10*b^3/n*(x^n)^5*c-286*b*c^10/n*(x^n)^21*a^3-1430*a^9/n*(x^n)
^8*b^2*c^3+6435/2*a^8/n*(x^n)^8*b^4*c^2-1716*a^7/n*(x^n)^8*b^6*c+429*a^10/n
*(x^n)^6*b^2*c^2-715*a^9/n*(x^n)^6*b^4*c-5148*a^7/n*(x^n)^12*b^2*c^5+15015*
a^6/n*(x^n)^12*b^4*c^4-12012*a^5/n*(x^n)^12*b^6*c^3+6435/2*a^4/n*(x^n)^12*b
^8*c^2-286*a^3/n*(x^n)^12*b^10*c+6006/n*(x^n)^14*a^6*b^2*c^6-18018/n*(x^n)^
14*a^5*b^4*c^5+15015/n*(x^n)^14*a^4*b^6*c^4-4290/n*(x^n)^14*a^3*b^8*c^3+429
/n*(x^n)^14*a^2*b^10*c^2-13/n*(x^n)^14*a*b^12*c
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2045 vs. $2(23) = 46$.
time = 0.35, size = 2045, normalized size = 81.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="maxima
")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n - a*c^1
3*x^(26*n)/n + 26*b^3*c^11*x^(25*n)/n - 13*a*b*c^12*x^(25*n)/n + 143/2*b^4*
c^10*x^(24*n)/n - 78*a*b^2*c^11*x^(24*n)/n + 13/2*a^2*c^12*x^(24*n)/n + 143
*b^5*c^9*x^(23*n)/n - 286*a*b^3*c^10*x^(23*n)/n + 78*a^2*b*c^11*x^(23*n)/n
+ 429/2*b^6*c^8*x^(22*n)/n - 715*a*b^4*c^9*x^(22*n)/n + 429*a^2*b^2*c^10*x^
(22*n)/n - 26*a^3*c^11*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n - 1287*a*b^5*
c^8*x^(21*n)/n + 1430*a^2*b^3*c^9*x^(21*n)/n - 286*a^3*b*c^10*x^(21*n)/n +
429/2*b^8*c^6*x^(20*n)/n - 1716*a*b^6*c^7*x^(20*n)/n + 6435/2*a^2*b^4*c^8*x
^(20*n)/n - 1430*a^3*b^2*c^9*x^(20*n)/n + 143/2*a^4*c^10*x^(20*n)/n + 143*b
^9*c^5*x^(19*n)/n - 1716*a*b^7*c^6*x^(19*n)/n + 5148*a^2*b^5*c^7*x^(19*n)/n
 - 4290*a^3*b^3*c^8*x^(19*n)/n + 715*a^4*b*c^9*x^(19*n)/n + 143/2*b^10*c^4*
x^(18*n)/n - 1287*a*b^8*c^5*x^(18*n)/n + 6006*a^2*b^6*c^6*x^(18*n)/n - 8580
*a^3*b^4*c^7*x^(18*n)/n + 6435/2*a^4*b^2*c^8*x^(18*n)/n - 143*a^5*c^9*x^(18
*n)/n + 26*b^11*c^3*x^(17*n)/n - 715*a*b^9*c^4*x^(17*n)/n + 5148*a^2*b^7*c^
5*x^(17*n)/n - 12012*a^3*b^5*c^6*x^(17*n)/n + 8580*a^4*b^3*c^7*x^(17*n)/n -
 1287*a^5*b*c^8*x^(17*n)/n + 13/2*b^12*c^2*x^(16*n)/n - 286*a*b^10*c^3*x^(1
6*n)/n + 6435/2*a^2*b^8*c^4*x^(16*n)/n - 12012*a^3*b^6*c^5*x^(16*n)/n + 150
15*a^4*b^4*c^6*x^(16*n)/n - 5148*a^5*b^2*c^7*x^(16*n)/n + 429/2*a^6*c^8*x^(
```

```
16*n)/n + b^13*c*x^(15*n)/n - 78*a*b^11*c^2*x^(15*n)/n + 1430*a^2*b^9*c^3*x
^(15*n)/n - 8580*a^3*b^7*c^4*x^(15*n)/n + 18018*a^4*b^5*c^5*x^(15*n)/n - 12
012*a^5*b^3*c^6*x^(15*n)/n + 1716*a^6*b*c^7*x^(15*n)/n + 1/14*b^14*x^(14*n)
/n - 13*a*b^12*c*x^(14*n)/n + 429*a^2*b^10*c^2*x^(14*n)/n - 4290*a^3*b^8*c^
3*x^(14*n)/n + 15015*a^4*b^6*c^4*x^(14*n)/n - 18018*a^5*b^4*c^5*x^(14*n)/n
+ 6006*a^6*b^2*c^6*x^(14*n)/n - 1716/7*a^7*c^7*x^(14*n)/n - a*b^13*x^(13*n)
/n + 78*a^2*b^11*c*x^(13*n)/n - 1430*a^3*b^9*c^2*x^(13*n)/n + 8580*a^4*b^7*
c^3*x^(13*n)/n - 18018*a^5*b^5*c^4*x^(13*n)/n + 12012*a^6*b^3*c^5*x^(13*n)/
n - 1716*a^7*b*c^6*x^(13*n)/n + 13/2*a^2*b^12*x^(12*n)/n - 286*a^3*b^10*c*x
^(12*n)/n + 6435/2*a^4*b^8*c^2*x^(12*n)/n - 12012*a^5*b^6*c^3*x^(12*n)/n +
15015*a^6*b^4*c^4*x^(12*n)/n - 5148*a^7*b^2*c^5*x^(12*n)/n + 429/2*a^8*c^6*
x^(12*n)/n - 26*a^3*b^11*x^(11*n)/n + 715*a^4*b^9*c*x^(11*n)/n - 5148*a^5*b
^7*c^2*x^(11*n)/n + 12012*a^6*b^5*c^3*x^(11*n)/n - 8580*a^7*b^3*c^4*x^(11*n
)/n + 1287*a^8*b*c^5*x^(11*n)/n + 143/2*a^4*b^10*x^(10*n)/n - 1287*a^5*b^8*
c*x^(10*n)/n + 6006*a^6*b^6*c^2*x^(10*n)/n - 8580*a^7*b^4*c^3*x^(10*n)/n +
6435/2*a^8*b^2*c^4*x^(10*n)/n - 143*a^9*c^5*x^(10*n)/n - 143*a^5*b^9*x^(9*n
)/n + 1716*a^6*b^7*c*x^(9*n)/n - 5148*a^7*b^5*c^2*x^(9*n)/n + 4290*a^8*b^3*
c^3*x^(9*n)/n - 715*a^9*b*c^4*x^(9*n)/n + 429/2*a^6*b^8*x^(8*n)/n - 1716*a^
7*b^6*c*x^(8*n)/n + 6435/2*a^8*b^4*c^2*x^(8*n)/n - 1430*a^9*b^2*c^3*x^(8*n)
/n + 143/2*a^10*c^4*x^(8*n)/n - 1716/7*a^7*b^7*x^(7*n)/n + 1287*a^8*b^5*c*x
^(7*n)/n - 1430*a^9*b^3*c^2*x^(7*n)/n + 286*a^10*b*c^3*x^(7*n)/n + 429/2*a^
8*b^6*x^(6*n)/n - 715*a^9*b^4*c*x^(6*n)/n + 429*a^10*b^2*c^2*x^(6*n)/n - 26
*a^11*c^3*x^(6*n)/n - 143*a^9*b^5*x^(5*n)/n + 286*a^10*b^3*c*x^(5*n)/n - 78
*a^11*b*c^2*x^(5*n)/n + 143/2*a^10*b^4*x^(4*n)/n - 78*a^11*b^2*c*x^(4*n)/n
+ 13/2*a^12*c^2*x^(4*n)/n - 26*a^11*b^3*x^(3*n)/n + 13*a^12*b*c*x^(3*n)/n +
 13/2*a^12*b^2*x^(2*n)/n - a^13*c*x^(2*n)/n - a^13*b*x^n/n
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1299 vs. $2(23) = 46$.
time = 0.40, size = 1299, normalized size = 51.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="fricas
")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) - 14*a^13*b*x^n + 7*(13*b^2*c^12 -
 2*a*c^13)*x^(26*n) + 182*(2*b^3*c^11 - a*b*c^12)*x^(25*n) + 91*(11*b^4*c^1
0 - 12*a*b^2*c^11 + a^2*c^12)*x^(24*n) + 182*(11*b^5*c^9 - 22*a*b^3*c^10 +
6*a^2*b*c^11)*x^(23*n) + 91*(33*b^6*c^8 - 110*a*b^4*c^9 + 66*a^2*b^2*c^10 -
 4*a^3*c^11)*x^(22*n) + 286*(12*b^7*c^7 - 63*a*b^5*c^8 + 70*a^2*b^3*c^9 - 1
4*a^3*b*c^10)*x^(21*n) + 1001*(3*b^8*c^6 - 24*a*b^6*c^7 + 45*a^2*b^4*c^8 -
20*a^3*b^2*c^9 + a^4*c^10)*x^(20*n) + 2002*(b^9*c^5 - 12*a*b^7*c^6 + 36*a^2
*b^5*c^7 - 30*a^3*b^3*c^8 + 5*a^4*b*c^9)*x^(19*n) + 1001*(b^10*c^4 - 18*a*b
```

```
^8*c^5 + 84*a^2*b^6*c^6 - 120*a^3*b^4*c^7 + 45*a^4*b^2*c^8 - 2*a^5*c^9)*x^(
18*n) + 182*(2*b^11*c^3 - 55*a*b^9*c^4 + 396*a^2*b^7*c^5 - 924*a^3*b^5*c^6
+ 660*a^4*b^3*c^7 - 99*a^5*b*c^8)*x^(17*n) + 91*(b^12*c^2 - 44*a*b^10*c^3 +
 495*a^2*b^8*c^4 - 1848*a^3*b^6*c^5 + 2310*a^4*b^4*c^6 - 792*a^5*b^2*c^7 +
33*a^6*c^8)*x^(16*n) + 14*(b^13*c - 78*a*b^11*c^2 + 1430*a^2*b^9*c^3 - 8580
*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6 + 1716*a^6*b*c^7)*x^(1
5*n) + (b^14 - 182*a*b^12*c + 6006*a^2*b^10*c^2 - 60060*a^3*b^8*c^3 + 21021
0*a^4*b^6*c^4 - 252252*a^5*b^4*c^5 + 84084*a^6*b^2*c^6 - 3432*a^7*c^7)*x^(1
4*n) - 14*(a*b^13 - 78*a^2*b^11*c + 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 1
8018*a^5*b^5*c^4 - 12012*a^6*b^3*c^5 + 1716*a^7*b*c^6)*x^(13*n) + 91*(a^2*b
^12 - 44*a^3*b^10*c + 495*a^4*b^8*c^2 - 1848*a^5*b^6*c^3 + 2310*a^6*b^4*c^4
 - 792*a^7*b^2*c^5 + 33*a^8*c^6)*x^(12*n) - 182*(2*a^3*b^11 - 55*a^4*b^9*c
+ 396*a^5*b^7*c^2 - 924*a^6*b^5*c^3 + 660*a^7*b^3*c^4 - 99*a^8*b*c^5)*x^(11
*n) + 1001*(a^4*b^10 - 18*a^5*b^8*c + 84*a^6*b^6*c^2 - 120*a^7*b^4*c^3 + 45
*a^8*b^2*c^4 - 2*a^9*c^5)*x^(10*n) - 2002*(a^5*b^9 - 12*a^6*b^7*c + 36*a^7*
b^5*c^2 - 30*a^8*b^3*c^3 + 5*a^9*b*c^4)*x^(9*n) + 1001*(3*a^6*b^8 - 24*a^7*
b^6*c + 45*a^8*b^4*c^2 - 20*a^9*b^2*c^3 + a^10*c^4)*x^(8*n) - 286*(12*a^7*b
^7 - 63*a^8*b^5*c + 70*a^9*b^3*c^2 - 14*a^10*b*c^3)*x^(7*n) + 91*(33*a^8*b^
6 - 110*a^9*b^4*c + 66*a^10*b^2*c^2 - 4*a^11*c^3)*x^(6*n) - 182*(11*a^9*b^5
 - 22*a^10*b^3*c + 6*a^11*b*c^2)*x^(5*n) + 91*(11*a^10*b^4 - 12*a^11*b^2*c
+ a^12*c^2)*x^(4*n) - 182*(2*a^11*b^3 - a^12*b*c)*x^(3*n) + 7*(13*a^12*b^2
- 2*a^13*c)*x^(2*n))/n
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1693 vs. $2(23) = 46$.
time = 3.77, size = 1693, normalized size = 67.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) - 14*a*c^13
*x^(26*n) + 364*b^3*c^11*x^(25*n) - 182*a*b*c^12*x^(25*n) + 1001*b^4*c^10*x
^(24*n) - 1092*a*b^2*c^11*x^(24*n) + 91*a^2*c^12*x^(24*n) + 2002*b^5*c^9*x^
(23*n) - 4004*a*b^3*c^10*x^(23*n) + 1092*a^2*b*c^11*x^(23*n) + 3003*b^6*c^8
```

*x^(22*n) - 10010*a*b^4*c^9*x^(22*n) + 6006*a^2*b^2*c^10*x^(22*n) - 364*a^3
*c^11*x^(22*n) + 3432*b^7*c^7*x^(21*n) - 18018*a*b^5*c^8*x^(21*n) + 20020*a
^2*b^3*c^9*x^(21*n) - 4004*a^3*b*c^10*x^(21*n) + 3003*b^8*c^6*x^(20*n) - 24
024*a*b^6*c^7*x^(20*n) + 45045*a^2*b^4*c^8*x^(20*n) - 20020*a^3*b^2*c^9*x^(
20*n) + 1001*a^4*c^10*x^(20*n) + 2002*b^9*c^5*x^(19*n) - 24024*a*b^7*c^6*x^
(19*n) + 72072*a^2*b^5*c^7*x^(19*n) - 60060*a^3*b^3*c^8*x^(19*n) + 10010*a^
4*b*c^9*x^(19*n) + 1001*b^10*c^4*x^(18*n) - 18018*a*b^8*c^5*x^(18*n) + 8408
4*a^2*b^6*c^6*x^(18*n) - 120120*a^3*b^4*c^7*x^(18*n) + 45045*a^4*b^2*c^8*x^
(18*n) - 2002*a^5*c^9*x^(18*n) + 364*b^11*c^3*x^(17*n) - 10010*a*b^9*c^4*x^
(17*n) + 72072*a^2*b^7*c^5*x^(17*n) - 168168*a^3*b^5*c^6*x^(17*n) + 120120*
a^4*b^3*c^7*x^(17*n) - 18018*a^5*b*c^8*x^(17*n) + 91*b^12*c^2*x^(16*n) - 40
04*a*b^10*c^3*x^(16*n) + 45045*a^2*b^8*c^4*x^(16*n) - 168168*a^3*b^6*c^5*x^
(16*n) + 210210*a^4*b^4*c^6*x^(16*n) - 72072*a^5*b^2*c^7*x^(16*n) + 3003*a^
6*c^8*x^(16*n) + 14*b^13*c*x^(15*n) - 1092*a*b^11*c^2*x^(15*n) + 20020*a^2*
b^9*c^3*x^(15*n) - 120120*a^3*b^7*c^4*x^(15*n) + 252252*a^4*b^5*c^5*x^(15*n
) - 168168*a^5*b^3*c^6*x^(15*n) + 24024*a^6*b*c^7*x^(15*n) + b^14*x^(14*n)
- 182*a*b^12*c*x^(14*n) + 6006*a^2*b^10*c^2*x^(14*n) - 60060*a^3*b^8*c^3*x^
(14*n) + 210210*a^4*b^6*c^4*x^(14*n) - 252252*a^5*b^4*c^5*x^(14*n) + 84084*
a^6*b^2*c^6*x^(14*n) - 3432*a^7*c^7*x^(14*n) - 14*a*b^13*x^(13*n) + 1092*a^
2*b^11*c*x^(13*n) - 20020*a^3*b^9*c^2*x^(13*n) + 120120*a^4*b^7*c^3*x^(13*n
) - 252252*a^5*b^5*c^4*x^(13*n) + 168168*a^6*b^3*c^5*x^(13*n) - 24024*a^7*b
*c^6*x^(13*n) + 91*a^2*b^12*x^(12*n) - 4004*a^3*b^10*c*x^(12*n) + 45045*a^4
*b^8*c^2*x^(12*n) - 168168*a^5*b^6*c^3*x^(12*n) + 210210*a^6*b^4*c^4*x^(12*
n) - 72072*a^7*b^2*c^5*x^(12*n) + 3003*a^8*c^6*x^(12*n) - 364*a^3*b^11*x^(1
1*n) + 10010*a^4*b^9*c*x^(11*n) - 72072*a^5*b^7*c^2*x^(11*n) + 168168*a^6*b
^5*c^3*x^(11*n) - 120120*a^7*b^3*c^4*x^(11*n) + 18018*a^8*b*c^5*x^(11*n) +
1001*a^4*b^10*x^(10*n) - 18018*a^5*b^8*c*x^(10*n) + 84084*a^6*b^6*c^2*x^(10
*n) - 120120*a^7*b^4*c^3*x^(10*n) + 45045*a^8*b^2*c^4*x^(10*n) - 2002*a^9*c
^5*x^(10*n) - 2002*a^5*b^9*x^(9*n) + 24024*a^6*b^7*c*x^(9*n) - 72072*a^7*b^
5*c^2*x^(9*n) + 60060*a^8*b^3*c^3*x^(9*n) - 10010*a^9*b*c^4*x^(9*n) + 3003*
a^6*b^8*x^(8*n) - 24024*a^7*b^6*c*x^(8*n) + 45045*a^8*b^4*c^2*x^(8*n) - 200
20*a^9*b^2*c^3*x^(8*n) + 1001*a^10*c^4*x^(8*n) - 3432*a^7*b^7*x^(7*n) + 180
18*a^8*b^5*c*x^(7*n) - 20020*a^9*b^3*c^2*x^(7*n) + 4004*a^10*b*c^3*x^(7*n)
+ 3003*a^8*b^6*x^(6*n) - 10010*a^9*b^4*c*x^(6*n) + 6006*a^10*b^2*c^2*x^(6*n
) - 364*a^11*c^3*x^(6*n) - 2002*a^9*b^5*x^(5*n) + 4004*a^10*b^3*c*x^(5*n) -
 1092*a^11*b*c^2*x^(5*n) + 1001*a^10*b^4*x^(4*n) - 1092*a^11*b^2*c*x^(4*n)
+ 91*a^12*c^2*x^(4*n) - 364*a^11*b^3*x^(3*n) + 182*a^12*b*c*x^(3*n) + 91*a^
12*b^2*x^(2*n) - 14*a^13*c*x^(2*n) - 14*a^13*b*x^n)/n

Mupad [B]
time = 5.78, size = 1401, normalized size = 56.04

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^13,x)
[Out] x^(n - 1)*((x^(11*n + 1)*((13*a^2*b^12)/2 + (429*a^8*c^6)/2 - 286*a^3*b^10*
c + (6435*a^4*b^8*c^2)/2 - 12012*a^5*b^6*c^3 + 15015*a^6*b^4*c^4 - 5148*a^7
*b^2*c^5))/n + (x^(15*n + 1)*((429*a^6*c^8)/2 + (13*b^12*c^2)/2 - 286*a*b^1
0*c^3 + (6435*a^2*b^8*c^4)/2 - 12012*a^3*b^6*c^5 + 15015*a^4*b^4*c^6 - 5148
*a^5*b^2*c^7))/n - (x^(12*n + 1)*(a*b^13 - 78*a^2*b^11*c + 1716*a^7*b*c^6 +
 1430*a^3*b^9*c^2 - 8580*a^4*b^7*c^3 + 18018*a^5*b^5*c^4 - 12012*a^6*b^3*c^
5))/n + (x^(14*n + 1)*(b^13*c - 78*a*b^11*c^2 + 1716*a^6*b*c^7 + 1430*a^2*b
^9*c^3 - 8580*a^3*b^7*c^4 + 18018*a^4*b^5*c^5 - 12012*a^5*b^3*c^6))/n + (x^
(5*n + 1)*((429*a^8*b^6)/2 - 26*a^11*c^3 - 715*a^9*b^4*c + 429*a^10*b^2*c^2
))/n - (x^(21*n + 1)*(26*a^3*c^11 - (429*b^6*c^8)/2 + 715*a*b^4*c^9 - 429*a
^2*b^2*c^10))/n + (x^(9*n + 1)*((143*a^4*b^10)/2 - 143*a^9*c^5 - 1287*a^5*b
^8*c + 6006*a^6*b^6*c^2 - 8580*a^7*b^4*c^3 + (6435*a^8*b^2*c^4)/2))/n - (x^
(17*n + 1)*(143*a^5*c^9 - (143*b^10*c^4)/2 + 1287*a*b^8*c^5 - 6006*a^2*b^6*
c^6 + 8580*a^3*b^4*c^7 - (6435*a^4*b^2*c^8)/2))/n + (x^(13*n + 1)*(b^14/14
- (1716*a^7*c^7)/7 + 429*a^2*b^10*c^2 - 4290*a^3*b^8*c^3 + 15015*a^4*b^6*c^
4 - 18018*a^5*b^4*c^5 + 6006*a^6*b^2*c^6 - 13*a*b^12*c))/n + (x^(7*n + 1)*(
(429*a^6*b^8)/2 + (143*a^10*c^4)/2 - 1716*a^7*b^6*c + (6435*a^8*b^4*c^2)/2
- 1430*a^9*b^2*c^3))/n + (x^(19*n + 1)*((143*a^4*c^10)/2 + (429*b^8*c^6)/2
- 1716*a*b^6*c^7 + (6435*a^2*b^4*c^8)/2 - 1430*a^3*b^2*c^9))/n + (c^14*x^(2
7*n + 1))/(14*n) - (a^12*x^(n + 1)*(a*c - (13*b^2)/2))/n + (a^10*x^(3*n + 1
)*((143*b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (c^10*x^(23*n + 1)*((143
*b^4)/2 + (13*a^2*c^2)/2 - 78*a*b^2*c))/n + (b*c^13*x^(26*n + 1))/n - (c^12
*x^(25*n + 1)*(a*c - (13*b^2)/2))/n - (a^13*b*x)/n - (143*a^7*b*x^(6*n + 1)
*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) + (143*b*c^7*x^
(20*n + 1)*(12*b^6 - 14*a^3*c^3 + 70*a^2*b^2*c^2 - 63*a*b^4*c))/(7*n) - (14
3*a^5*b*x^(8*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 - 30*a^3*b^2*c^3 - 12
*a*b^6*c))/n + (143*b*c^5*x^(18*n + 1)*(b^8 + 5*a^4*c^4 + 36*a^2*b^4*c^2 -
30*a^3*b^2*c^3 - 12*a*b^6*c))/n - (13*a^3*b*x^(10*n + 1)*(2*b^10 - 99*a^5*c
^5 + 396*a^2*b^6*c^2 - 924*a^3*b^4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n +
 (13*b*c^3*x^(16*n + 1)*(2*b^10 - 99*a^5*c^5 + 396*a^2*b^6*c^2 - 924*a^3*b^
4*c^3 + 660*a^4*b^2*c^4 - 55*a*b^8*c))/n - (13*a^9*b*x^(4*n + 1)*(11*b^4 +
6*a^2*c^2 - 22*a*b^2*c))/n + (13*b*c^9*x^(22*n + 1)*(11*b^4 + 6*a^2*c^2 - 2
2*a*b^2*c))/n + (13*a^11*b*x^(2*n + 1)*(a*c - 2*b^2))/n - (13*b*c^11*x^(24*
n + 1)*(a*c - 2*b^2))/n)
```

3.101 $\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx$

Optimal. Leaf size=15

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

```
[Out] 1/14*(c*x^2+b*x)^14
```

Rubi [A]
time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {643}

$$\frac{1}{14}\left(bx+cx^2\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b*x + c*x^2)^14/14
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^{13}\,dx = \frac{1}{14}\left(bx+cx^2\right)^{14}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 172 vs. 2(15) = 30.
time = 0.00, size = 172, normalized size = 11.47

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13}{2}b^{12}c^2x^{16}+26b^{11}c^3x^{17}+\frac{143}{2}b^{10}c^4x^{18}+143b^9c^5x^{19}+\frac{429}{2}b^8c^6x^{20}+\frac{1716}{7}b^7c^7x^{21}+\frac{429}{2}b^6c^8x^{22}+143b^5c^9x^{23}+\frac{143}{2}b^4c^{10}x^{24}+26b^3c^{11}x^{25}+\frac{13}{2}b^2c^{12}x^{26}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^13,x]
```

```
[Out] (b^14*x^14)/14 + b^13*c*x^15 + (13*b^12*c^2*x^16)/2 + 26*b^11*c^3*x^17 + (1
43*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (429*b^8*c^6*x^20)/2 + (1716*b^7*c
```

```
^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*c^9*x^23 + (143*b^4*c^10*x^24)/
2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2 + b*c^13*x^27 + (c^14*x^28)/14
```

Maple [A]
time = 0.20, size = 14, normalized size = 0.93

method	result
gosper	$\frac{(cx+b)^{14}x^{14}}{14}$
default	$\frac{(c\,x^2+bx)^{14}}{14}$
norman	$b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^{17}+\frac{143}{2}x^{18}b^{10}c^4+143b^9c^5x^{19}+$
risch	$b\,c^{13}x^{27}+\frac{1}{14}c^{14}x^{28}+\frac{1}{14}x^{14}b^{14}+b^{13}c\,x^{15}+\frac{13}{2}x^{16}b^{12}c^2+26b^{11}c^3x^{17}+\frac{143}{2}x^{18}b^{10}c^4+143b^9c^5x^{19}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^13,x,method=_RETURNVERBOSE)
[Out] 1/14*(c*x^2+b*x)^14
```

Maxima [A]
time = 0.29, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="maxima")
[Out] 1/14*(c*x^2 + b*x)^14
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(13) = 26$.
time = 0.33, size = 154, normalized size = 10.27

$$\frac{1}{14}\,c^{14}x^{28}+bc^{13}x^{27}+\frac{13}{2}\,b^2c^{12}x^{26}+26\,b^3c^{11}x^{25}+\frac{143}{2}\,b^4c^{10}x^{24}+143\,b^5c^9x^{23}+\frac{429}{2}\,b^6c^8x^{22}+\frac{1716}{7}\,b^7c^7x^{21}+\frac{429}{2}\,b^8c^6x^{20}+143\,b^9c^5x^{19}+\frac{143}{2}\,b^{10}c^4x^{18}+26\,b^{11}c^3x^{17}+\frac{13}{2}\,b^{12}c^2x^{16}+b^{13}cx^{15}+\frac{1}{14}\,b^{14}x^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="fricas")
[Out] 1/14*c^14*x^28 + b*c^13*x^27 + 13/2*b^2*c^12*x^26 + 26*b^3*c^11*x^25 + 143/
2*b^4*c^10*x^24 + 143*b^5*c^9*x^23 + 429/2*b^6*c^8*x^22 + 1716/7*b^7*c^7*x^
21 + 429/2*b^8*c^6*x^20 + 143*b^9*c^5*x^19 + 143/2*b^10*c^4*x^18 + 26*b^11*
c^3*x^17 + 13/2*b^12*c^2*x^16 + b^13*c*x^15 + 1/14*b^14*x^14
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(10) = 20$.
time = 0.05, size = 175, normalized size = 11.67

$$\frac{b^{14}x^{14}}{14}+b^{13}cx^{15}+\frac{13b^{12}c^2x^{16}}{2}+26b^{11}c^3x^{17}+\frac{143b^{10}c^4x^{18}}{2}+143b^9c^5x^{19}+\frac{429b^8c^6x^{20}}{2}+\frac{1716b^7c^7x^{21}}{7}+\frac{429b^6c^8x^{22}}{2}+143b^5c^9x^{23}+\frac{143b^4c^{10}x^{24}}{2}+26b^3c^{11}x^{25}+\frac{13b^2c^{12}x^{26}}{2}+bc^{13}x^{27}+\frac{c^{14}x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**13,x)
```

```
[Out] b**14*x**14/14 + b**13*c*x**15 + 13*b**12*c**2*x**16/2 + 26*b**11*c**3*x**1
7 + 143*b**10*c**4*x**18/2 + 143*b**9*c**5*x**19 + 429*b**8*c**6*x**20/2 +
1716*b**7*c**7*x**21/7 + 429*b**6*c**8*x**22/2 + 143*b**5*c**9*x**23 + 143*
b**4*c**10*x**24/2 + 26*b**3*c**11*x**25 + 13*b**2*c**12*x**26/2 + b*c**13*
x**27 + c**14*x**28/14
```

Giac [**A**]
time = 2.89, size = 13, normalized size = 0.87

$$\frac{1}{14}\left(cx^2+bx\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^13,x, algorithm="giac")
```

```
[Out] 1/14*(c*x^2 + b*x)^14
```

Mupad [**B**]
time = 2.09, size = 154, normalized size = 10.27

$$\frac{b^{14}\,x^{14}}{14}+b^{13}\,c\,x^{15}+\frac{13\,b^{12}\,c^2\,x^{16}}{2}+26\,b^{11}\,c^3\,x^{17}+\frac{143\,b^{10}\,c^4\,x^{18}}{2}+143\,b^9\,c^5\,x^{19}+\frac{429\,b^8\,c^6\,x^{20}}{2}+\frac{1716\,b^7\,c^7\,x^{21}}{7}+\frac{429\,b^6\,c^8\,x^{22}}{2}+143\,b^5\,c^9\,x^{23}+\frac{143\,b^4\,c^{10}\,x^{24}}{2}+26\,b^3\,c^{11}\,x^{25}+\frac{13\,b^2\,c^{12}\,x^{26}}{2}+b\,c^{13}\,x^{27}+\frac{c^{14}\,x^{28}}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^13*(b + 2*c*x),x)
```

```
[Out] (b^14*x^14)/14 + (c^14*x^28)/14 + b^13*c*x^15 + b*c^13*x^27 + (13*b^12*c^2*
x^16)/2 + 26*b^11*c^3*x^17 + (143*b^10*c^4*x^18)/2 + 143*b^9*c^5*x^19 + (42
9*b^8*c^6*x^20)/2 + (1716*b^7*c^7*x^21)/7 + (429*b^6*c^8*x^22)/2 + 143*b^5*
c^9*x^23 + (143*b^4*c^10*x^24)/2 + 26*b^3*c^11*x^25 + (13*b^2*c^12*x^26)/2
```

3.102 $\int x(b+2cx^2)\left(bx^2+cx^4\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

[Out] 1/28*x^28*(c*x^2+b)^14

Rubi [A]
time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1598, 457, 75}

$$\frac{1}{28}x^{28}\left(b+cx^2\right)^{14}$$

Antiderivative was successfully verified.

[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]

[Out] (x^28*(b + c*x^2)^14)/28

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^{13}\,dx &= \int x^{27}\left(b+cx^2\right)^{13}\left(b+2cx^2\right)\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^2\right) \\
&= \frac{1}{28}x^{28}\left(b+cx^2\right)^{14}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 182 vs. 2(16) = 32.
time = 0.00, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{1}{2}b^{13}cx^{30}+\frac{13}{4}b^{12}c^2x^{32}+13b^{11}c^3x^{34}+\frac{143}{4}b^{10}c^4x^{36}+\frac{143}{2}b^9c^5x^{38}+\frac{429}{4}b^8c^6x^{40}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^6c^8x^{44}+\frac{143}{2}b^5c^9x^{46}+\frac{143}{4}b^4c^{10}x^{48}+13b^3c^{11}x^{50}+\frac{13}{4}b^2c^{12}x^{52}+\frac{1}{2}bc^{13}x^{54}+\frac{c^{14}x^{56}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x]
```

```
[Out] (b^14*x^28)/28 + (b^13*c*x^30)/2 + (13*b^12*c^2*x^32)/4 + 13*b^11*c^3*x^34
+ (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x^38)/2 + (429*b^8*c^6*x^40)/4 + (85
8*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4 + (143*b^5*c^9*x^46)/2 + (143*b^4*
c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^2*c^12*x^52)/4 + (b*c^13*x^54)/2 +
(c^14*x^56)/28
```

Maple [**A**]
time = 0.23, size = 24, normalized size = 1.50

method	result
gosper	$\frac{x^{28}\left(c\,x^2+b\right)^{14}}{28}$
default	$\frac{\left(b^2-\left(2c\,x^2+b\right)^2\right)^{14}}{7516192768c^{14}}$
risch	$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}b\,c^{13}x^{54}+\frac{13}{4}x^{52}b^2c^{12}+\frac{143}{2}x^{38}b^9c^5+\frac{429}{4}x^{40}b^8c^6+\frac{858}{7}x^{42}b^7c^7+\frac{429}{4}x^{44}b^6c^8+\frac{143}{2}x^{46}b^5c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7516192768*(b^2-(2*c*x^2+b)^2)^14/c^14
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.
time = 0.29, size = 156, normalized size = 9.75

$$\frac{1}{28}c^{14}x^{56}+\frac{1}{2}bc^{13}x^{54}+\frac{13}{4}b^2c^{12}x^{52}+13\,b^3c^{11}x^{50}+\frac{143}{4}b^4c^{10}x^{48}+\frac{143}{2}b^5c^9x^{46}+\frac{429}{4}b^6c^8x^{44}+\frac{858}{7}b^7c^7x^{42}+\frac{429}{4}b^8c^6x^{40}+\frac{143}{2}b^9c^5x^{38}+\frac{143}{4}b^{10}c^4x^{36}+13\,b^{11}c^3x^{34}+\frac{13}{4}b^{12}c^2x^{32}+\frac{1}{2}b^{13}cx^{30}+\frac{1}{28}b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="maxima")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.33, size = 156, normalized size = 9.75

$$\frac{1}{28}\,c^{14}x^{56}+\frac{1}{2}\,bc^{13}x^{54}+\frac{13}{4}\,b^{2}c^{12}x^{52}+13\,b^{3}c^{11}x^{50}+\frac{143}{4}\,b^{4}c^{10}x^{48}+\frac{143}{2}\,b^{5}c^{9}x^{46}+\frac{429}{4}\,b^{6}c^{8}x^{44}+\frac{858}{7}\,b^{7}c^{7}x^{42}+\frac{429}{4}\,b^{8}c^{6}x^{40}+\frac{143}{2}\,b^{9}c^{5}x^{38}+\frac{143}{4}\,b^{10}c^{4}x^{36}+13\,b^{11}c^{3}x^{34}+\frac{13}{4}\,b^{12}c^{2}x^{32}+\frac{1}{2}\,b^{13}cx^{30}+\frac{1}{28}\,b^{14}x^{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="fricas")
```

```
[Out] 1/28*c^14*x^56 + 1/2*b*c^13*x^54 + 13/4*b^2*c^12*x^52 + 13*b^3*c^11*x^50 +
143/4*b^4*c^10*x^48 + 143/2*b^5*c^9*x^46 + 429/4*b^6*c^8*x^44 + 858/7*b^7*c
^7*x^42 + 429/4*b^8*c^6*x^40 + 143/2*b^9*c^5*x^38 + 143/4*b^10*c^4*x^36 + 1
3*b^11*c^3*x^34 + 13/4*b^12*c^2*x^32 + 1/2*b^13*c*x^30 + 1/28*b^14*x^28
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(12) = 24$.
time = 0.05, size = 182, normalized size = 11.38

$$\frac{b^{14}x^{28}}{28}+\frac{b^{13}cx^{30}}{2}+\frac{13b^{12}c^{2}x^{32}}{4}+13b^{11}c^{3}x^{34}+\frac{143b^{10}c^{4}x^{36}}{4}+\frac{143b^{9}c^{5}x^{38}}{2}+\frac{429b^{8}c^{6}x^{40}}{4}+\frac{858b^{7}c^{7}x^{42}}{7}+\frac{429b^{6}c^{8}x^{44}}{4}+\frac{143b^{5}c^{9}x^{46}}{2}+\frac{143b^{4}c^{10}x^{48}}{4}+13b^{3}c^{11}x^{50}+\frac{13b^{2}c^{12}x^{52}}{4}+\frac{bc^{13}x^{54}}{2}+\frac{c^{14}x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**13,x)
```

```
[Out] b**14*x**28/28 + b**13*c*x**30/2 + 13*b**12*c**2*x**32/4 + 13*b**11*c**3*x*
*34 + 143*b**10*c**4*x**36/4 + 143*b**9*c**5*x**38/2 + 429*b**8*c**6*x**40/
4 + 858*b**7*c**7*x**42/7 + 429*b**6*c**8*x**44/4 + 143*b**5*c**9*x**46/2 +
 143*b**4*c**10*x**48/4 + 13*b**3*c**11*x**50 + 13*b**2*c**12*x**52/4 + b*c
**13*x**54/2 + c**14*x**56/28
```

Giac [**A**]
time = 4.07, size = 15, normalized size = 0.94

$$\frac{1}{28}\left(cx^{4}+bx^{2}\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^13,x, algorithm="giac")
```

```
[Out] 1/28*(c*x^4 + b*x^2)^14
```

Mupad [B]
time = 2.08, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{28}}{28}+\frac{b^{13}\,c\,x^{30}}{2}+\frac{13\,b^{12}\,c^2\,x^{32}}{4}+13\,b^{11}\,c^3\,x^{34}+\frac{143\,b^{10}\,c^4\,x^{36}}{4}+\frac{143\,b^9\,c^5\,x^{38}}{2}+\frac{429\,b^8\,c^6\,x^{40}}{4}+\frac{858\,b^7\,c^7\,x^{42}}{7}+\frac{429\,b^6\,c^8\,x^{44}}{4}+\frac{143\,b^5\,c^9\,x^{46}}{2}+\frac{143\,b^4\,c^{10}\,x^{48}}{4}+13\,b^3\,c^{11}\,x^{50}+\frac{13\,b^2\,c^{12}\,x^{52}}{4}+\frac{b\,c^{13}\,x^{54}}{2}+\frac{c^{14}\,x^{56}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^13,x)
```

```
[Out] (b^14*x^28)/28 + (c^14*x^56)/28 + (b^13*c*x^30)/2 + (b*c^13*x^54)/2 + (13*b
^12*c^2*x^32)/4 + 13*b^11*c^3*x^34 + (143*b^10*c^4*x^36)/4 + (143*b^9*c^5*x
^38)/2 + (429*b^8*c^6*x^40)/4 + (858*b^7*c^7*x^42)/7 + (429*b^6*c^8*x^44)/4
 + (143*b^5*c^9*x^46)/2 + (143*b^4*c^10*x^48)/4 + 13*b^3*c^11*x^50 + (13*b^
2*c^12*x^52)/4
```

3.103 $\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^{13}\,dx$

Optimal. Leaf size=16

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

```
[Out] 1/42*x^42*(c*x^3+b)^14
```

Rubi [A]
time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1598, 457, 75}

$$\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (x^42*(b + c*x^3)^14)/42
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^{13}\,dx=\int x^{41}\left(b+cx^3\right)^{13}\left(b+2cx^3\right)\,dx$$
$$=\frac{1}{3}\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^3\right)$$
$$=\frac{1}{42}x^{42}\left(b+cx^3\right)^{14}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 186 vs. 2(16) = 32.
time = 0.00, size = 186, normalized size = 11.62

$$\frac{b^{14}x^{42}}{42}+\frac{1}{3}b^{13}cx^{45}+\frac{13}{6}b^{12}c^2x^{48}+\frac{26}{3}b^{11}c^3x^{51}+\frac{143}{6}b^{10}c^4x^{54}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{2}b^8c^6x^{60}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^6c^8x^{66}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{6}b^4c^{10}x^{72}+\frac{26}{3}b^3c^{11}x^{75}+\frac{13}{6}b^2c^{12}x^{78}+\frac{1}{3}bc^{13}x^{81}+\frac{c^{14}x^{84}}{42}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x]
```

```
[Out] (b^14*x^42)/42 + (b^13*c*x^45)/3 + (13*b^12*c^2*x^48)/6 + (26*b^11*c^3*x^51
)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c^5*x^57)/3 + (143*b^8*c^6*x^60)/2 +
 (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^66)/2 + (143*b^5*c^9*x^69)/3 + (143*
b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3 + (13*b^2*c^12*x^78)/6 + (b*c^13*x^
81)/3 + (c^14*x^84)/42
```

Maple [A]
time = 0.26, size = 24, normalized size = 1.50

method	result
gosper	$\frac{x^{42}\left(c\,x^3+b\right)^{14}}{42}$
default	$\frac{\left(b^2-\left(2c\,x^3+b\right)^2\right)^{14}}{11274289152c^{14}}$
risch	$\frac{26}{3}x^{75}b^3c^{11}+\frac{13}{6}x^{78}b^2c^{12}+\frac{1}{3}b\,c^{13}x^{81}+\frac{1}{42}c^{14}x^{84}+\frac{143}{2}x^{66}b^6c^8+\frac{143}{3}x^{69}b^5c^9+\frac{143}{6}x^{72}b^4c^{10}+\frac{143}{2}x^{60}b^8c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11274289152*(b^2-(2*c*x^3+b)^2)^14/c^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(14) = 28.
time = 0.28, size = 156, normalized size = 9.75

$$\frac{1}{42}c^{14}x^{84}+\frac{1}{3}bc^{13}x^{81}+\frac{13}{6}b^2c^{12}x^{78}+\frac{26}{3}b^3c^{11}x^{75}+\frac{143}{6}b^4c^{10}x^{72}+\frac{143}{3}b^5c^9x^{69}+\frac{143}{2}b^6c^8x^{66}+\frac{572}{7}b^7c^7x^{63}+\frac{143}{2}b^8c^6x^{60}+\frac{143}{3}b^9c^5x^{57}+\frac{143}{6}b^{10}c^4x^{54}+\frac{26}{3}b^{11}c^3x^{51}+\frac{13}{6}b^{12}c^2x^{48}+\frac{1}{3}b^{13}cx^{45}+\frac{1}{42}b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="maxima")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. $2(14) = 28$.
time = 0.33, size = 156, normalized size = 9.75

$$\frac{1}{42}\,c^{14}x^{84}+\frac{1}{3}\,bc^{13}x^{81}+\frac{13}{6}\,b^{2}c^{12}x^{78}+\frac{26}{3}\,b^{3}c^{11}x^{75}+\frac{143}{6}\,b^{4}c^{10}x^{72}+\frac{143}{3}\,b^{5}c^{9}x^{69}+\frac{143}{2}\,b^{6}c^{8}x^{66}+\frac{572}{7}\,b^{7}c^{7}x^{63}+\frac{143}{2}\,b^{8}c^{6}x^{60}+\frac{143}{3}\,b^{9}c^{5}x^{57}+\frac{143}{6}\,b^{10}c^{4}x^{54}+\frac{26}{3}\,b^{11}c^{3}x^{51}+\frac{13}{6}\,b^{12}c^{2}x^{48}+\frac{1}{3}\,b^{13}cx^{45}+\frac{1}{42}\,b^{14}x^{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="fricas")
```

```
[Out] 1/42*c^14*x^84 + 1/3*b*c^13*x^81 + 13/6*b^2*c^12*x^78 + 26/3*b^3*c^11*x^75
+ 143/6*b^4*c^10*x^72 + 143/3*b^5*c^9*x^69 + 143/2*b^6*c^8*x^66 + 572/7*b^7
*c^7*x^63 + 143/2*b^8*c^6*x^60 + 143/3*b^9*c^5*x^57 + 143/6*b^10*c^4*x^54 +
 26/3*b^11*c^3*x^51 + 13/6*b^12*c^2*x^48 + 1/3*b^13*c*x^45 + 1/42*b^14*x^42
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(12) = 24$.
time = 0.04, size = 185, normalized size = 11.56

$$\frac{b^{14}x^{42}}{42}+\frac{b^{13}cx^{45}}{3}+\frac{13b^{12}c^{2}x^{48}}{6}+\frac{26b^{11}c^{3}x^{51}}{3}+\frac{143b^{10}c^{4}x^{54}}{6}+\frac{143b^{9}c^{5}x^{57}}{3}+\frac{143b^{8}c^{6}x^{60}}{2}+\frac{572b^{7}c^{7}x^{63}}{7}+\frac{143b^{6}c^{8}x^{66}}{2}+\frac{143b^{5}c^{9}x^{69}}{3}+\frac{143b^{4}c^{10}x^{72}}{6}+\frac{26b^{3}c^{11}x^{75}}{3}+\frac{13b^{2}c^{12}x^{78}}{6}+\frac{bc^{13}x^{81}}{3}+\frac{c^{14}x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**13,x)
```

```
[Out] b**14*x**42/42 + b**13*c*x**45/3 + 13*b**12*c**2*x**48/6 + 26*b**11*c**3*x*
*51/3 + 143*b**10*c**4*x**54/6 + 143*b**9*c**5*x**57/3 + 143*b**8*c**6*x**6
0/2 + 572*b**7*c**7*x**63/7 + 143*b**6*c**8*x**66/2 + 143*b**5*c**9*x**69/3
 + 143*b**4*c**10*x**72/6 + 26*b**3*c**11*x**75/3 + 13*b**2*c**12*x**78/6 +
 b*c**13*x**81/3 + c**14*x**84/42
```

Giac [A]
time = 3.50, size = 15, normalized size = 0.94

$$\frac{1}{42}\left(cx^{6}+bx^{3}\right)^{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^13,x, algorithm="giac")
```

```
[Out] 1/42*(c*x^6 + b*x^3)^14
```

Mupad [B]
time = 2.08, size = 156, normalized size = 9.75

$$\frac{b^{14}\,x^{42}}{42}+\frac{b^{13}\,c\,x^{45}}{3}+\frac{13\,b^{12}\,c^2\,x^{48}}{6}+\frac{26\,b^{11}\,c^3\,x^{51}}{3}+\frac{143\,b^{10}\,c^4\,x^{54}}{6}+\frac{143\,b^9\,c^5\,x^{57}}{3}+\frac{143\,b^8\,c^6\,x^{60}}{2}+\frac{572\,b^7\,c^7\,x^{63}}{7}+\frac{143\,b^6\,c^8\,x^{66}}{2}+\frac{143\,b^5\,c^9\,x^{69}}{3}+\frac{143\,b^4\,c^{10}\,x^{72}}{6}+\frac{26\,b^3\,c^{11}\,x^{75}}{3}+\frac{13\,b^2\,c^{12}\,x^{78}}{6}+\frac{b\,c^{13}\,x^{81}}{3}+\frac{c^{14}\,x^{84}}{42}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^13,x)
```

```
[Out] (b^14*x^42)/42 + (c^14*x^84)/42 + (b^13*c*x^45)/3 + (b*c^13*x^81)/3 + (13*b
^12*c^2*x^48)/6 + (26*b^11*c^3*x^51)/3 + (143*b^10*c^4*x^54)/6 + (143*b^9*c
^5*x^57)/3 + (143*b^8*c^6*x^60)/2 + (572*b^7*c^7*x^63)/7 + (143*b^6*c^8*x^6
6)/2 + (143*b^5*c^9*x^69)/3 + (143*b^4*c^10*x^72)/6 + (26*b^3*c^11*x^75)/3
+ (13*b^2*c^12*x^78)/6
```

3.104 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx$

Optimal. Leaf size=21

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

```
[Out] 1/14*x^(14*n)*(b+c*x^n)^14/n
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1598, 457, 75}

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^{13}\,dx=\int x^{-1+14n}(b+cx^n)^{13}\,(b+2cx^n)\,dx$$
$$=\frac{\text{Subst}\left(\int x^{13}(b+cx)^{13}(b+2cx)\,dx,x,x^n\right)}{n}$$
$$=\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Mathematica [A]
time = 0.03, size = 21, normalized size = 1.00

$$\frac{x^{14n}(b+cx^n)^{14}}{14n}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x]
```

```
[Out] (x^(14*n)*(b + c*x^n)^14)/(14*n)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.
time = 0.18, size = 230, normalized size = 10.95

method	result
risch	$\frac{c^{14}x^{28n}}{14n}+\frac{b\,c^{13}x^{27n}}{n}+\frac{13c^{12}x^{26n}b^{2}}{2n}+\frac{26b^{3}c^{11}x^{25n}}{n}+\frac{143c^{10}x^{24n}b^{4}}{2n}+\frac{143b^{5}c^{9}x^{23n}}{n}+\frac{429c^{8}x^{22n}b^{6}}{2n}+\frac{1716b^{7}c^{7}x^{21n}}{7n}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*c^14/n*(x^n)^28+b*c^13/n*(x^n)^27+13/2*b^2*c^12/n*(x^n)^26+26*b^3*c^11
/n*(x^n)^25+143/2*b^4*c^10/n*(x^n)^24+143*b^5*c^9/n*(x^n)^23+429/2*b^6*c^8/
n*(x^n)^22+1716/7*b^7*c^7/n*(x^n)^21+429/2*b^8*c^6/n*(x^n)^20+143*b^9*c^5/n
*(x^n)^19+143/2*b^10*c^4/n*(x^n)^18+26*b^11*c^3/n*(x^n)^17+13/2*b^12*c^2/n*
(x^n)^16+b^13*c/n*(x^n)^15+1/14*b^14/n*(x^n)^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 229 vs. 2(19) = 38.
time = 0.29, size = 229, normalized size = 10.90

$$\frac{c^{14}x^{28\,n}}{14\,n}+\frac{bc^{13}x^{27\,n}}{n}+\frac{13\,b^{2}c^{12}x^{26\,n}}{2\,n}+\frac{26\,b^{3}c^{11}x^{25\,n}}{n}+\frac{143\,b^{4}c^{10}x^{24\,n}}{2\,n}+\frac{143\,b^{5}c^{9}x^{23\,n}}{n}+\frac{429\,b^{6}c^{8}x^{22\,n}}{2\,n}+\frac{1716\,b^{7}c^{7}x^{21\,n}}{7\,n}+\frac{429\,b^{8}c^{6}x^{20\,n}}{2\,n}+\frac{143\,b^{9}c^{5}x^{19\,n}}{n}+\frac{143\,b^{10}c^{4}x^{18\,n}}{2\,n}+\frac{26\,b^{11}c^{3}x^{17\,n}}{n}+\frac{13\,b^{12}c^{2}x^{16\,n}}{2\,n}+\frac{b^{13}cx^{15\,n}}{n}+\frac{b^{14}x^{14\,n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="maxima")
```

```
[Out] 1/14*c^14*x^(28*n)/n + b*c^13*x^(27*n)/n + 13/2*b^2*c^12*x^(26*n)/n + 26*b^
3*c^11*x^(25*n)/n + 143/2*b^4*c^10*x^(24*n)/n + 143*b^5*c^9*x^(23*n)/n + 42
9/2*b^6*c^8*x^(22*n)/n + 1716/7*b^7*c^7*x^(21*n)/n + 429/2*b^8*c^6*x^(20*n)
/n + 143*b^9*c^5*x^(19*n)/n + 143/2*b^10*c^4*x^(18*n)/n + 26*b^11*c^3*x^(17
*n)/n + 13/2*b^12*c^2*x^(16*n)/n + b^13*c*x^(15*n)/n + 1/14*b^14*x^(14*n)/n
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.
time = 0.37, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^{2}c^{12}x^{26\,n}+364\,b^{3}c^{11}x^{25\,n}+1001\,b^{4}c^{10}x^{24\,n}+2002\,b^{5}c^{9}x^{23\,n}+3003\,b^{6}c^{8}x^{22\,n}+3432\,b^{7}c^{7}x^{21\,n}+3003\,b^{8}c^{6}x^{20\,n}+2002\,b^{9}c^{5}x^{19\,n}+1001\,b^{10}c^{4}x^{18\,n}+364\,b^{11}c^{3}x^{17\,n}+91\,b^{12}c^{2}x^{16\,n}+14\,b^{13}cx^{15\,n}+b^{14}x^{14\,n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="fricas")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**13,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(19) = 38$.
time = 4.67, size = 189, normalized size = 9.00

$$\frac{c^{14}x^{28\,n}+14\,bc^{13}x^{27\,n}+91\,b^{2}c^{12}x^{26\,n}+364\,b^{3}c^{11}x^{25\,n}+1001\,b^{4}c^{10}x^{24\,n}+2002\,b^{5}c^{9}x^{23\,n}+3003\,b^{6}c^{8}x^{22\,n}+3432\,b^{7}c^{7}x^{21\,n}+3003\,b^{8}c^{6}x^{20\,n}+2002\,b^{9}c^{5}x^{19\,n}+1001\,b^{10}c^{4}x^{18\,n}+364\,b^{11}c^{3}x^{17\,n}+91\,b^{12}c^{2}x^{16\,n}+14\,b^{13}cx^{15\,n}+b^{14}x^{14\,n}}{14\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^13,x, algorithm="giac")
```

```
[Out] 1/14*(c^14*x^(28*n) + 14*b*c^13*x^(27*n) + 91*b^2*c^12*x^(26*n) + 364*b^3*c
^11*x^(25*n) + 1001*b^4*c^10*x^(24*n) + 2002*b^5*c^9*x^(23*n) + 3003*b^6*c^
8*x^(22*n) + 3432*b^7*c^7*x^(21*n) + 3003*b^8*c^6*x^(20*n) + 2002*b^9*c^5*x
^(19*n) + 1001*b^10*c^4*x^(18*n) + 364*b^11*c^3*x^(17*n) + 91*b^12*c^2*x^(1
6*n) + 14*b^13*c*x^(15*n) + b^14*x^(14*n))/n
```

Mupad [B]
time = 2.63, size = 229, normalized size = 10.90

$$\frac{b^{14}x^{14n}}{14n}+\frac{c^{14}x^{28n}}{14n}+\frac{13b^{12}c^2x^{16n}}{2n}+\frac{26b^{11}c^3x^{17n}}{n}+\frac{143b^{10}c^4x^{18n}}{2n}+\frac{143b^9c^5x^{19n}}{n}+\frac{429b^8c^6x^{20n}}{2n}+\frac{1716b^7c^7x^{21n}}{7n}+\frac{429b^6c^8x^{22n}}{2n}+\frac{143b^5c^9x^{23n}}{n}+\frac{143b^4c^{10}x^{24n}}{2n}+\frac{26b^3c^{11}x^{25n}}{n}+\frac{13b^2c^{12}x^{26n}}{2n}+\frac{b^{13}cx^{15n}}{n}+\frac{bc^{13}x^{27n}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^13,x)
```

```
[Out] (b^14*x^(14*n))/(14*n) + (c^14*x^(28*n))/(14*n) + (13*b^12*c^2*x^(16*n))/(2
*n) + (26*b^11*c^3*x^(17*n))/n + (143*b^10*c^4*x^(18*n))/(2*n) + (143*b^9*c
^5*x^(19*n))/n + (429*b^8*c^6*x^(20*n))/(2*n) + (1716*b^7*c^7*x^(21*n))/(7*
n) + (429*b^6*c^8*x^(22*n))/(2*n) + (143*b^5*c^9*x^(23*n))/n + (143*b^4*c^1
0*x^(24*n))/(2*n) + (26*b^3*c^11*x^(25*n))/n + (13*b^2*c^12*x^(26*n))/(2*n)
 + (b^13*c*x^(15*n))/n + (b*c^13*x^(27*n))/n
```

3.105 $\int \frac{b+2cx}{a+bx+cx^2} \, dx$

Optimal. Leaf size=11

$$\log \left(a+bx+cx^2\right)$$

```
[Out] ln(c*x^2+b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {642}

$$\log \left(a+bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(a + b*x + c*x^2),x]
```

```
[Out] Log[a + b*x + c*x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{a+bx+cx^2} \, dx = \log \left(a+bx+cx^2\right)$$

Mathematica [A]
time = 0.00, size = 10, normalized size = 0.91

$$\log(a+x(b+cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2),x]
```

```
[Out] Log[a + x*(b + c*x)]
```

Maple [A]
time = 0.34, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\ln\left(c\,x^{2}+bx+a\right)$	12
default	$\ln\left(c\,x^{2}+bx+a\right)$	12
norman	$\ln\left(c\,x^{2}+bx+a\right)$	12
risch	$\ln\left(c\,x^{2}+bx+a\right)$	12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] ln(c*x^2+b*x+a)
```

Maxima [A]
time = 0.28, size = 11, normalized size = 1.00

$$\log\left(cx^{2}+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x + a)
```

Fricas [A]
time = 0.35, size = 11, normalized size = 1.00

$$\log\left(cx^{2}+bx+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x + a)
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.91

$$\log\left(a+bx+cx^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a),x)
```

```
[Out] log(a + b*x + c*x**2)
```

Giac [A]
time = 4.39, size = 12, normalized size = 1.09

$$\log\left(\left|cx^{2}+bx+a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(c*x^2 + b*x + a))
```

Mupad [B]
time = 1.96, size = 11, normalized size = 1.00

$$\ln\left(c\,x^2+b\,x+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(a + b*x + c*x^2),x)
```

```
[Out] log(a + b*x + c*x^2)
```

3.106 $\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

```
[Out] 1/2*ln(c*x^4+b*x^2+a)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1261, 642}

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(b+2cx^2)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\log\left(a+bx^2+cx^4\right)\end{aligned}$$

Mathematica [A]
time = 0.00, size = 17, normalized size = 1.00

$$\frac{1}{2}\log\left(a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a + b*x^2 + c*x^4]/2
```

Maple [A]
time = 0.01, size = 16, normalized size = 0.94

method	result	size
default	$\frac{\ln(c\,x^4+b\,x^2+a)}{2}$	16
norman	$\frac{\ln(c\,x^4+b\,x^2+a)}{2}$	16
risch	$\frac{\ln(c\,x^4+b\,x^2+a)}{2}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(c*x^4+b*x^2+a)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.88

$$\frac{1}{2}\log\left(cx^4+bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

Fricas [A]
time = 0.34, size = 15, normalized size = 0.88

$$\frac{1}{2}\log\left(cx^4+bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 + a)
```

Sympy [A]
time = 0.14, size = 14, normalized size = 0.82

$$\frac{\log\left(a+bx^2+cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a),x)
```

```
[Out] log(a + b*x**2 + c*x**4)/2
```

Giac [A]
time = 5.37, size = 16, normalized size = 0.94

$$\frac{1}{2}\log\left(\left|cx^4+bx^2+a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^4 + b*x^2 + a))
```

Mupad [B]
time = 1.96, size = 15, normalized size = 0.88

$$\frac{\ln\left(c\,x^4+b\,x^2+a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] log(a + b*x^2 + c*x^4)/2
```

3.107 $\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx$

Optimal. Leaf size=17

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

```
[Out] 1/3*ln(c*x^6+b*x^3+a)
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1482, 642}

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(b+2cx^3)}{a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^3\right)\\ &= \frac{1}{3}\log\left(a+bx^3+cx^6\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 17, normalized size = 1.00

$$\frac{1}{3}\log\left(a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a + b*x^3 + c*x^6]/3
```

Maple [A]
time = 0.02, size = 16, normalized size = 0.94

method	result	size
default	$\frac{\ln(c\,x^6+b\,x^3+a)}{3}$	16
norman	$\frac{\ln(c\,x^6+b\,x^3+a)}{3}$	16
risch	$\frac{\ln(c\,x^6+b\,x^3+a)}{3}$	16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(c*x^6+b*x^3+a)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 0.88

$$\frac{1}{3}\log\left(cx^6+bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

Fricas [A]
time = 0.40, size = 15, normalized size = 0.88

$$\frac{1}{3}\log\left(cx^6+bx^3+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 + a)
```

Sympy [A]
time = 0.24, size = 14, normalized size = 0.82

$$\frac{\log\left(a+bx^3+cx^6\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a),x)
```

```
[Out] log(a + b*x**3 + c*x**6)/3
```

Giac [A]
time = 3.44, size = 16, normalized size = 0.94

$$\frac{1}{3} \log \left(\left|cx^6 + bx^3 + a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^6 + b*x^3 + a))
```

Mupad [B]
time = 0.05, size = 15, normalized size = 0.88

$$\frac{\ln\left(c\,x^6 + b\,x^3 + a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6),x)
```

```
[Out] log(a + b*x^3 + c*x^6)/3
```

3.108 $\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=19

$$\frac{\log(a+bx^n+cx^{2n})}{n}$$

```
[Out] ln(a+b*x^n+c*x^(2*n))/n
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1482, 642}

$$\frac{\log(a+bx^n+cx^{2n})}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}(b+2cx^n)}{a+bx^n+cx^{2n}}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^n\right)}{n}\\ &= \frac{\log(a+bx^n+cx^{2n})}{n}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 19, normalized size = 1.00

$$\frac{\log(a+bx^n+cx^{2n})}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a + b*x^n + c*x^(2*n)]/n
```

Maple [A]
time = 0.03, size = 24, normalized size = 1.26

method	result	size
norman	$\frac{\ln\left(a+b\,\mathrm{e}^{n\ln(x)}+c\,\mathrm{e}^{2n\ln(x)}\right)}{n}$	24
risch	$\frac{\ln\left(x^{2n}+\frac{b\,x^{n}}{c}+\frac{a}{c}\right)}{n}$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln(a+b*exp(n*ln(x))+c*exp(n*ln(x))^2)
```

Maxima [A]
time = 0.31, size = 23, normalized size = 1.21

$$\frac{\log\left(\frac{cx^{2\,n}+bx^{n}+a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log((c*x^(2*n) + b*x^n + a)/c)/n
```

Fricas [A]
time = 0.34, size = 19, normalized size = 1.00

$$\frac{\log\left(cx^{2\,n}+bx^{n}+a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.38, size = 19, normalized size = 1.00

$$\frac{\log\left(cx^{2n}+bx^{n}+a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] log(c*x^(2*n) + b*x^n + a)/n
```

Mupad [B]
time = 2.32, size = 121, normalized size = 6.37

$$-\frac{2\,b\,\mathrm{atan}\left(\frac{b}{\sqrt{4\,a\,c-b^2}}+\frac{2\,c\,x^n}{\sqrt{4\,a\,c-b^2}}\right)-\ln\left(a+b\,x^n+c\,x^{2\,n}\right)\,\sqrt{4\,a\,c-b^2}}{n\,\sqrt{4\,a\,c-b^2}}-\frac{2\,b\,\mathrm{atanh}\left(\frac{b+2\,c\,x^n}{\sqrt{b^2-4\,a\,c}}\right)}{n\,\sqrt{b^2-4\,a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] - (2*b*atan(b/(4*a*c - b^2)^(1/2) + (2*c*x^n)/(4*a*c - b^2)^(1/2)) - log(a
+ b*x^n + c*x^(2*n))*(4*a*c - b^2)^(1/2))/(n*(4*a*c - b^2)^(1/2)) - (2*b*at
anh((b + 2*c*x^n)/(b^2 - 4*a*c)^(1/2)))/(n*(b^2 - 4*a*c)^(1/2))
```

3.109 $\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{7\left(a+bx+cx^2\right)^7}$$

```
[Out] -1/7/(c*x^2+b*x+a)^7
```

Rubi [A]
time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{643\}$

$$-\frac{1}{7\left(a+bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(a + b*x + c*x^2)^7
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8}\,dx=-\frac{1}{7\left(a+bx+cx^2\right)^7}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.94

$$-\frac{1}{7(a+x(b+cx))^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(a + b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(a + x*(b + c*x))^7
```

Maple [A]
time = 0.26, size = 15, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{7(c\,x^{2}+bx+a)^{7}}$	15
derivativedivides	$-\frac{1}{7(c\,x^{2}+bx+a)^{7}}$	15
default	$-\frac{1}{7(c\,x^{2}+bx+a)^{7}}$	15
norman	$-\frac{1}{7(c\,x^{2}+bx+a)^{7}}$	15
risch	$-\frac{1}{7(c\,x^{2}+bx+a)^{7}}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/(c*x^2+b*x+a)^7
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.88

$$-\frac{1}{7\,(cx^2+bx+a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. 2(14) = 28.
time = 0.39, size = 350, normalized size = 21.88

$$-\frac{1}{7\,(c^7x^{14}+7\,bc^6x^{13}+7\,(3\,b^2c^5+ac^6)x^{12}+7\,(5\,b^3c^4+6\,abc^5)x^{11}+7\,(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5)x^{10}+7\,(3\,b^5c^2+20\,ab^3c^3+15\,a^2bc^4)x^9+7\,(b^6c+15\,ab^4c^2+30\,a^2b^2c^3+5\,a^3c^4)x^8+7\,a^6bx+(b^7+42\,ab^5c+210\,a^2b^3c^2+140\,a^3bc^3)x^7+a^7+7\,(ab^6+15\,a^2b^4c+30\,a^3b^2c^2+5\,a^4c^3)x^6+7\,(3\,a^2b^5+20\,a^3b^3c+15\,a^4bc^2)x^5+7\,(5\,a^3b^4+15\,a^4b^2c+3\,a^5c^2)x^4+7\,(5\,a^4b^3+6\,a^5bc)x^3+7\,(3\,a^5b^2+a^6c)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 + a*c^6)*x^12 + 7*(5*b^3*c^4 +
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 + 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2
+ 140*a^3*b*c^3)*x^7 + a^7 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 + 7*(5*a^3*b^
4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^3 + 7*(3*a^
5*b^2 + a^6*c)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(15) = 30$.
time = 3.26, size = 359, normalized size = 22.44

$$-\frac{1}{7a^7 + 49a^6bx + 49bc^6x^{13} + 7c^7x^{14} + x^{12}\cdot(49ac^6 + 147b^2c^5) + x^{11}\cdot(294abc^5 + 245b^3c^4) + x^{10}\cdot(147a^2c^5 + 735ab^2c^4 + 245b^4c^3) + x^9\cdot(735a^2bc^4 + 980ab^3c^3 + 147b^5c^2) + x^8\cdot(245a^3c^4 + 1470a^2b^2c^3 + 735ab^4c^2 + 49b^6c) + x^7\cdot(980a^3bc^3 + 1470a^2b^3c^2 + 294ab^5c + 7b^7) + x^6\cdot(245a^4c^3 + 1470a^3b^2c^2 + 735a^2b^4c + 49ab^6) + x^5\cdot(735a^4bc^2 + 980a^3b^3c + 147a^2b^5) + x^4\cdot(147a^5c^2 + 735a^4b^2c + 245a^3b^4) + x^3\cdot(294a^5bc + 245a^4b^3) + x^2\cdot(49a^6c + 147a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x+a)**8,x)
```

```
[Out] -1/(7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(49*a*c**
6 + 147*b**2*c**5) + x**11*(294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a**2
*c**5 + 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 + 980*a*b*
*3*c**3 + 147*b**5*c**2) + x**8*(245*a**3*c**4 + 1470*a**2*b**2*c**3 + 735*
a*b**4*c**2 + 49*b**6*c) + x**7*(980*a**3*b*c**3 + 1470*a**2*b**3*c**2 + 29
4*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 + 1470*a**3*b**2*c**2 + 735*a**2
*b**4*c + 49*a*b**6) + x**5*(735*a**4*b*c**2 + 980*a**3*b**3*c + 147*a**2*b
**5) + x**4*(147*a**5*c**2 + 735*a**4*b**2*c + 245*a**3*b**4) + x**3*(294*a
**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c + 147*a**5*b**2))
```

Giac [A]
time = 2.93, size = 14, normalized size = 0.88

$$-\frac{1}{7\,(cx^2 + bx + a)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x+a)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x + a)^7
```

Mupad [B]
time = 3.62, size = 358, normalized size = 22.38

$$-\frac{1}{7\,(x^5\,(105\,a^4\,b\,c^2 + 140\,a^3\,b^3\,c + 21\,a^2\,b^5) + x^9\,(105\,a^2\,b\,c^4 + 140\,a\,b^3\,c^3 + 21\,b^5\,c^2) + x^7\,(140\,a^3\,b\,c^3 + 210\,a^2\,b^3\,c^2 + 42\,a\,b^5\,c + b^7) + x^3\,(42\,c\,a^5\,b + 35\,a^4\,b^3) + x^{11}\,(35\,b^3\,c^4 + 42\,a\,b\,c^5) + x^4\,(21\,a^5\,c^2 + 105\,a^4\,b^2\,c + 35\,a^3\,b^4) + x^{10}\,(21\,a^2\,c^5 + 105\,a\,b^2\,c^4 + 35\,b^4\,c^3) + a^7 + x^6\,(35\,a^4\,c^3 + 210\,a^3\,b^2\,c^2 + 105\,a^2\,b^4\,c + 7\,a\,b^6) + x^8\,(35\,a^3\,c^4 + 210\,a^2\,b^2\,c^3 + 105\,a\,b^4\,c^2 + 7\,b^6\,c) + c^7\,x^{14} + x^2\,(7\,c\,a^6 + 21\,a^5\,b^2) + x^{12}\,(21\,b^2\,c^5 + 7\,a\,c^6) + 7\,b\,c^6\,x^{13} + 7\,a^6\,b\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(a + b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*(x^5*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 +
 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^3*c^
2 + 42*a*b^5*c) + x^3*(35*a^4*b^3 + 42*a^5*b*c) + x^11*(35*b^3*c^4 + 42*a*b
*c^5) + x^4*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^10*(21*a^2*c^5 +
35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^6*(7*a*b^6 + 35*a^4*c^3 + 105*a^2*b^4
*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 210*a^2
*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c + 21*a^5*b^2) + x^12*(7*a*c^6 + 21*b^2*
c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

3.110 $\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

```
[Out] -1/14/(c*x^4+b*x^2+a)^7
```

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1261, 643}

$$-\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(a + b*x^2 + c*x^4)^7
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(b+2cx^2)}{(a+bx^2+cx^4)^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx,x,x^2\right)\\ &= -\frac{1}{14\left(a+bx^2+cx^4\right)^7}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{14\left(a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(a + b*x^2 + c*x^4)^7
```

Maple [A]
time = 0.20, size = 17, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{14\left(c\,x^{4}+b\,x^{2}+a\right)^{7}}$	17
default	$-\frac{1}{14\left(c\,x^{4}+b\,x^{2}+a\right)^{7}}$	17
norman	$-\frac{1}{14\left(c\,x^{4}+b\,x^{2}+a\right)^{7}}$	17
risch	$-\frac{1}{14\left(c\,x^{4}+b\,x^{2}+a\right)^{7}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/14/(c*x^4+b*x^2+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(16) = 32$.
time = 0.40, size = 352, normalized size = 19.56

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+7\left(3\,b^{2}c^{5}+ac^{6}\right)x^{24}+7\left(5\,b^{3}c^{4}+6\,abc^{5}\right)x^{22}+7\left(5\,b^{4}c^{3}+15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{20}+7\left(3\,b^{5}c^{2}+20\,ab^{3}c^{3}+15\,a^{2}bc^{4}\right)x^{18}+7\left(b^{6}c+15\,ab^{4}c^{2}+30\,a^{2}b^{2}c^{3}+5\,a^{3}c^{4}\right)x^{16}+\left(b^{7}+42\,ab^{5}c+210\,a^{2}b^{3}c^{2}+140\,a^{3}bc^{3}\right)x^{14}+7\left(ab^{6}+15\,a^{2}b^{4}c+30\,a^{3}b^{2}c^{2}+5\,a^{4}c^{3}\right)x^{12}+7\left(3\,a^{2}b^{5}+20\,a^{3}b^{3}c+15\,a^{4}bc^{2}\right)x^{10}+7\,a^{6}bx^{2}+7\left(5\,a^{3}b^{4}+15\,a^{4}b^{2}c+3\,a^{5}c^{2}\right)x^{8}+a^{7}+7\left(5\,a^{4}b^{3}+6\,a^{5}bc\right)x^{6}+7\left(3\,a^{5}b^{2}+a^{6}c\right)x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*
b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 +
 7*(3*a^5*b^2 + a^6*c)*x^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(16) = 32$.
time = 0.39, size = 352, normalized size = 19.56

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+7\left(3\,b^{2}c^{5}+ac^{6}\right)x^{24}+7\left(5\,b^{3}c^{4}+6\,abc^{5}\right)x^{22}+7\left(5\,b^{4}c^{3}+15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{20}+7\left(3\,b^{5}c^{2}+20\,ab^{3}c^{3}+15\,a^{2}bc^{4}\right)x^{18}+7\left(b^{6}c+15\,ab^{4}c^{2}+30\,a^{2}b^{2}c^{3}+5\,a^{3}c^{4}\right)x^{16}+\left(b^{7}+42\,ab^{5}c+210\,a^{2}b^{3}c^{2}+140\,a^{3}bc^{3}\right)x^{14}+7\left(ab^{6}+15\,a^{2}b^{4}c+30\,a^{3}b^{2}c^{2}+5\,a^{4}c^{3}\right)x^{12}+7\left(3\,a^{2}b^{5}+20\,a^{3}b^{3}c+15\,a^{4}bc^{2}\right)x^{10}+7\,a^{6}bx^{2}+7\left(5\,a^{3}b^{4}+15\,a^{4}b^{2}c+3\,a^{5}c^{2}\right)x^{8}+a^{7}+7\left(5\,a^{4}b^{3}+6\,a^{5}bc\right)x^{6}+7\left(3\,a^{5}b^{2}+a^{6}c\right)x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 + a*c^6)*x^24 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^16 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^14 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 + 7*(5*a^3*
b^4 + 15*a^4*b^2*c + 3*a^5*c^2)*x^8 + a^7 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^6 +
 7*(3*a^5*b^2 + a^6*c)*x^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(17) = 34$.
time = 4.88, size = 360, normalized size = 20.00

$$-\frac{1}{14a^7 + 98a^6bx^2 + 98bc^6x^{26} + 14c^7x^{28} + x^{24}\cdot(98ac^6 + 294b^2c^5) + x^{22}\cdot(588abc^5 + 490b^3c^4) + x^{20}\cdot(294a^2c^5 + 1470ab^2c^4 + 490b^4c^3) + x^{18}\cdot(1470a^2bc^4 + 1960ab^3c^3 + 294b^5c^2) + x^{16}\cdot(490a^3c^4 + 2940a^2b^2c^3 + 1470ab^4c^2 + 98b^6c) + x^{14}\cdot(1960a^3bc^3 + 2940a^2b^3c^2 + 588ab^5c + 14b^7) + x^{12}\cdot(490a^4c^3 + 2940a^3b^2c^2 + 1470a^2b^4c + 98ab^6) + x^{10}\cdot(1470a^4bc^2 + 1960a^3b^3c + 294a^2b^5) + x^8\cdot(294a^5c^2 + 1470a^4b^2c + 490a^3b^4) + x^6\cdot(588a^5bc + 490a^4b^3) + x^4\cdot(98a^6c + 294a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2+a)**8,x)
```

```
[Out] -1/(14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(98*
a*c**6 + 294*b**2*c**5) + x**22*(588*a*b*c**5 + 490*b**3*c**4) + x**20*(294
*a**2*c**5 + 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4 +
1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(490*a**3*c**4 + 2940*a**2*b**2*c
**3 + 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(1960*a**3*b*c**3 + 2940*a**2*b
**3*c**2 + 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 + 2940*a**3*b**2*
c**2 + 1470*a**2*b**4*c + 98*a*b**6) + x**10*(1470*a**4*b*c**2 + 1960*a**3*
b**3*c + 294*a**2*b**5) + x**8*(294*a**5*c**2 + 1470*a**4*b**2*c + 490*a**3
*b**4) + x**6*(588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c + 294*a**5*b
**2))
```

Giac [A]
time = 5.94, size = 16, normalized size = 0.89

$$-\frac{1}{14\left(cx^4+bx^2+a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2+a)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2 + a)^7
```

Mupad [B]
time = 12.16, size = 360, normalized size = 20.00

$$-\frac{1}{14\,\left(x^{10}\,\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{18}\,\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{14}\,\left(140\,a^3\,b\,c^3+210\,a^2\,b^3\,c^2+42\,a\,b^5\,c+b^7\right)+x^6\,\left(42\,c\,a^5\,b+35\,a^4\,b^3\right)+x^{22}\,\left(35\,b^3\,c^4+42\,a\,b\,c^5\right)+x^8\,\left(21\,a^5\,c^2+105\,a^4\,b^2\,c+35\,a^3\,b^4\right)+x^{20}\,\left(21\,a^2\,c^5+105\,a\,b^2\,c^4+35\,b^4\,c^3\right)+a^7+x^{12}\,\left(35\,a^4\,c^3+210\,a^3\,b^2\,c^2+105\,a^2\,b^4\,c+7\,a\,b^6\right)+x^{16}\,\left(35\,a^3\,c^4+210\,a^2\,b^2\,c^3+105\,a\,b^4\,c^2+7\,b^6\,c\right)+c^7\,x^{28}+x^4\,\left(7\,c\,a^6+21\,a^5\,b^2\right)+x^{24}\,\left(21\,b^2\,c^5+7\,a\,c^6\right)+7\,a^6\,b\,x^2+7\,b\,c^6\,x^{26}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(a + b*x^2 + c*x^4)^8,x)
```

```
[Out] -1/(14*(x^10*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c^
2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 + 42*a*b^5*c) + x^6*(35*a^4*b^3 + 42*a^5*b*c) + x^22*(35*b^3*c^4 + 42
*a*b*c^5) + x^8*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^20*(21*a^2*c^
5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^12*(7*a*b^6 + 35*a^4*c^3 + 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c + 21*a^5*b^2) + x^24*(7*a*c^6 + 2
1*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

3.111 $\int \frac{x^2\left(b+2cx^3\right)}{\left(a+bx^3+cx^6\right)^8}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

[Out] -1/21/(c*x^6+b*x^3+a)^7

Rubi [A]
time = 0.01, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1482, 643}

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]

[Out] -1/21*1/(a + b*x^3 + c*x^6)^7

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(b+2cx^3)}{\left(a+bx^3+cx^6\right)^8}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{\left(a+bx+cx^2\right)^8}\,dx,x,x^3\right)\\
&= -\frac{1}{21\left(a+bx^3+cx^6\right)^7}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 18, normalized size = 1.00

$$-\frac{1}{21\left(a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(a + b*x^3 + c*x^6)^7
```

Maple [A]
time = 0.08, size = 17, normalized size = 0.94

method	result	size
gosper	$-\frac{1}{21\left(c\,x^{6}+b\,x^{3}+a\right)^{7}}$	17
default	$-\frac{1}{21\left(c\,x^{6}+b\,x^{3}+a\right)^{7}}$	17
risch	$-\frac{1}{21\left(c\,x^{6}+b\,x^{3}+a\right)^{7}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/21/(c*x^6+b*x^3+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(16) = 32.
time = 0.40, size = 352, normalized size = 19.56

$$-\frac{1}{21\,\left(c^{7} x^{42} + 7\,b c^{6} x^{39} + 7\,{\left(3\,b^{2} c^{5} + a c^{6}\right)} x^{36} + 7\,{\left(5\,b^{3} c^{4} + 6\,a b c^{5}\right)} x^{33} + 7\,{\left(5\,b^{4} c^{3} + 15\,a b^{2} c^{4} + 3\,a^{2} c^{5}\right)} x^{30} + 7\,{\left(3\,b^{5} c^{2} + 20\,a b^{3} c^{3} + 15\,a^{2} b c^{4}\right)} x^{27} + 7\,{\left(b^{6} c + 15\,a b^{4} c^{2} + 30\,a^{2} b^{2} c^{3} + 5\,a^{3} c^{4}\right)} x^{24} + {\left(b^{7} + 42\,a b^{5} c + 210\,a^{2} b^{3} c^{2} + 140\,a^{3} b c^{3}\right)} x^{21} + 7\,{\left(a b^{6} + 15\,a^{2} b^{4} c + 30\,a^{3} b^{2} c^{2} + 5\,a^{4} c^{3}\right)} x^{18} + 7\,{\left(3\,a^{2} b^{5} + 20\,a^{3} b^{3} c + 15\,a^{4} b c^{2}\right)} x^{15} + 7\,{\left(5\,a^{3} b^{4} + 15\,a^{4} b^{2} c + 3\,a^{5} c^{2}\right)} x^{12} + 7\,a^{6} b x^{3} + 7\,{\left(5\,a^{4} b^{3} + 6\,a^{5} b c\right)} x^{9} + a^{7} + 7\,{\left(3\,a^{5} b^{2} + a^{6} c\right)} x^{6}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(16) = 32.
time = 0.39, size = 352, normalized size = 19.56

$$-\frac{1}{21\,\left(c^{7} x^{42} + 7\,b c^{6} x^{39} + 7\,{\left(3\,b^{2} c^{5} + a c^{6}\right)} x^{36} + 7\,{\left(5\,b^{3} c^{4} + 6\,a b c^{5}\right)} x^{33} + 7\,{\left(5\,b^{4} c^{3} + 15\,a b^{2} c^{4} + 3\,a^{2} c^{5}\right)} x^{30} + 7\,{\left(3\,b^{5} c^{2} + 20\,a b^{3} c^{3} + 15\,a^{2} b c^{4}\right)} x^{27} + 7\,{\left(b^{6} c + 15\,a b^{4} c^{2} + 30\,a^{2} b^{2} c^{3} + 5\,a^{3} c^{4}\right)} x^{24} + {\left(b^{7} + 42\,a b^{5} c + 210\,a^{2} b^{3} c^{2} + 140\,a^{3} b c^{3}\right)} x^{21} + 7\,{\left(a b^{6} + 15\,a^{2} b^{4} c + 30\,a^{3} b^{2} c^{2} + 5\,a^{4} c^{3}\right)} x^{18} + 7\,{\left(3\,a^{2} b^{5} + 20\,a^{3} b^{3} c + 15\,a^{4} b c^{2}\right)} x^{15} + 7\,{\left(5\,a^{3} b^{4} + 15\,a^{4} b^{2} c + 3\,a^{5} c^{2}\right)} x^{12} + 7\,a^{6} b x^{3} + 7\,{\left(5\,a^{4} b^{3} + 6\,a^{5} b c\right)} x^{9} + a^{7} + 7\,{\left(3\,a^{5} b^{2} + a^{6} c\right)} x^{6}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 + a*c^6)*x^36 + 7*(5*b^3*c^4
+ 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 + 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 + 5*a^3*c^4)*x^24 + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3
*b*c^3)*x^21 + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 + 7*(5*a^3*b^4 + 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 + 6*a^5*b*c)*x^9 + a^7
+ 7*(3*a^5*b^2 + a^6*c)*x^6)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(17) = 34$.
time = 15.87, size = 360, normalized size = 20.00

$$-\frac{1}{21a^7 + 147a^6bx^3 + 147bc^6x^{39} + 21c^7x^{42} + x^{36}\cdot(147ac^6 + 441b^2c^5) + x^{33}\cdot(882abc^5 + 735b^3c^4) + x^{30}\cdot(441a^2c^5 + 2205ab^2c^4 + 735b^4c^3) + x^{27}\cdot(2205a^2bc^4 + 2940ab^3c^3 + 441b^5c^2) + x^{24}\cdot(735a^3c^4 + 4410a^2b^2c^3 + 2205ab^4c^2 + 147b^6c) + x^{21}\cdot(2940a^3bc^3 + 4410a^2b^3c^2 + 882ab^5c + 21b^7) + x^{18}\cdot(735a^4c^3 + 4410a^3b^2c^2 + 2205a^2b^4c + 147ab^6) + x^{15}\cdot(2205a^4bc^2 + 2940a^3b^3c + 441a^2b^5) + x^{12}\cdot(441a^5c^2 + 2205a^4b^2c + 735a^3b^4) + x^9\cdot(882a^5bc + 735a^4b^3) + x^6\cdot(147a^6c + 441a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3+a)**8,x)
```

```
[Out] -1/(21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(1
47*a*c**6 + 441*b**2*c**5) + x**33*(882*a*b*c**5 + 735*b**3*c**4) + x**30*(
441*a**2*c**5 + 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c**4
 + 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(735*a**3*c**4 + 4410*a**2*b**
2*c**3 + 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(2940*a**3*b*c**3 + 4410*a*
*2*b**3*c**2 + 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 + 4410*a**3*b
**2*c**2 + 2205*a**2*b**4*c + 147*a*b**6) + x**15*(2205*a**4*b*c**2 + 2940*
a**3*b**3*c + 441*a**2*b**5) + x**12*(441*a**5*c**2 + 2205*a**4*b**2*c + 73
5*a**3*b**4) + x**9*(882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*c + 441
*a**5*b**2))
```

Giac [A]
time = 6.16, size = 16, normalized size = 0.89

$$-\frac{1}{21\left(cx^6 + bx^3 + a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3+a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3 + a)^7
```

Mupad [B]
time = 18.21, size = 360, normalized size = 20.00

$$-\frac{1}{21\,\left(x^{15}\,\left(105\,a^4\,b\,c^2+140\,a^3\,b^3\,c+21\,a^2\,b^5\right)+x^{27}\,\left(105\,a^2\,b\,c^4+140\,a\,b^3\,c^3+21\,b^5\,c^2\right)+x^{21}\,\left(140\,a^3\,b\,c^3+210\,a^2\,b^3\,c^2+42\,a\,b^5\,c+b^7\right)+x^9\,\left(42\,c\,a^5\,b+35\,a^4\,b^3\right)+x^{33}\,\left(35\,b^3\,c^4+42\,a\,b\,c^5\right)+x^{12}\,\left(21\,a^5\,c^2+105\,a^4\,b^2\,c+35\,a^3\,b^4\right)+x^{30}\,\left(21\,a^2\,c^5+105\,a\,b^2\,c^4+35\,b^4\,c^3\right)+a^7+x^{18}\,\left(35\,a^4\,c^3+210\,a^3\,b^2\,c^2+105\,a^2\,b^4\,c+7\,a\,b^6\right)+x^{24}\,\left(35\,a^3\,c^4+210\,a^2\,b^2\,c^3+105\,a\,b^4\,c^2+7\,b^6\,c\right)+c^7\,x^{42}+x^6\,\left(7\,c\,a^6+21\,a^5\,b^2\right)+x^{36}\,\left(21\,b^2\,c^5+7\,a\,c^6\right)+7\,a^6\,b\,x^3+7\,b\,c^6\,x^{39}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(a + b*x^3 + c*x^6)^8,x)
```

```
[Out] -1/(21*(x^15*(21*a^2*b^5 + 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c^
2 + 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 + 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 + 42*a*b^5*c) + x^9*(35*a^4*b^3 + 42*a^5*b*c) + x^33*(35*b^3*c^4 + 42
*a*b*c^5) + x^12*(35*a^3*b^4 + 21*a^5*c^2 + 105*a^4*b^2*c) + x^30*(21*a^2*c
^5 + 35*b^4*c^3 + 105*a*b^2*c^4) + a^7 + x^18*(7*a*b^6 + 35*a^4*c^3 + 105*a
^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c + 35*a^3*c^4 + 105*a*b^4*c^2 +
210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c + 21*a^5*b^2) + x^36*(7*a*c^6 +
21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

3.112 $\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

```
[Out] -1/7/n/(a+b*x^n+c*x^(2*n))^7
```

Rubi [A]
time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1482, 643}

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*(a + b*x^n + c*x^(2*n))^7)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}(b+2cx^n)}{(a+bx^n+cx^{2n})^8}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{(a+bx+cx^2)^8}\,dx, x, x^n\right)}{n}\\ &= -\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 23, normalized size = 1.00

$$-\frac{1}{7n\left(a+bx^n+cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*(a + b*x^n + c*x^(2*n))^7)
```

Maple [A]
time = 0.07, size = 22, normalized size = 0.96

method	result	size
risch	$-\frac{1}{7n\left(a+b\,x^{n}+c\,x^{2n}\right)^{7}}$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/7/n/(a+b*x^n+c*(x^n)^2)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. 2(21) = 42.
time = 0.74, size = 416, normalized size = 18.09

$$-\frac{1}{7\left(c^7nx^{14n}+7\,bc^6nx^{13n}+7\,a^6bnx^n+a^7n+7\left(3\,b^2c^5n+ac^6n\right)x^{12n}+7\left(5\,b^3c^4n+6\,abc^5n\right)x^{11n}+7\left(5\,b^4c^3n+15\,ab^2c^4n+3\,a^2c^5n\right)x^{10n}+7\left(3\,b^5c^2n+20\,ab^3c^3n+15\,a^2bc^4n\right)x^{9n}+7\left(b^6cn+15\,ab^4c^2n+30\,a^2b^2c^3n+5\,a^3c^4n\right)x^{8n}+\left(b^7n+42\,ab^5cn+210\,a^2b^3c^2n+140\,a^3bc^3n\right)x^{7n}+7\left(ab^6n+15\,a^2b^4cn+30\,a^3b^2c^2n+5\,a^4c^3n\right)x^{6n}+7\left(3\,a^2b^5n+20\,a^3b^3cn+15\,a^4bc^2n\right)x^{5n}+7\left(5\,a^3b^4n+15\,a^4b^2cn+3\,a^5c^2n\right)x^{4n}+7\left(5\,a^4b^3n+6\,a^5bcn\right)x^{3n}+7\left(3\,a^5b^2n+a^6cn\right)x^{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5*n + a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n + 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n + 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n + 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n + 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n + 5*a^3*c^4*n)*x^(8*n) + (b^7*n + 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n + 140*a^3*b*c^3*n)*x^(7*n) + 7*(a*b^6*n + 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n + 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n + 20*a^3*b^3*c*n + 15*a^4*b*c^2
*n)*x^(5*n) + 7*(5*a^3*b^4*n + 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n + 6*a^5*b*c*n)*x^(3*n) + 7*(3*a^5*b^2*n + a^6*c*n)*x^(2*n))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(21) = 42.
time = 0.40, size = 394, normalized size = 17.13

$$-\frac{1}{7\left(c^7nx^{14n}+7\,bc^6nx^{13n}+7\,a^6bnx^n+a^7n+7\left(3\,b^2c^5+ac^6\right)nx^{12n}+7\left(5\,b^3c^4+6\,abc^5\right)nx^{11n}+7\left(5\,b^4c^3+15\,ab^2c^4+3\,a^2c^5\right)nx^{10n}+7\left(3\,b^5c^2+20\,ab^3c^3+15\,a^2bc^4\right)nx^{9n}+7\left(b^6c+15\,ab^4c^2+30\,a^2b^2c^3+5\,a^3c^4\right)nx^{8n}+\left(b^7+42\,ab^5c+210\,a^2b^3c^2+140\,a^3bc^3\right)nx^{7n}+7\left(ab^6+15\,a^2b^4c+30\,a^3b^2c^2+5\,a^4c^3\right)nx^{6n}+7\left(3\,a^2b^5+20\,a^3b^3c+15\,a^4bc^2\right)nx^{5n}+7\left(5\,a^3b^4+15\,a^4b^2c+3\,a^5c^2\right)nx^{4n}+7\left(5\,a^4b^3+6\,a^5bc\right)nx^{3n}+7\left(3\,a^5b^2+a^6c\right)nx^{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n + a^7*n + 7*(3*b^
2*c^5 + a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 + 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 + 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 + 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c + 15*a*b^4*c^2 + 30*a^2*b^2*c^3 + 5*a^3
*c^4)*n*x^(8*n) + (b^7 + 42*a*b^5*c + 210*a^2*b^3*c^2 + 140*a^3*b*c^3)*n*x^
(7*n) + 7*(a*b^6 + 15*a^2*b^4*c + 30*a^3*b^2*c^2 + 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 + 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) + 7*(5*a^3*b^4 + 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 + 6*a^5*b*c)*n*x^(3*n) + 7*(3
*a^5*b^2 + a^6*c)*n*x^(2*n))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.15, size = 21, normalized size = 0.91

$$-\frac{1}{7\left(cx^{2n}+bx^n+a\right)^7n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n + a)^7*n)
```

Mupad [B]
time = 23.01, size = 496, normalized size = 21.57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(a + b*x^n + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*a^7*n + 7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n + 49*a*b^
6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) + 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*n)
 + 49*b*c^6*n*x^(13*n) + 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) + 24
5*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) + 147*a^5*c^2*n*x^(4*n) + 245*a
```

```
^4*c^3*n*x^(6*n) + 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 147*b^5
*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 147*b^2*
c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) + 980*a^3*b^3*c*n*x^(5*n) + 735*a^
4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) + 980*a^3*b*c^3*n*x^(7*n) + 735
*a*b^4*c^2*n*x^(8*n) + 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x^(9*n) +
735*a*b^2*c^4*n*x^(10*n) + 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*b^3*c^2*n*
x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) + 294*a^5*b*c*n*x^(3*n) + 294*a*b^5*c*
n*x^(7*n) + 294*a*b*c^5*n*x^(11*n))
```

3.113 $\int \frac{b+2cx}{-a+bx+cx^2}\,dx$

Optimal. Leaf size=13

$$\log\left(a-bx-cx^2\right)$$

```
[Out] ln(-c*x^2-b*x+a)
```

Rubi [A]
time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {642}

$$\log\left(a-bx-cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[a - b*x - c*x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{-a+bx+cx^2}\,dx = \log\left(a-bx-cx^2\right)$$

Mathematica [A]
time = 0.00, size = 12, normalized size = 0.92

$$\log(-a+x(b+cx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2),x]
```

```
[Out] Log[-a + x*(b + c*x)]
```

Maple [A]
time = 0.22, size = 14, normalized size = 1.08

method	result	size
derivativedivides	$\ln\left(c\,x^{2}+bx-a\right)$	14
default	$\ln\left(-c\,x^{2}-bx+a\right)$	14
norman	$\ln\left(-c\,x^{2}-bx+a\right)$	14
risch	$\ln\left(-c\,x^{2}-bx+a\right)$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x-a),x,method=_RETURNVERBOSE)
[Out] ln(-c*x^2-b*x+a)
```

Maxima [A]
time = 0.28, size = 13, normalized size = 1.00

$$\log\left(cx^{2}+bx-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="maxima")
[Out] log(c*x^2 + b*x - a)
```

Fricas [A]
time = 0.34, size = 13, normalized size = 1.00

$$\log\left(cx^{2}+bx-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="fricas")
[Out] log(c*x^2 + b*x - a)
```

Sympy [A]
time = 0.06, size = 10, normalized size = 0.77

$$\log\left(-a+bx+cx^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a),x)
[Out] log(-a + b*x + c*x**2)
```

Giac [A]
time = 2.77, size = 14, normalized size = 1.08

$$\log\left(\left|cx^{2}+bx-a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a),x, algorithm="giac")
```

```
[Out] log(abs(c*x^2 + b*x - a))
```

Mupad [B]
time = 0.05, size = 13, normalized size = 1.00

$$\ln\left(c\,x^2+b\,x-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x - a + c*x^2),x)
```

```
[Out] log(b*x - a + c*x^2)
```

3.114 $\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

```
[Out] 1/2*ln(-c*x^4-b*x^2+a)
```

Rubi [A]
time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1261, 642}

$$\frac{1}{2}\log\left(a-bx^2-cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[a - b*x^2 - c*x^4]/2
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(b+2cx^2)}{-a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\log\left(a-bx^2-cx^4\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{2}\log\left(-a+bx^2+cx^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4),x]
```

```
[Out] Log[-a + b*x^2 + c*x^4]/2
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.95

method	result	size
default	$\frac{\ln(-c x^4-b x^2+a)}{2}$	18
norman	$\frac{\ln(-c x^4-b x^2+a)}{2}$	18
risch	$\frac{\ln(-c x^4-b x^2+a)}{2}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(-c*x^4-b*x^2+a)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.89

$$\frac{1}{2}\log\left(cx^4+bx^2-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

Fricas [A]
time = 0.34, size = 17, normalized size = 0.89

$$\frac{1}{2}\log\left(cx^4+bx^2-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^4 + b*x^2 - a)
```

Sympy [A]
time = 0.17, size = 14, normalized size = 0.74

$$\frac{\log\left(-a+bx^2+cx^4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a),x)
```

```
[Out] log(-a + b*x**2 + c*x**4)/2
```

Giac [A]
time = 3.80, size = 18, normalized size = 0.95

$$\frac{1}{2}\log\left(\left|cx^4+bx^2-a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^4 + b*x^2 - a))
```

Mupad [B]
time = 0.05, size = 17, normalized size = 0.89

$$\frac{\ln\left(c\,x^4+b\,x^2-a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4),x)
```

```
[Out] log(b*x^2 - a + c*x^4)/2
```

3.115 $\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6} dx$

Optimal. Leaf size=19

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

```
[Out] 1/3*ln(-c*x^6-b*x^3+a)
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = $\{1482, 642\}$

$$\frac{1}{3}\log\left(a-bx^3-cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[a - b*x^3 - c*x^6]/3
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(b+2cx^3)}{-a+bx^3+cx^6}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2}\,dx, x, x^3\right)\\ &= \frac{1}{3}\log\left(a-bx^3-cx^6\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 19, normalized size = 1.00

$$\frac{1}{3}\log\left(-a+bx^3+cx^6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6),x]
```

```
[Out] Log[-a + b*x^3 + c*x^6]/3
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.95

method	result	size
default	$\frac{\ln\left(-c\,x^{6}-b\,x^{3}+a\right)}{3}$	18
norman	$\frac{\ln\left(-c\,x^{6}-b\,x^{3}+a\right)}{3}$	18
risch	$\frac{\ln\left(-c\,x^{6}-b\,x^{3}+a\right)}{3}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*ln(-c*x^6-b*x^3+a)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 0.89

$$\frac{1}{3}\,\log\left(cx^{6}+bx^{3}-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

Fricas [A]
time = 0.33, size = 17, normalized size = 0.89

$$\frac{1}{3}\,\log\left(cx^{6}+bx^{3}-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^6 + b*x^3 - a)
```

Sympy [A]
time = 0.21, size = 14, normalized size = 0.74

$$\frac{\log\left(-a+bx^{3}+cx^{6}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a),x)
```

```
[Out] log(-a + b*x**3 + c*x**6)/3
```

Giac [A]
time = 4.08, size = 18, normalized size = 0.95

$$\frac{1}{3}\log\left(\left|cx^6+bx^3-a\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^6 + b*x^3 - a))
```

Mupad [B]
time = 0.06, size = 17, normalized size = 0.89

$$\frac{\ln\left(c\,x^6+b\,x^3-a\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6),x)
```

```
[Out] log(b*x^3 - a + c*x^6)/3
```

3.116 $\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} \, dx$

Optimal. Leaf size=21

$$\frac{\log(a-bx^n-cx^{2n})}{n}$$

[Out] ln(a-b*x^n-c*x^(2*n))/n

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1482, 642}

$$\frac{\log(a-bx^n-cx^{2n})}{n}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]

[Out] Log[a - b*x^n - c*x^(2*n)]/n

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^{-1+n}(b+2cx^n)}{-a+bx^n+cx^{2n}} \, dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{-a+bx+cx^2} \, dx, x, x^n\right)}{n}\\ &= \frac{\log(a-bx^n-cx^{2n})}{n}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 21, normalized size = 1.00

$$\frac{\log(a-bx^n-cx^{2n})}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[a - b*x^n - c*x^(2*n)]/n
```

Maple [A]
time = 0.03, size = 26, normalized size = 1.24

method	result	size
norman	$\frac{\ln\left(-c\,{\mathrm e}^{2 n \ln\left(x\right)}-b\,{\mathrm e}^{n \ln\left(x\right)}+a\right)}{n}$	26
risch	$\frac{\ln\left(x^{2 n}+\frac{b\,x^{n}}{c}-\frac{a}{c}\right)}{n}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/n*ln(-c*exp(n*ln(x))^2-b*exp(n*ln(x))+a)
```

Maxima [A]
time = 0.32, size = 25, normalized size = 1.19

$$\frac{\log\left(\frac{cx^{2\,n}+bx^{n}-a}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] log((c*x^(2*n) + b*x^n - a)/c)/n
```

Fricas [A]
time = 0.35, size = 21, normalized size = 1.00

$$\frac{\log\left(cx^{2\,n}+bx^{n}-a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.71, size = 21, normalized size = 1.00

$$\frac{\log\left(cx^{2n} + bx^{n} - a\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] log(c*x^(2*n) + b*x^n - a)/n
```

Mupad [B]
time = 2.68, size = 199, normalized size = 9.48

$$\ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,\left(b+2\,c\,x^n\right)\right)\,\left(\frac{1}{n}+\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)+\ln\left(\frac{2\,c\,x^n}{n}-\left(\frac{1}{n}-\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)\,\left(b+2\,c\,x^n\right)\right)\,\left(\frac{1}{n}-\frac{b\,\sqrt{b^2+4\,a\,c}}{n\,b^2+4\,a\,c\,n}\right)-\frac{2\,b\,\mathrm{atanh}\left(\frac{b+2\,c\,x^n}{\sqrt{b^2+4\,a\,c}}\right)}{n\,\sqrt{b^2+4\,a\,c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n)),x)
```

```
[Out] log((2*c*x^n)/n - (1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2*
c*x^n))*(1/n + (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) + log((2*c*x^n)/n
 - (1/n - (b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n))*(b + 2*c*x^n))*(1/n -
(b*(4*a*c + b^2)^(1/2))/(b^2*n + 4*a*c*n)) - (2*b*atanh((b + 2*c*x^n)/(4*a*
c + b^2)^(1/2)))/(n*(4*a*c + b^2)^(1/2))
```

3.117 $\int \frac{b+2cx}{(-a+bx+cx^2)^8} dx$

Optimal. Leaf size=18

$$\frac{1}{7 (a-b x-c x^2)^7}$$

[Out] 1/7/(-c*x^2-b*x+a)^7

Rubi [A]
time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{643\}$

$$\frac{1}{7 (a-b x-c x^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]

[Out] 1/(7*(a - b*x - c*x^2)^7)

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2 c x}{(-a+b x+c x^2)^8} \, dx = \frac{1}{7 (a-b x-c x^2)^7}$$

Mathematica [A]
time = 0.01, size = 16, normalized size = 0.89

$$\frac{1}{7(a-x(b+cx))^7}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)/(-a + b*x + c*x^2)^8,x]

[Out] 1/(7*(a - x*(b + c*x))^7)

Maple [A]
time = 0.26, size = 17, normalized size = 0.94

method	result	size
gosper	$\frac{1}{7(-c\,x^{2}-bx+a)^{7}}$	17
derivativedivides	$-\frac{1}{7(c\,x^{2}+bx-a)^{7}}$	17
default	$\frac{1}{7(-c\,x^{2}-bx+a)^{7}}$	17
norman	$\frac{1}{7(-c\,x^{2}-bx+a)^{7}}$	17
risch	$\frac{1}{7(-c\,x^{2}-bx+a)^{7}}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x-a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/(-c*x^2-b*x+a)^7
```

Maxima [A]
time = 0.28, size = 16, normalized size = 0.89

$$-\frac{1}{7\left(cx^{2}+bx-a\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x - a)^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. 2(16) = 32.
time = 0.36, size = 354, normalized size = 19.67

$$-\frac{1}{7\left(c^{7}x^{14}+7\,bc^{6}x^{13}+7\left(3\,b^{2}c^{5}-ac^{6}\right)x^{12}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)x^{11}+7\left(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)x^{10}+7\left(3\,b^{5}c^{2}-20\,ab^{3}c^{3}+15\,a^{2}bc^{4}\right)x^{9}+7\left(b^{6}c-15\,ab^{4}c^{2}+30\,a^{2}b^{2}c^{3}-5\,a^{3}c^{4}\right)x^{8}+7\,a^{6}bx+\left(b^{7}-42\,ab^{5}c+210\,a^{2}b^{3}c^{2}-140\,a^{3}bc^{3}\right)x^{7}-a^{7}-7\left(ab^{6}-15\,a^{2}b^{4}c+30\,a^{3}b^{2}c^{2}-5\,a^{4}c^{3}\right)x^{6}+7\left(3\,a^{2}b^{5}-20\,a^{3}b^{3}c+15\,a^{4}bc^{2}\right)x^{5}-7\left(5\,a^{3}b^{4}-15\,a^{4}b^{2}c+3\,a^{5}c^{2}\right)x^{4}+7\left(5\,a^{4}b^{3}-6\,a^{5}bc\right)x^{3}-7\left(3\,a^{5}b^{2}-a^{6}c\right)x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 7*(3*b^2*c^5 - a*c^6)*x^12 + 7*(5*b^3*c^4 -
 6*a*b*c^5)*x^11 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^10 + 7*(3*b^5
*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^9 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*
b^2*c^3 - 5*a^3*c^4)*x^8 + 7*a^6*b*x + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2
- 140*a^3*b*c^3)*x^7 - a^7 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a
^4*c^3)*x^6 + 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^5 - 7*(5*a^3*b^
4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^4 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^3 - 7*(3*a^
5*b^2 - a^6*c)*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(14) = 28$.
time = 3.20, size = 359, normalized size = 19.94

$$-\frac{1}{-7a^7 + 49a^6bx + 49bc^6x^{13} + 7c^7x^{14} + x^{12}\left(-49ac^6 + 147b^2c^5\right) + x^{11}\left(-294abc^5 + 245b^3c^4\right) + x^{10}\cdot\left(147a^2c^5 - 735ab^2c^4 + 245b^4c^3\right) + x^9\cdot\left(735a^2bc^4 - 980ab^3c^3 + 147b^5c^2\right) + x^8\left(-245a^3c^4 + 1470a^2b^2c^3 - 735ab^4c^2 + 49b^6c\right) + x^7\left(-980a^3bc^3 + 1470a^2b^3c^2 - 294ab^5c + 7b^7\right) + x^6\cdot\left(245a^4c^3 - 1470a^3b^2c^2 + 735a^2b^4c - 49ab^6\right) + x^5\cdot\left(735a^4bc^2 - 980a^3b^3c + 147a^2b^5\right) + x^4\left(-147a^5c^2 + 735a^4b^2c - 245a^3b^4\right) + x^3\left(-294a^5bc + 245a^4b^3\right) + x^2\cdot\left(49a^6c - 147a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x-a)**8,x)
```

```
[Out] -1/(-7*a**7 + 49*a**6*b*x + 49*b*c**6*x**13 + 7*c**7*x**14 + x**12*(-49*a*c
**6 + 147*b**2*c**5) + x**11*(-294*a*b*c**5 + 245*b**3*c**4) + x**10*(147*a
**2*c**5 - 735*a*b**2*c**4 + 245*b**4*c**3) + x**9*(735*a**2*b*c**4 - 980*a
*b**3*c**3 + 147*b**5*c**2) + x**8*(-245*a**3*c**4 + 1470*a**2*b**2*c**3 -
735*a*b**4*c**2 + 49*b**6*c) + x**7*(-980*a**3*b*c**3 + 1470*a**2*b**3*c**2
 - 294*a*b**5*c + 7*b**7) + x**6*(245*a**4*c**3 - 1470*a**3*b**2*c**2 + 735
*a**2*b**4*c - 49*a*b**6) + x**5*(735*a**4*b*c**2 - 980*a**3*b**3*c + 147*a
**2*b**5) + x**4*(-147*a**5*c**2 + 735*a**4*b**2*c - 245*a**3*b**4) + x**3*
(-294*a**5*b*c + 245*a**4*b**3) + x**2*(49*a**6*c - 147*a**5*b**2))
```

Giac [A]
time = 4.95, size = 16, normalized size = 0.89

$$-\frac{1}{7\left(cx^2+bx-a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x-a)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x - a)^7
```

Mupad [B]
time = 5.22, size = 358, normalized size = 19.89

$$-\frac{1}{7\left(x^5\left(105a^4bc^2 - 140a^3b^3c + 21a^2b^5\right) + x^9\left(105a^2bc^4 - 140ab^3c^3 + 21b^5c^2\right) + x^7\left(-140a^3bc^3 + 210a^2b^3c^2 - 42ab^5c + b^7\right) + x^3\left(35a^4b^3 - 42a^5bc\right) + x^{11}\left(35b^3c^4 - 42abc^5\right) - x^4\left(21a^5c^2 - 105a^4b^2c + 35a^3b^4\right) + x^{10}\left(21a^2c^5 - 105ab^2c^4 + 35b^4c^3\right) - a^7 - x^6\left(-35a^4c^3 + 210a^3b^2c^2 - 105a^2b^4c + 7ab^6\right) + x^8\left(-35a^3c^4 + 210a^2b^2c^3 - 105ab^4c^2 + 7b^6c\right) + c^7x^{14} + x^2\left(7a^6c - 21a^5b^2\right) - x^{12}\left(7ac^6 - 21b^2c^5\right) + 7bc^6x^{13} + 7a^6bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x - a + c*x^2)^8,x)
```

```
[Out] -1/(7*(x^5*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^9*(21*b^5*c^2 -
 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^7*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^3*c^
2 - 42*a*b^5*c) + x^3*(35*a^4*b^3 - 42*a^5*b*c) + x^11*(35*b^3*c^4 - 42*a*b
*c^5) - x^4*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^10*(21*a^2*c^5 +
35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^6*(7*a*b^6 - 35*a^4*c^3 - 105*a^2*b^4
*c + 210*a^3*b^2*c^2) + x^8*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 210*a^2
*b^2*c^3) + c^7*x^14 + x^2*(7*a^6*c - 21*a^5*b^2) - x^12*(7*a*c^6 - 21*b^2*
c^5) + 7*b*c^6*x^13 + 7*a^6*b*x))
```

3.118 $\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx$

Optimal. Leaf size=20

$$\frac{1}{14\left(a-bx^2-cx^4\right)^7}$$

[Out] 1/14/(-c*x^4-b*x^2+a)^7

Rubi [A]

time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1261, 643}

$$\frac{1}{14\left(a-bx^2-cx^4\right)^7}$$

Antiderivative was successfully verified.

[In] Int[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]

[Out] 1/(14*(a - b*x^2 - c*x^4)^7)

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(b+2cx^2)}{(-a+bx^2+cx^4)^8}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx, x, x^2\right)\\ &= \frac{1}{14\left(a-bx^2-cx^4\right)^7}\end{aligned}$$

Mathematica [A]

time = 0.01, size = 20, normalized size = 1.00

$$-\frac{1}{14\left(-a+bx^2+cx^4\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(-a + b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(-a + b*x^2 + c*x^4)^7
```

Maple [A]
time = 0.21, size = 19, normalized size = 0.95

method	result	size
gosper	$\frac{1}{14\left(-c\,x^{4}-b\,x^{2}+a\right)^{7}}$	19
default	$\frac{1}{14\left(-c\,x^{4}-b\,x^{2}+a\right)^{7}}$	19
norman	$\frac{1}{14\left(-c\,x^{4}-b\,x^{2}+a\right)^{7}}$	19
risch	$\frac{1}{14\left(-c\,x^{4}-b\,x^{2}+a\right)^{7}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14/(-c*x^4-b*x^2+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(18) = 36.
time = 0.41, size = 356, normalized size = 17.80

$$-\frac{1}{14\,(c^7x^{28} + 7\,bc^6x^{26} + 7\,(3\,b^2c^5 - ac^6)x^{24} + 7\,(5\,b^3c^4 - 6\,abc^5)x^{22} + 7\,(5\,b^4c^3 - 15\,ab^2c^4 + 3\,a^2c^5)x^{20} + 7\,(3\,b^5c^2 - 20\,ab^3c^3 + 15\,a^2bc^4)x^{18} + 7\,(b^6c - 15\,ab^4c^2 + 30\,a^2b^2c^3 - 5\,a^3c^4)x^{16} + (b^7 - 42\,ab^5c + 210\,a^2b^3c^2 - 140\,a^3bc^3)x^{14} - 7\,(ab^6 - 15\,a^2b^4c + 30\,a^3b^2c^2 - 5\,a^4c^3)x^{12} + 7\,(3\,a^2b^5 - 20\,a^3b^3c + 15\,a^4bc^2)x^{10} + 7\,a^6bx^2 - 7\,(5\,a^3b^4 - 15\,a^4b^2c + 3\,a^5c^2)x^8 - a^7 + 7\,(5\,a^4b^3 - 6\,a^5bc)x^6 - 7\,(3\,a^5b^2 - a^6c)x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(18) = 36.
time = 0.37, size = 356, normalized size = 17.80

$$-\frac{1}{14\,(c^7x^{28} + 7\,bc^6x^{26} + 7\,(3\,b^2c^5 - ac^6)x^{24} + 7\,(5\,b^3c^4 - 6\,abc^5)x^{22} + 7\,(5\,b^4c^3 - 15\,ab^2c^4 + 3\,a^2c^5)x^{20} + 7\,(3\,b^5c^2 - 20\,ab^3c^3 + 15\,a^2bc^4)x^{18} + 7\,(b^6c - 15\,ab^4c^2 + 30\,a^2b^2c^3 - 5\,a^3c^4)x^{16} + (b^7 - 42\,ab^5c + 210\,a^2b^3c^2 - 140\,a^3bc^3)x^{14} - 7\,(ab^6 - 15\,a^2b^4c + 30\,a^3b^2c^2 - 5\,a^4c^3)x^{12} + 7\,(3\,a^2b^5 - 20\,a^3b^3c + 15\,a^4bc^2)x^{10} + 7\,a^6bx^2 - 7\,(5\,a^3b^4 - 15\,a^4b^2c + 3\,a^5c^2)x^8 - a^7 + 7\,(5\,a^4b^3 - 6\,a^5bc)x^6 - 7\,(3\,a^5b^2 - a^6c)x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 7*(3*b^2*c^5 - a*c^6)*x^24 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^22 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^20 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^18 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^16 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^14 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^12 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^10 + 7*a^6*b*x^2 - 7*(5*a^3*
b^4 - 15*a^4*b^2*c + 3*a^5*c^2)*x^8 - a^7 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^6 -
 7*(3*a^5*b^2 - a^6*c)*x^4)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(15) = 30$.
time = 5.17, size = 360, normalized size = 18.00

$$-\frac{1}{-14a^7 + 98a^6bx^2 + 98bc^6x^{26} + 14c^7x^{28} + x^{24}\left(-98ac^6 + 294b^2c^5\right) + x^{22}\left(-588abc^5 + 490b^3c^4\right) + x^{20}\cdot\left(294a^2c^5 - 1470ab^2c^4 + 490b^4c^3\right) + x^{18}\cdot\left(1470a^2bc^4 - 1960ab^3c^3 + 294b^5c^2\right) + x^{16}\left(-490a^3c^4 + 2940a^2b^2c^3 - 1470ab^4c^2 + 98b^6c\right) + x^{14}\left(-1960a^3bc^3 + 2940a^2b^3c^2 - 588ab^5c + 14b^7\right) + x^{12}\cdot\left(490a^4c^3 - 2940a^3b^2c^2 + 1470a^2b^4c - 98ab^6\right) + x^{10}\cdot\left(1470a^4bc^2 - 1960a^3b^3c + 294a^2b^5\right) + x^8\left(-294a^5c^2 + 1470a^4b^2c - 490a^3b^4\right) + x^6\left(-588a^5bc + 490a^4b^3\right) + x^4\cdot\left(98a^6c - 294a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2-a)**8,x)
```

```
[Out] -1/(-14*a**7 + 98*a**6*b*x**2 + 98*b*c**6*x**26 + 14*c**7*x**28 + x**24*(-9
8*a*c**6 + 294*b**2*c**5) + x**22*(-588*a*b*c**5 + 490*b**3*c**4) + x**20*(
294*a**2*c**5 - 1470*a*b**2*c**4 + 490*b**4*c**3) + x**18*(1470*a**2*b*c**4
 - 1960*a*b**3*c**3 + 294*b**5*c**2) + x**16*(-490*a**3*c**4 + 2940*a**2*b*
*2*c**3 - 1470*a*b**4*c**2 + 98*b**6*c) + x**14*(-1960*a**3*b*c**3 + 2940*a
**2*b**3*c**2 - 588*a*b**5*c + 14*b**7) + x**12*(490*a**4*c**3 - 2940*a**3*
b**2*c**2 + 1470*a**2*b**4*c - 98*a*b**6) + x**10*(1470*a**4*b*c**2 - 1960*
a**3*b**3*c + 294*a**2*b**5) + x**8*(-294*a**5*c**2 + 1470*a**4*b**2*c - 49
0*a**3*b**4) + x**6*(-588*a**5*b*c + 490*a**4*b**3) + x**4*(98*a**6*c - 294
*a**5*b**2))
```

Giac [A]
time = 6.01, size = 18, normalized size = 0.90

$$-\frac{1}{14\left(cx^4 + bx^2 - a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2-a)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2 - a)^7
```

Mupad [B]
time = 11.04, size = 360, normalized size = 18.00

$$-\frac{1}{14\left(x^{10}\left(105a^4bc^2 - 140a^3b^3c + 21a^2b^5\right) + x^{18}\left(105a^2bc^4 - 140ab^3c^3 + 21b^5c^2\right) + x^{14}\left(-140a^3bc^3 + 210a^2b^3c^2 - 42ab^5c + b^7\right) + x^6\left(35a^4b^3 - 42a^5bc\right) + x^{22}\left(35b^3c^4 - 42abc^5\right) - x^8\left(21a^5c^2 - 105a^4b^2c + 35a^3b^4\right) + x^{20}\left(21a^2c^5 - 105ab^2c^4 + 35b^4c^3\right) - a^7 - x^{12}\left(-35a^4c^3 + 210a^3b^2c^2 - 105a^2b^4c + 7ab^6\right) + x^{16}\left(-35a^3c^4 + 210a^2b^2c^3 - 105ab^4c^2 + 7b^6c\right) + c^7x^{28} + x^4\left(7a^6c - 21a^5b^2\right) - x^{24}\left(7ac^6 - 21b^2c^5\right) + 7a^6bx^2 + 7bc^6x^{26}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 - a + c*x^4)^8,x)
```

```
[Out] -1/(14*(x^10*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^18*(21*b^5*c^
2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^14*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 - 42*a*b^5*c) + x^6*(35*a^4*b^3 - 42*a^5*b*c) + x^22*(35*b^3*c^4 - 42
*a*b*c^5) - x^8*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^20*(21*a^2*c^
5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^12*(7*a*b^6 - 35*a^4*c^3 - 105*a^
2*b^4*c + 210*a^3*b^2*c^2) + x^16*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 + 2
10*a^2*b^2*c^3) + c^7*x^28 + x^4*(7*a^6*c - 21*a^5*b^2) - x^24*(7*a*c^6 - 2
1*b^2*c^5) + 7*a^6*b*x^2 + 7*b*c^6*x^26))
```

3.119 $\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx$

Optimal. Leaf size=20

$$\frac{1}{21(a-bx^3-cx^6)^7}$$

```
[Out] 1/21/(-c*x^6-b*x^3+a)^7
```

Rubi [A]
time = 0.02, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1482, 643}

$$\frac{1}{21(a-bx^3-cx^6)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] 1/(21*(a - b*x^3 - c*x^6)^7)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(b+2cx^3)}{(-a+bx^3+cx^6)^8}\,dx &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx, x, x^3\right)\\
&= \frac{1}{21(a-bx^3-cx^6)^7}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 1.00

$$-\frac{1}{21\left(-a+bx^3+cx^6\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(-a + b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(-a + b*x^3 + c*x^6)^7
```

Maple [A]
time = 0.08, size = 19, normalized size = 0.95

method	result	size
gosper	$\frac{1}{21\left(-c\,x^{6}-b\,x^{3}+a\right)^{7}}$	19
default	$\frac{1}{21\left(-c\,x^{6}-b\,x^{3}+a\right)^{7}}$	19
risch	$\frac{1}{21\left(-c\,x^{6}-b\,x^{3}+a\right)^{7}}$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/21/(-c*x^6-b*x^3+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(18) = 36$.
time = 0.42, size = 356, normalized size = 17.80

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+7\left(3\,b^2c^5-ac^6\right)x^{36}+7\left(5\,b^3c^4-6\,abc^5\right)x^{33}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{30}+7\left(3\,b^5c^2-20\,ab^3c^3+15\,a^2bc^4\right)x^{27}+7\left(b^6c-15\,ab^4c^2+30\,a^2b^2c^3-5\,a^3c^4\right)x^{24}+\left(b^7-42\,ab^5c+210\,a^2b^3c^2-140\,a^3bc^3\right)x^{21}-7\left(ab^6-15\,a^2b^4c+30\,a^3b^2c^2-5\,a^4c^3\right)x^{18}+7\left(3\,a^2b^5-20\,a^3b^3c+15\,a^4bc^2\right)x^{15}-7\left(5\,a^3b^4-15\,a^4b^2c+3\,a^5c^2\right)x^{12}+7\,a^6bx^3+7\left(5\,a^4b^3-6\,a^5bc\right)x^9-a^7-7\left(3\,a^5b^2-a^6c\right)x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(18) = 36$.
time = 0.41, size = 356, normalized size = 17.80

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+7\left(3\,b^2c^5-ac^6\right)x^{36}+7\left(5\,b^3c^4-6\,abc^5\right)x^{33}+7\left(5\,b^4c^3-15\,ab^2c^4+3\,a^2c^5\right)x^{30}+7\left(3\,b^5c^2-20\,ab^3c^3+15\,a^2bc^4\right)x^{27}+7\left(b^6c-15\,ab^4c^2+30\,a^2b^2c^3-5\,a^3c^4\right)x^{24}+\left(b^7-42\,ab^5c+210\,a^2b^3c^2-140\,a^3bc^3\right)x^{21}-7\left(ab^6-15\,a^2b^4c+30\,a^3b^2c^2-5\,a^4c^3\right)x^{18}+7\left(3\,a^2b^5-20\,a^3b^3c+15\,a^4bc^2\right)x^{15}-7\left(5\,a^3b^4-15\,a^4b^2c+3\,a^5c^2\right)x^{12}+7\,a^6bx^3+7\left(5\,a^4b^3-6\,a^5bc\right)x^9-a^7-7\left(3\,a^5b^2-a^6c\right)x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 7*(3*b^2*c^5 - a*c^6)*x^36 + 7*(5*b^3*c^4
- 6*a*b*c^5)*x^33 + 7*(5*b^4*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*x^30 + 7*(3*b^
5*c^2 - 20*a*b^3*c^3 + 15*a^2*b*c^4)*x^27 + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^
2*b^2*c^3 - 5*a^3*c^4)*x^24 + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3
*b*c^3)*x^21 - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*x^18 +
 7*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*x^15 - 7*(5*a^3*b^4 - 15*a^4*b
^2*c + 3*a^5*c^2)*x^12 + 7*a^6*b*x^3 + 7*(5*a^4*b^3 - 6*a^5*b*c)*x^9 - a^7
- 7*(3*a^5*b^2 - a^6*c)*x^6)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(15) = 30$.
time = 15.97, size = 360, normalized size = 18.00

$$-\frac{1}{-21a^7 + 147a^6bx^3 + 147bc^6x^{39} + 21c^7x^{42} + x^{36}\left(-147ac^6 + 441b^2c^5\right) + x^{33}\left(-882abc^5 + 735b^3c^4\right) + x^{30}\cdot\left(441a^2c^5 - 2205ab^2c^4 + 735b^4c^3\right) + x^{27}\cdot\left(2205a^2bc^4 - 2940ab^3c^3 + 441b^5c^2\right) + x^{24}\left(-735a^3c^4 + 4410a^2b^2c^3 - 2205ab^4c^2 + 147b^6c\right) + x^{21}\left(-2940a^3bc^3 + 4410a^2b^3c^2 - 882ab^5c + 21b^7\right) + x^{18}\cdot\left(735a^4c^3 - 4410a^3b^2c^2 + 2205a^2b^4c - 147ab^6\right) + x^{15}\cdot\left(2205a^4bc^2 - 2940a^3b^3c + 441a^2b^5\right) + x^{12}\left(-441a^5c^2 + 2205a^4b^2c - 735a^3b^4\right) + x^9\left(-882a^5bc + 735a^4b^3\right) + x^6\cdot\left(147a^6c - 441a^5b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3-a)**8,x)
```

```
[Out] -1/(-21*a**7 + 147*a**6*b*x**3 + 147*b*c**6*x**39 + 21*c**7*x**42 + x**36*(
-147*a*c**6 + 441*b**2*c**5) + x**33*(-882*a*b*c**5 + 735*b**3*c**4) + x**3
0*(441*a**2*c**5 - 2205*a*b**2*c**4 + 735*b**4*c**3) + x**27*(2205*a**2*b*c
**4 - 2940*a*b**3*c**3 + 441*b**5*c**2) + x**24*(-735*a**3*c**4 + 4410*a**2
*b**2*c**3 - 2205*a*b**4*c**2 + 147*b**6*c) + x**21*(-2940*a**3*b*c**3 + 44
10*a**2*b**3*c**2 - 882*a*b**5*c + 21*b**7) + x**18*(735*a**4*c**3 - 4410*a
**3*b**2*c**2 + 2205*a**2*b**4*c - 147*a*b**6) + x**15*(2205*a**4*b*c**2 -
2940*a**3*b**3*c + 441*a**2*b**5) + x**12*(-441*a**5*c**2 + 2205*a**4*b**2*
c - 735*a**3*b**4) + x**9*(-882*a**5*b*c + 735*a**4*b**3) + x**6*(147*a**6*
c - 441*a**5*b**2))
```

Giac [**A**]
time = 6.78, size = 18, normalized size = 0.90

$$-\frac{1}{21\left(cx^6 + bx^3 - a\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3-a)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3 - a)^7
```

Mupad [**B**]
time = 16.60, size = 360, normalized size = 18.00

$$-\frac{1}{21\left(x^{15}\left(105a^4bc^2 - 140a^3b^3c + 21a^2b^5\right) + x^{27}\left(105a^2bc^4 - 140ab^3c^3 + 21b^5c^2\right) + x^{21}\left(-140a^3bc^3 + 210a^2b^3c^2 - 42ab^5c + b^7\right) + x^9\left(35a^4b^3 - 42a^5bc\right) + x^{33}\left(35b^3c^4 - 42abc^5\right) - x^{12}\left(21a^5c^2 - 105a^4b^2c + 35a^3b^4\right) + x^{30}\left(21a^2c^5 - 105ab^2c^4 + 35b^4c^3\right) - a^7 - x^{18}\left(-35a^4c^3 + 210a^3b^2c^2 - 105a^2b^4c + 7ab^6\right) + x^{24}\left(-35a^3c^4 + 210a^2b^2c^3 - 105ab^4c^2 + 7b^6c\right) + c^7x^{42} + x^6\left(7a^6c - 21a^5b^2\right) - x^{36}\left(7ac^6 - 21b^2c^5\right) + 7a^6bx^3 + 7bc^6x^{39}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 - a + c*x^6)^8,x)
```

```
[Out] -1/(21*(x^15*(21*a^2*b^5 - 140*a^3*b^3*c + 105*a^4*b*c^2) + x^27*(21*b^5*c^
2 - 140*a*b^3*c^3 + 105*a^2*b*c^4) + x^21*(b^7 - 140*a^3*b*c^3 + 210*a^2*b^
3*c^2 - 42*a*b^5*c) + x^9*(35*a^4*b^3 - 42*a^5*b*c) + x^33*(35*b^3*c^4 - 42
*a*b*c^5) - x^12*(35*a^3*b^4 + 21*a^5*c^2 - 105*a^4*b^2*c) + x^30*(21*a^2*c
^5 + 35*b^4*c^3 - 105*a*b^2*c^4) - a^7 - x^18*(7*a*b^6 - 35*a^4*c^3 - 105*a
^2*b^4*c + 210*a^3*b^2*c^2) + x^24*(7*b^6*c - 35*a^3*c^4 - 105*a*b^4*c^2 +
210*a^2*b^2*c^3) + c^7*x^42 + x^6*(7*a^6*c - 21*a^5*b^2) - x^36*(7*a*c^6 -
21*b^2*c^5) + 7*a^6*b*x^3 + 7*b*c^6*x^39))
```

3.120 $\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx$

Optimal. Leaf size=25

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

```
[Out] 1/7/n/(a-b*x^n-c*x^(2*n))^7
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1482, 643}

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{-1+n}(b+2cx^n)}{(-a+bx^n+cx^{2n})^8}\,dx &= \frac{\text{Subst}\left(\int \frac{b+2cx}{(-a+bx+cx^2)^8}\,dx,x,x^n\right)}{n}\\
&= \frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 25, normalized size = 1.00

$$\frac{1}{7n\left(a-bx^n-cx^{2n}\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(-a + b*x^n + c*x^(2*n))^8,x]
```

```
[Out] 1/(7*n*(a - b*x^n - c*x^(2*n))^7)
```

Maple [A]
time = 0.06, size = 24, normalized size = 0.96

method	result	size
risch	$\frac{1}{7n\left(a-b\,x^{n}-c\,x^{2n}\right)^{7}}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7/n/(-c*(x^n)^2-b*x^n+a)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 419 vs. 2(23) = 46.
time = 0.79, size = 419, normalized size = 16.76

$$-\frac{1}{7\left(c^{7}nx^{14n}+7bc^{6}nx^{13n}+7a^{6}bnx^{n}-a^{7}n+7\left(3b^{2}c^{5}n-ac^{6}n\right)x^{12n}+7\left(5b^{3}c^{4}n-6abc^{5}n\right)x^{11n}+7\left(5b^{4}c^{3}n-15ab^{2}c^{4}n+3a^{2}c^{5}n\right)x^{10n}+7\left(3b^{5}c^{2}n-20ab^{3}c^{3}n+15a^{2}bc^{4}n\right)x^{9n}+7\left(b^{6}cn-15ab^{4}c^{2}n+30a^{2}b^{2}c^{3}n-5a^{3}c^{4}n\right)x^{8n}+\left(b^{7}n-42ab^{5}cn+210a^{2}b^{3}c^{2}n-140a^{3}bc^{3}n\right)x^{7n}-7\left(ab^{6}n-15a^{2}b^{4}cn+30a^{3}b^{2}c^{2}n-5a^{4}c^{3}n\right)x^{6n}+7\left(3a^{2}b^{5}n-20a^{3}b^{3}cn+15a^{4}bc^{2}n\right)x^{5n}-7\left(5a^{3}b^{4}n-15a^{4}b^{2}cn+3a^{5}c^{2}n\right)x^{4n}+7\left(5a^{4}b^{3}n-6a^{5}bcn\right)x^{3n}-7\left(3a^{5}b^{2}n-a^{6}cn\right)x^{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="maxima"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5*n - a*c^6*n)*x^(12*n) + 7*(5*b^3*c^4*n - 6*a*b*c^5*n)*x^(11*n) + 7*(5
*b^4*c^3*n - 15*a*b^2*c^4*n + 3*a^2*c^5*n)*x^(10*n) + 7*(3*b^5*c^2*n - 20*a
*b^3*c^3*n + 15*a^2*b*c^4*n)*x^(9*n) + 7*(b^6*c*n - 15*a*b^4*c^2*n + 30*a^2
*b^2*c^3*n - 5*a^3*c^4*n)*x^(8*n) + (b^7*n - 42*a*b^5*c*n + 210*a^2*b^3*c^2
*n - 140*a^3*b*c^3*n)*x^(7*n) - 7*(a*b^6*n - 15*a^2*b^4*c*n + 30*a^3*b^2*c^
2*n - 5*a^4*c^3*n)*x^(6*n) + 7*(3*a^2*b^5*n - 20*a^3*b^3*c*n + 15*a^4*b*c^2
*n)*x^(5*n) - 7*(5*a^3*b^4*n - 15*a^4*b^2*c*n + 3*a^5*c^2*n)*x^(4*n) + 7*(5
*a^4*b^3*n - 6*a^5*b*c*n)*x^(3*n) - 7*(3*a^5*b^2*n - a^6*c*n)*x^(2*n))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(23) = 46.
time = 0.36, size = 397, normalized size = 15.88

$$-\frac{1}{7\left(c^{7}nx^{14n}+7\,bc^{6}nx^{13n}+7\,a^{6}bnx^{n}-a^{7}n+7\left(3\,b^{2}c^{5}-ac^{6}\right)nx^{12n}+7\left(5\,b^{3}c^{4}-6\,abc^{5}\right)nx^{11n}+7\left(5\,b^{4}c^{3}-15\,ab^{2}c^{4}+3\,a^{2}c^{5}\right)nx^{10n}+7\left(3\,b^{5}c^{2}-20\,ab^{3}c^{3}+15\,a^{2}bc^{4}\right)nx^{9n}+7\left(b^{6}c-15\,ab^{4}c^{2}+30\,a^{2}b^{2}c^{3}-5\,a^{3}c^{4}\right)nx^{8n}+\left(b^{7}-42\,ab^{5}c+210\,a^{2}b^{3}c^{2}-140\,a^{3}bc^{3}\right)nx^{7n}-7\left(ab^{6}-15\,a^{2}b^{4}c+30\,a^{3}b^{2}c^{2}-5\,a^{4}c^{3}\right)nx^{6n}+7\left(3\,a^{2}b^{5}-20\,a^{3}b^{3}c+15\,a^{4}bc^{2}\right)nx^{5n}-7\left(5\,a^{3}b^{4}-15\,a^{4}b^{2}c+3\,a^{5}c^{2}\right)nx^{4n}+7\left(5\,a^{4}b^{3}-6\,a^{5}bc\right)nx^{3n}-7\left(3\,a^{5}b^{2}-a^{6}c\right)nx^{2n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="fricas"
)
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 7*a^6*b*n*x^n - a^7*n + 7*(3*b^
2*c^5 - a*c^6)*n*x^(12*n) + 7*(5*b^3*c^4 - 6*a*b*c^5)*n*x^(11*n) + 7*(5*b^4
*c^3 - 15*a*b^2*c^4 + 3*a^2*c^5)*n*x^(10*n) + 7*(3*b^5*c^2 - 20*a*b^3*c^3 +
 15*a^2*b*c^4)*n*x^(9*n) + 7*(b^6*c - 15*a*b^4*c^2 + 30*a^2*b^2*c^3 - 5*a^3
*c^4)*n*x^(8*n) + (b^7 - 42*a*b^5*c + 210*a^2*b^3*c^2 - 140*a^3*b*c^3)*n*x^
(7*n) - 7*(a*b^6 - 15*a^2*b^4*c + 30*a^3*b^2*c^2 - 5*a^4*c^3)*n*x^(6*n) + 7
*(3*a^2*b^5 - 20*a^3*b^3*c + 15*a^4*b*c^2)*n*x^(5*n) - 7*(5*a^3*b^4 - 15*a^
4*b^2*c + 3*a^5*c^2)*n*x^(4*n) + 7*(5*a^4*b^3 - 6*a^5*b*c)*n*x^(3*n) - 7*(3
*a^5*b^2 - a^6*c)*n*x^(2*n))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(-a+b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.65, size = 23, normalized size = 0.92

$$-\frac{1}{7\left(cx^{2n}+bx^{n}-a\right)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(-a+b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n - a)^7*n)
```

Mupad [B]
time = 22.40, size = 496, normalized size = 19.84

$$-\frac{1}{7b^7nx^{7n}-7a^7n+7c^7nx^{14n}+49a^6bnx^{n}-49ab^6nx^{6n}+49a^6cnx^{2n}-49ac^6nx^{12n}+49b^6cnx^{8n}+49bc^6nx^{13n}-147a^5b^2nx^{2n}+245a^4b^3nx^{3n}-245a^3b^4nx^{4n}+147a^2b^5nx^{5n}-147a^5c^2nx^{4n}+245a^4c^3nx^{6n}-245a^3c^4nx^{8n}+147a^2c^5nx^{10n}+147b^5c^2nx^{9n}+245b^4c^3nx^{10n}+245b^3c^4nx^{11n}+147b^2c^5nx^{12n}+735a^4b^2cnx^{4n}-980a^3b^3cnx^{5n}+735a^4bc^2nx^{5n}+735a^2b^4cnx^{6n}-980a^3bc^3nx^{7n}-735ab^4c^2nx^{8n}-980ab^3c^3nx^{9n}+735a^2bc^4nx^{9n}-735ab^2c^4nx^{10n}-1470a^3b^2c^2nx^{6n}+1470a^2b^3c^2nx^{7n}+1470a^2b^2c^3nx^{8n}-294a^5bcnx^{3n}-294ab^5cnx^{7n}-294abc^5nx^{11n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n - a + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*b^7*n*x^(7*n) - 7*a^7*n + 7*c^7*n*x^(14*n) + 49*a^6*b*n*x^n - 49*a*b^
6*n*x^(6*n) + 49*a^6*c*n*x^(2*n) - 49*a*c^6*n*x^(12*n) + 49*b^6*c*n*x^(8*n)
 + 49*b*c^6*n*x^(13*n) - 147*a^5*b^2*n*x^(2*n) + 245*a^4*b^3*n*x^(3*n) - 24
```

```
5*a^3*b^4*n*x^(4*n) + 147*a^2*b^5*n*x^(5*n) - 147*a^5*c^2*n*x^(4*n) + 245*a
^4*c^3*n*x^(6*n) - 245*a^3*c^4*n*x^(8*n) + 147*a^2*c^5*n*x^(10*n) + 147*b^5
*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(11*n) + 147*b^2*
c^5*n*x^(12*n) + 735*a^4*b^2*c*n*x^(4*n) - 980*a^3*b^3*c*n*x^(5*n) + 735*a^
4*b*c^2*n*x^(5*n) + 735*a^2*b^4*c*n*x^(6*n) - 980*a^3*b*c^3*n*x^(7*n) - 735
*a*b^4*c^2*n*x^(8*n) - 980*a*b^3*c^3*n*x^(9*n) + 735*a^2*b*c^4*n*x^(9*n) -
735*a*b^2*c^4*n*x^(10*n) - 1470*a^3*b^2*c^2*n*x^(6*n) + 1470*a^2*b^3*c^2*n*
x^(7*n) + 1470*a^2*b^2*c^3*n*x^(8*n) - 294*a^5*b*c*n*x^(3*n) - 294*a*b^5*c*
n*x^(7*n) - 294*a*b*c^5*n*x^(11*n))
```

3.121 $\int \frac{b+2cx}{bx+cx^2} \, dx$

Optimal. Leaf size=10

$$\log \left(bx+cx^2\right)$$

```
[Out] ln(c*x^2+b*x)
```

Rubi [A]
time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {642}

$$\log \left(bx+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[b*x + c*x^2]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log \left(bx+cx^2\right)$$

Mathematica [A]
time = 0.00, size = 9, normalized size = 0.90

$$\log(x)+\log(b+cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

```
[Out] Log[x] + Log[b + c*x]
```

Maple [A]
time = 0.22, size = 9, normalized size = 0.90

method	result	size
default	$\ln(x(cx+b))$	9
norman	$\ln(x)+\ln(cx+b)$	10
derivativedivides	$\ln(c\,x^2+bx)$	11
risch	$\ln(c\,x^2+bx)$	11

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x*(c*x+b))
```

Maxima [A]
time = 0.28, size = 10, normalized size = 1.00

$$\log(cx^2+bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
```

```
[Out] log(c*x^2 + b*x)
```

Fricas [A]
time = 0.34, size = 10, normalized size = 1.00

$$\log(cx^2+bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

```
[Out] log(c*x^2 + b*x)
```

Sympy [A]
time = 0.05, size = 8, normalized size = 0.80

$$\log(bx+cx^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x),x)
```

```
[Out] log(b*x + c*x**2)
```

Giac [A]
time = 3.95, size = 11, normalized size = 1.10

$$\log(|cx^2+bx|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

```
[Out] log(abs(c*x^2 + b*x))
```

Mupad [B]
time = 0.05, size = 8, normalized size = 0.80

$$\ln\left(x\left(b+c\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2),x)
```

```
[Out] log(x*(b + c*x))
```

3.122 $\int \frac{x(b+2cx^2)}{bx^2+cx^4} dx$

Optimal. Leaf size=16

$$\frac{1}{2}\log\left(bx^2+cx^4\right)$$

```
[Out] 1/2*ln(c*x^4+b*x^2)
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1598, 457, 78}

$$\frac{1}{2}\log\left(b+cx^2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
```

```
[Out] Log[x] + Log[b + c*x^2]/2
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(b+2cx^2)}{bx^2+cx^4}\,dx &= \int \frac{b+2cx^2}{x\left(b+cx^2\right)}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx, x, x^2\right) \\
&= \log(x)+\frac{1}{2}\log\left(b+cx^2\right)
\end{aligned}
$$

Mathematica [**A**]
time = 0.01, size = 15, normalized size = 0.94

$$\log(x)+\frac{1}{2}\log\left(b+cx^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x]
[Out] Log[x] + Log[b + c*x^2]/2
```

Maple [**A**]
time = 0.15, size = 14, normalized size = 0.88

method	result	size
default	$\ln(x)+\frac{\ln(cx^2+b)}{2}$	14
norman	$\ln(x)+\frac{\ln(cx^2+b)}{2}$	14
risch	$\ln(x)+\frac{\ln(cx^2+b)}{2}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/2*ln(c*x^2+b)
```

Maxima [**A**]
time = 0.27, size = 17, normalized size = 1.06

$$\frac{1}{2}\log\left(cx^2+b\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c*x^2 + b) + 1/2*log(x^2)
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.81

$$\frac{1}{2}\log\left(cx^2+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*log(c*x^2 + b) + log(x)
```

Sympy [A]
time = 0.10, size = 12, normalized size = 0.75

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2),x)
```

```
[Out] log(x) + log(b/c + x**2)/2
```

Giac [A]
time = 4.41, size = 15, normalized size = 0.94

$$\frac{1}{2}\log\left(\left|cx^4+bx^2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c*x^4 + b*x^2))
```

Mupad [B]
time = 0.06, size = 13, normalized size = 0.81

$$\frac{\ln\left(c\,x^2+b\right)}{2}+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4),x)
```

```
[Out] log(b + c*x^2)/2 + log(x)
```

3.123 $\int \frac{x^2(b+2cx^3)}{bx^3+cx^6} dx$

Optimal. Leaf size=16

$$\frac{1}{3}\log\left(bx^3+cx^6\right)$$

```
[Out] 1/3*ln(c*x^6+b*x^3)
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1598, 457, 78}

$$\frac{1}{3}\log\left(b+cx^3\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(b+2cx^3)}{bx^3+cx^6}\,dx &= \int \frac{b+2cx^3}{x\,(b+cx^3)}\,dx\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^3\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx,x,x^3\right)\\ &= \log(x)+\frac{1}{3}\log\left(b+cx^3\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 0.94

$$\log(x)+\frac{1}{3}\log\left(b+cx^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x]
```

```
[Out] Log[x] + Log[b + c*x^3]/3
```

Maple [A]
time = 0.16, size = 14, normalized size = 0.88

method	result	size
default	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
norman	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14
risch	$\ln(x)+\frac{\ln(c\,x^3+b)}{3}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x,method=_RETURNVERBOSE)
```

```
[Out] ln(x)+1/3*ln(c*x^3+b)
```

Maxima [A]
time = 0.27, size = 17, normalized size = 1.06

$$\frac{1}{3}\log\left(cx^3+b\right)+\frac{1}{3}\log\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="maxima")
```

```
[Out] 1/3*log(c*x^3 + b) + 1/3*log(x^3)
```

Fricas [A]
time = 0.34, size = 13, normalized size = 0.81

$$\frac{1}{3}\,\log\left(cx^3+b\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="fricas")
```

```
[Out] 1/3*log(c*x^3 + b) + log(x)
```

Sympy [A]
time = 0.08, size = 12, normalized size = 0.75

$$\log\left(x\right)+\frac{\log\left(\frac{b}{c}+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3),x)
```

```
[Out] log(x) + log(b/c + x**3)/3
```

Giac [A]
time = 4.92, size = 15, normalized size = 0.94

$$\frac{1}{3}\,\log\left(\left|cx^6+bx^3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(c*x^6 + b*x^3))
```

Mupad [B]
time = 1.99, size = 13, normalized size = 0.81

$$\frac{\ln\left(c\,x^3+b\right)}{3}+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6),x)
```

```
[Out] log(b + c*x^3)/3 + log(x)
```

3.124 $\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=15

$$\log(x) + \frac{\log(b + cx^n)}{n}$$

```
[Out] ln(x)+ln(b+c*x^n)/n
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1598, 457, 78}

$$\frac{\log(b + cx^n)}{n} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
```

```
[Out] Log[x] + Log[b + c*x^n]/n
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+n}(b+2cx^n)}{bx^n+cx^{2n}}\,dx = \int \frac{b+2cx^n}{x\,(b+cx^n)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x(b+cx)}\,dx,x,x^n\right)}{n}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{c}{b+cx}\right)\,dx,x,x^n\right)}{n}$$
$$= \log(x)+\frac{\log\left(b+cx^n\right)}{n}$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.27

$$\frac{\log\left(x^n\right)+\log\left(n(b+cx^n)\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x]
[Out] (Log[x^n] + Log[n*(b + c*x^n)])/n
```

Maple [A]
time = 0.24, size = 18, normalized size = 1.20

method	result	size
norman	$\ln\left(x\right)+\frac{\ln\left(c\,e^{n\ln\left(x\right)}+b\right)}{n}$	18
risch	$\ln\left(x\right)+\frac{\ln\left(x^n+\frac{b}{c}\right)}{n}$	18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x,method=_RETURNVERBOSE)
[Out] ln(x)+1/n*ln(c*exp(n*ln(x))+b)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.
time = 0.29, size = 47, normalized size = 3.13

$$b\left(\frac{\log\left(x\right)}{b}-\frac{\log\left(\frac{cx^n+b}{c}\right)}{bn}\right)+\frac{2\,\log\left(\frac{cx^n+b}{c}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] b*(log(x)/b - log((c*x^n + b)/c)/(b*n)) + 2*log((c*x^n + b)/c)/n
```

Fricas [A]
time = 0.35, size = 17, normalized size = 1.13

$$\frac{n\log\left(x\right)+\log\left(cx^{n}+b\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] (n*log(x) + log(c*x^n + b))/n
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(12) = 24$.
time = 6.88, size = 54, normalized size = 3.60

$$\begin{cases} \log\left(x\right) & \text{for}\ c=0\wedge n=0\\ \frac{(b+2c)\log\left(x\right)}{b+c} & \text{for}\ n=0\\ -\frac{n\log\left(x^{-n}\right)}{n^{2}-n}+\frac{\log\left(x^{-n}\right)}{n^{2}-n} & \text{for}\ c=0\\ \frac{\log\left(x^{n}\right)}{n}+\frac{\log\left(\frac{b}{c}+x^{n}\right)}{n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n)),x)
```

```
[Out] Piecewise((log(x), Eq(c, 0) & Eq(n, 0)), ((b + 2*c)*log(x)/(b + c), Eq(n, 0
)), (-n*log(x**(-n))/(n**2 - n) + log(x**(-n))/(n**2 - n), Eq(c, 0)), (log(
x**n)/n + log(b/c + x**n)/n, True))
```

Giac [A]
time = 5.58, size = 17, normalized size = 1.13

$$\frac{\log\left(|cx^{n}+b|\right)}{n}+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] log(abs(c*x^n + b))/n + log(abs(x))
```

Mupad [B]
time = 2.23, size = 28, normalized size = 1.87

$$\frac{2\left(\ln\left(b+c\,x^{n}\right)-\mathrm{atanh}\left(\frac{2\,c\,x^{n}}{b}+1\right)\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n)),x)
[Out] (2*(log(b + c*x^n) - atanh((2*c*x^n)/b + 1)))/n
```

3.125 $\int \frac{b+2cx}{(bx+cx^2)^8} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

```
[Out] -1/7/(c*x^2+b*x)^7
```

Rubi [A]
time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {643}

$$-\frac{1}{7\left(bx+cx^2\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(b*x + c*x^2)^7
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{b+2cx}{\left(bx+cx^2\right)^8} \, dx = -\frac{1}{7\left(bx+cx^2\right)^7}$$

Mathematica [A]
time = 0.01, size = 14, normalized size = 0.93

$$-\frac{1}{7x^7(b+cx)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)/(b*x + c*x^2)^8,x]
```

```
[Out] -1/7*1/(x^7*(b + c*x)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(13) = 26$.
time = 0.16, size = 177, normalized size = 11.80

method	result
gosper	$-\frac{1}{7x^7(cx+b)^7}$
norman	$-\frac{1}{7x^7(cx+b)^7}$
risch	$-\frac{1}{7x^7(cx+b)^7}$
derivativedivides	$-\frac{1}{7(c\,x^2+bx)^7}$
default	$\frac{132c^7}{b^{13}(cx+b)} + \frac{66c^7}{b^{12}(cx+b)^2} + \frac{30c^7}{b^{11}(cx+b)^3} + \frac{12c^7}{b^{10}(cx+b)^4} + \frac{4c^7}{b^9(cx+b)^5} + \frac{c^7}{b^8(cx+b)^6} + \frac{c^7}{7b^7(cx+b)^7} - \frac{1}{7b^7x^7} -$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)/(c*x^2+b*x)^8,x,method=_RETURNVERBOSE)
```

```
[Out] 132/b^13*c^7/(c*x+b)+66/b^12*c^7/(c*x+b)^2+30/b^11*c^7/(c*x+b)^3+12/b^10*c^
7/(c*x+b)^4+4/b^9*c^7/(c*x+b)^5+c^7/b^8/(c*x+b)^6+1/7*c^7/b^7/(c*x+b)^7-1/7
/b^7/x^7-132/b^13*c^6/x+66/b^12*c^5/x^2-30/b^11*c^4/x^3+12/b^10*c^3/x^4-4/b
^9*c^2/x^5+1/b^8*c/x^6
```

Maxima [A]
time = 0.27, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2+bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="maxima")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(13) = 26$.
time = 0.36, size = 81, normalized size = 5.40

$$-\frac{1}{7\left(c^7x^{14}+7\,bc^6x^{13}+21\,b^2c^5x^{12}+35\,b^3c^4x^{11}+35\,b^4c^3x^{10}+21\,b^5c^2x^9+7\,b^6cx^8+b^7x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*x^14 + 7*b*c^6*x^13 + 21*b^2*c^5*x^12 + 35*b^3*c^4*x^11 + 35*b^4*
c^3*x^10 + 21*b^5*c^2*x^9 + 7*b^6*c*x^8 + b^7*x^7)
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(14) = 28$.
time = 0.47, size = 87, normalized size = 5.80

$$-\frac{1}{7b^7x^7 + 49b^6cx^8 + 147b^5c^2x^9 + 245b^4c^3x^{10} + 245b^3c^4x^{11} + 147b^2c^5x^{12} + 49bc^6x^{13} + 7c^7x^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x**2+b*x)**8,x)
```

```
[Out] -1/(7*b**7*x**7 + 49*b**6*c*x**8 + 147*b**5*c**2*x**9 + 245*b**4*c**3*x**10
 + 245*b**3*c**4*x**11 + 147*b**2*c**5*x**12 + 49*b*c**6*x**13 + 7*c**7*x**
14)
```

Giac [**A**]
time = 2.99, size = 13, normalized size = 0.87

$$-\frac{1}{7\left(cx^2 + bx\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)/(c*x^2+b*x)^8,x, algorithm="giac")
```

```
[Out] -1/7/(c*x^2 + b*x)^7
```

Mupad [**B**]
time = 4.30, size = 12, normalized size = 0.80

$$-\frac{1}{7\,x^7\,{\left(b+c\,x\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)/(b*x + c*x^2)^8,x)
```

```
[Out] -1/(7*x^7*(b + c*x)^7)
```

3.126 $\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8} \, dx$

Optimal. Leaf size=16

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

[Out] -1/14/x^14/(c*x^2+b)^7

Rubi [A]
time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1598, 457, 75}

$$-\frac{1}{14x^{14}(b+cx^2)^7}$$

Antiderivative was successfully verified.

[In] Int[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]

[Out] -1/14*1/(x^14*(b + c*x^2)^7)

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}\int \frac{x(b+2cx^2)}{(bx^2+cx^4)^8}\,dx &= \int \frac{b+2cx^2}{x^{15}\,(b+cx^2)^8}\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^2\right)\\ &= -\frac{1}{14x^{14}\,(b+cx^2)^7}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$-\frac{1}{14x^{14}\,(b+cx^2)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x]
```

```
[Out] -1/14*1/(x^14*(b + c*x^2)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(14) = 28$.
time = 0.16, size = 197, normalized size = 12.31

method	result
gosper	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
norman	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
risch	$-\frac{1}{14x^{14}(c\,x^2+b)^7}$
default	$-\frac{c^8\left(-\frac{4b^4}{c\left(c\,x^2+b\right)^5}-\frac{b^6}{7c\left(c\,x^2+b\right)^7}-\frac{66b}{c\left(c\,x^2+b\right)^2}-\frac{132}{c\left(c\,x^2+b\right)}-\frac{b^5}{c\left(c\,x^2+b\right)^6}-\frac{12b^3}{c\left(c\,x^2+b\right)^4}-\frac{30b^2}{c\left(c\,x^2+b\right)^3}\right)}{2b^{13}}-\frac{1}{14b^7x^{14}}-\frac{66c^6}{b^{13}x^2}+\frac{33c^5}{b^{12}x^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*c^8/b^13*(-4*b^4/c/(c*x^2+b)^5-1/7*b^6/c/(c*x^2+b)^7-66*b/c/(c*x^2+b)^
2-132/c/(c*x^2+b)-b^5/c/(c*x^2+b)^6-12*b^3/c/(c*x^2+b)^4-30/c*b^2/(c*x^2+b)
^3)-1/14/b^7/x^14-66/b^13*c^6/x^2+33/b^12*c^5/x^4-15/b^11*c^4/x^6+6/b^10*c^
3/x^8-2/b^9*c^2/x^10+1/2/b^8*c/x^12
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.

time = 0.29, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="maxima")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.
time = 0.35, size = 81, normalized size = 5.06

$$-\frac{1}{14\left(c^{7}x^{28}+7\,bc^{6}x^{26}+21\,b^{2}c^{5}x^{24}+35\,b^{3}c^{4}x^{22}+35\,b^{4}c^{3}x^{20}+21\,b^{5}c^{2}x^{18}+7\,b^{6}cx^{16}+b^{7}x^{14}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="fricas")
```

```
[Out] -1/14/(c^7*x^28 + 7*b*c^6*x^26 + 21*b^2*c^5*x^24 + 35*b^3*c^4*x^22 + 35*b^4
*c^3*x^20 + 21*b^5*c^2*x^18 + 7*b^6*c*x^16 + b^7*x^14)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.
time = 0.67, size = 87, normalized size = 5.44

$$-\frac{1}{14b^{7}x^{14}+98b^{6}cx^{16}+294b^{5}c^{2}x^{18}+490b^{4}c^{3}x^{20}+490b^{3}c^{4}x^{22}+294b^{2}c^{5}x^{24}+98bc^{6}x^{26}+14c^{7}x^{28}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)/(c*x**4+b*x**2)**8,x)
```

```
[Out] -1/(14*b**7*x**14 + 98*b**6*c*x**16 + 294*b**5*c**2*x**18 + 490*b**4*c**3*x
**20 + 490*b**3*c**4*x**22 + 294*b**2*c**5*x**24 + 98*b*c**6*x**26 + 14*c**
7*x**28)
```

Giac [A]
time = 3.46, size = 15, normalized size = 0.94

$$-\frac{1}{14\left(cx^{4}+bx^{2}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)/(c*x^4+b*x^2)^8,x, algorithm="giac")
```

```
[Out] -1/14/(c*x^4 + b*x^2)^7
```

Mupad [B]
time = 2.33, size = 14, normalized size = 0.88

$$-\frac{1}{14\,x^{14}\,{\left(c\,x^2+b\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(b + 2*c*x^2))/(b*x^2 + c*x^4)^8,x)
```

```
[Out] -1/(14*x^14*(b + c*x^2)^7)
```

3.127 $\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

```
[Out] -1/21/x^21/(c*x^3+b)^7
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1598, 457, 75}

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^2(b+2cx^3)}{(bx^3+cx^6)^8}\,dx = \int \frac{b+2cx^3}{x^{22}\left(b+cx^3\right)^8}\,dx$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^3\right)$$
$$= -\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$-\frac{1}{21x^{21}\left(b+cx^3\right)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x]
```

```
[Out] -1/21*1/(x^21*(b + c*x^3)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(14) = 28$.
time = 0.22, size = 197, normalized size = 12.31

method	result
gosper	$-\frac{1}{21x^{21}\left(c\,x^3+b\right)^7}$
risch	$-\frac{1}{21x^{21}\left(c\,x^3+b\right)^7}$
default	$-\frac{c^8\left(-\frac{4b^4}{c\left(c\,x^3+b\right)^5}-\frac{b^6}{7c\left(c\,x^3+b\right)^7}-\frac{66b}{c\left(c\,x^3+b\right)^2}-\frac{132}{c\left(c\,x^3+b\right)}-\frac{b^5}{c\left(c\,x^3+b\right)^6}-\frac{12b^3}{c\left(c\,x^3+b\right)^4}-\frac{30b^2}{c\left(c\,x^3+b\right)^3}\right)}{3b^{13}}-\frac{1}{21b^7x^{21}}-\frac{44c^6}{b^{13}x^3}+\frac{22c^5}{b^{12}x^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*c^8/b^13*(-4*b^4/c/(c*x^3+b)^5-1/7*b^6/c/(c*x^3+b)^7-66*b/c/(c*x^3+b)^
2-132/c/(c*x^3+b)-b^5/c/(c*x^3+b)^6-12*b^3/c/(c*x^3+b)^4-30/c*b^2/(c*x^3+b)
^3)-1/21/b^7/x^21-44/b^13*c^6/x^3+22/b^12*c^5/x^6-10/b^11*c^4/x^9+4/b^10*c^
3/x^12-4/3/b^9*c^2/x^15+1/3/b^8*c/x^18
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.
time = 0.31, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^7x^{42}+7\,bc^6x^{39}+21\,b^2c^5x^{36}+35\,b^3c^4x^{33}+35\,b^4c^3x^{30}+21\,b^5c^2x^{27}+7\,b^6cx^{24}+b^7x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="maxima")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(14) = 28$.
time = 0.34, size = 81, normalized size = 5.06

$$-\frac{1}{21\left(c^{7} x^{42}+7 \, b c^{6} x^{39}+21 \, b^{2} c^{5} x^{36}+35 \, b^{3} c^{4} x^{33}+35 \, b^{4} c^{3} x^{30}+21 \, b^{5} c^{2} x^{27}+7 \, b^{6} c x^{24}+b^{7} x^{21}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="fricas")
```

```
[Out] -1/21/(c^7*x^42 + 7*b*c^6*x^39 + 21*b^2*c^5*x^36 + 35*b^3*c^4*x^33 + 35*b^4
*c^3*x^30 + 21*b^5*c^2*x^27 + 7*b^6*c*x^24 + b^7*x^21)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(15) = 30$.
time = 0.93, size = 87, normalized size = 5.44

$$-\frac{1}{21b^{7}x^{21}+147b^{6}cx^{24}+441b^{5}c^{2}x^{27}+735b^{4}c^{3}x^{30}+735b^{3}c^{4}x^{33}+441b^{2}c^{5}x^{36}+147bc^{6}x^{39}+21c^{7}x^{42}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)/(c*x**6+b*x**3)**8,x)
```

```
[Out] -1/(21*b**7*x**21 + 147*b**6*c*x**24 + 441*b**5*c**2*x**27 + 735*b**4*c**3*
x**30 + 735*b**3*c**4*x**33 + 441*b**2*c**5*x**36 + 147*b*c**6*x**39 + 21*c
**7*x**42)
```

Giac [A]
time = 3.80, size = 15, normalized size = 0.94

$$-\frac{1}{21\left(cx^{6}+bx^{3}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)/(c*x^6+b*x^3)^8,x, algorithm="giac")
```

```
[Out] -1/21/(c*x^6 + b*x^3)^7
```

Mupad [B]
time = 5.07, size = 14, normalized size = 0.88

$$-\frac{1}{21\,x^{21}\,{\left(c\,x^{3}+b\right)}^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(b + 2*c*x^3))/(b*x^3 + c*x^6)^8,x)
```

```
[Out] -1/(21*x^21*(b + c*x^3)^7)
```

3.128 $\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8} dx$

Optimal. Leaf size=21

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

```
[Out] -1/7/n/(x^(7*n))/(b+c*x^n)^7
```

Rubi [A]
time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1598, 457, 75}

$$-\frac{x^{-7n}}{7n(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Rule 75

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\int \frac{x^{-1+n}(b+2cx^n)}{(bx^n+cx^{2n})^8}\,dx = \int \frac{x^{-1-7n}(b+2cx^n)}{(b+cx^n)^8}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{b+2cx}{x^8(b+cx)^8}\,dx, x, x^n\right)}{n}$$
$$= -\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Mathematica [A]
time = 0.07, size = 21, normalized size = 1.00

$$-\frac{x^{-7n}}{7n\,(b+cx^n)^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1 + n)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x]
```

```
[Out] -1/7*1/(n*x^(7*n)*(b + c*x^n)^7)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(21) = 42.
time = 0.19, size = 203, normalized size = 9.67

method	result
risch	$-\frac{132c^6x^{-n}}{b^{13}n}+\frac{66c^5x^{-2n}}{b^{12}n}-\frac{30c^4x^{-3n}}{b^{11}n}+\frac{12c^3x^{-4n}}{b^{10}n}-\frac{4c^2x^{-5n}}{b^9n}+\frac{c\,x^{-6n}}{b^8n}-\frac{x^{-7n}}{7b^7n}+\frac{c^7\left(924x^{6n}c^6+6006b\,c^5x^{5n}+16380b^2c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x,method=_RETURNVERBOSE)
```

```
[Out] -132/b^13*c^6/n/(x^n)+66/b^12*c^5/n/(x^n)^2-30/b^11*c^4/n/(x^n)^3+12/b^10*c
^3/n/(x^n)^4-4/b^9*c^2/n/(x^n)^5+1/b^8*c/n/(x^n)^6-1/7/b^7/n/(x^n)^7+1/7*c^
7*(924*(x^n)^6*c^6+6006*b*c^5*(x^n)^5+16380*b^2*c^4*(x^n)^4+24024*b^3*c^3*(
x^n)^3+20020*b^4*c^2*(x^n)^2+9009*b^5*c*x^n+1716*b^6)/b^13/n/(b+c*x^n)^7
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 612 vs. 2(21) = 42.
time = 0.35, size = 612, normalized size = 29.14

$$-\frac{1}{105}b\left(\frac{360360\,c^{13}x^{13n}+2342340\,bc^{12}x^{12n}+6426420\,b^2c^{11}x^{11n}+9579570\,b^3c^{10}x^{10n}+8270262\,b^4c^9x^{9n}+4018014\,b^5c^8x^{8n}+934362\,b^6c^7x^{7n}+45045\,b^7c^6x^{6n}-5005\,b^8c^5x^{5n}+1001\,b^9c^4x^{4n}-273\,b^{10}c^3x^{3n}+91\,b^{11}c^2x^{2n}-35\,b^{12}cx^n+15\,b^{13}}{b^{14}c^7nx^{14n}+7\,b^{15}c^6nx^{13n}+21\,b^{16}c^5nx^{12n}+35\,b^{17}c^4nx^{11n}+35\,b^{18}c^3nx^{10n}+21\,b^{19}c^2nx^{9n}+7\,b^{20}cnx^{8n}+b^{21}nx^{7n}}+\frac{360360\,c^7\log(x)}{b^{15}}-\frac{360360\,c^7\log\left(\frac{cx^n+b}{c}\right)}{b^{15}n}\right)+\frac{1}{105}c\left(\frac{360360\,c^{12}x^{12n}+2342340\,bc^{11}x^{11n}+6426420\,b^2c^{10}x^{10n}+9579570\,b^3c^9x^{9n}+8270262\,b^4c^8x^{8n}+4018014\,b^5c^7x^{7n}+934362\,b^6c^6x^{6n}+45045\,b^7c^5x^{5n}-5005\,b^8c^4x^{4n}+1001\,b^9c^3x^{3n}-273\,b^{10}c^2x^{2n}+91\,b^{11}cx^n-35\,b^{12}}{b^{13}c^7nx^{13n}+7\,b^{14}c^6nx^{12n}+21\,b^{15}c^5nx^{11n}+35\,b^{16}c^4nx^{10n}+35\,b^{17}c^3nx^{9n}+21\,b^{18}c^2nx^{8n}+7\,b^{19}cnx^{7n}+b^{20}nx^{6n}}+\frac{360360\,c^6\log(x)}{b^{14}}-\frac{360360\,c^6\log\left(\frac{cx^n+b}{c}\right)}{b^{14}n}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="maxima")
```

```
[Out] -1/105*b*((360360*c^13*x^(13*n) + 2342340*b*c^12*x^(12*n) + 6426420*b^2*c^1
1*x^(11*n) + 9579570*b^3*c^10*x^(10*n) + 8270262*b^4*c^9*x^(9*n) + 4018014*
b^5*c^8*x^(8*n) + 934362*b^6*c^7*x^(7*n) + 45045*b^7*c^6*x^(6*n) - 5005*b^8
*c^5*x^(5*n) + 1001*b^9*c^4*x^(4*n) - 273*b^10*c^3*x^(3*n) + 91*b^11*c^2*x^
(2*n) - 35*b^12*c*x^n + 15*b^13)/(b^14*c^7*n*x^(14*n) + 7*b^15*c^6*n*x^(13*
n) + 21*b^16*c^5*n*x^(12*n) + 35*b^17*c^4*n*x^(11*n) + 35*b^18*c^3*n*x^(10*
n) + 21*b^19*c^2*n*x^(9*n) + 7*b^20*c*n*x^(8*n) + b^21*n*x^(7*n)) + 360360*
c^7*log(x)/b^15 - 360360*c^7*log((c*x^n + b)/c)/(b^15*n)) + 1/105*c*((36036
0*c^12*x^(12*n) + 2342340*b*c^11*x^(11*n) + 6426420*b^2*c^10*x^(10*n) + 957
9570*b^3*c^9*x^(9*n) + 8270262*b^4*c^8*x^(8*n) + 4018014*b^5*c^7*x^(7*n) +
934362*b^6*c^6*x^(6*n) + 45045*b^7*c^5*x^(5*n) - 5005*b^8*c^4*x^(4*n) + 100
1*b^9*c^3*x^(3*n) - 273*b^10*c^2*x^(2*n) + 91*b^11*c*x^n - 35*b^12)/(b^13*c
^7*n*x^(13*n) + 7*b^14*c^6*n*x^(12*n) + 21*b^15*c^5*n*x^(11*n) + 35*b^16*c^
4*n*x^(10*n) + 35*b^17*c^3*n*x^(9*n) + 21*b^18*c^2*n*x^(8*n) + 7*b^19*c*n*x
^(7*n) + b^20*n*x^(6*n)) + 360360*c^6*log(x)/b^14 - 360360*c^6*log((c*x^n +
 b)/c)/(b^14*n))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(21) = 42$.
time = 0.39, size = 105, normalized size = 5.00

$$-\frac{1}{7\left(c^{7}nx^{14\,n}+7\,bc^{6}nx^{13\,n}+21\,b^{2}c^{5}nx^{12\,n}+35\,b^{3}c^{4}nx^{11\,n}+35\,b^{4}c^{3}nx^{10\,n}+21\,b^{5}c^{2}nx^{9\,n}+7\,b^{6}cnx^{8\,n}+b^{7}nx^{7\,n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="fricas")
```

```
[Out] -1/7/(c^7*n*x^(14*n) + 7*b*c^6*n*x^(13*n) + 21*b^2*c^5*n*x^(12*n) + 35*b^3*
c^4*n*x^(11*n) + 35*b^4*c^3*n*x^(10*n) + 21*b^5*c^2*n*x^(9*n) + 7*b^6*c*n*x
^(8*n) + b^7*n*x^(7*n))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)/(b*x**n+c*x**(2*n))**8,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.64, size = 20, normalized size = 0.95

$$-\frac{1}{7\left(cx^{2\,n}+bx^{n}\right)^{7}n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)/(b*x^n+c*x^(2*n))^8,x, algorithm="giac")
```

```
[Out] -1/7/((c*x^(2*n) + b*x^n)^7*n)
```

Mupad [B]
time = 2.36, size = 107, normalized size = 5.10

$$-\frac{1}{7\,b^7\,n\,x^{7\,n}+7\,c^7\,n\,x^{14\,n}+49\,b^6\,c\,n\,x^{8\,n}+49\,b\,c^6\,n\,x^{13\,n}+147\,b^5\,c^2\,n\,x^{9\,n}+245\,b^4\,c^3\,n\,x^{10\,n}+245\,b^3\,c^4\,n\,x^{11\,n}+147\,b^2\,c^5\,n\,x^{12\,n}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(n - 1)*(b + 2*c*x^n))/(b*x^n + c*x^(2*n))^8,x)
```

```
[Out] -1/(7*b^7*n*x^(7*n) + 7*c^7*n*x^(14*n) + 49*b^6*c*n*x^(8*n) + 49*b*c^6*n*x^
(13*n) + 147*b^5*c^2*n*x^(9*n) + 245*b^4*c^3*n*x^(10*n) + 245*b^3*c^4*n*x^(
11*n) + 147*b^2*c^5*n*x^(12*n))
```

3.129 $\int (b+2cx)\left(a+bx+cx^2\right)^p \, dx$

Optimal. Leaf size=20

$$\frac{\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

```
[Out] (c*x^2+b*x+a)^(1+p)/(1+p)
```

Rubi [A]
time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {643}

$$\frac{\left(a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
[Out] (a + b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(a+bx+cx^2\right)^p \, dx = \frac{\left(a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A]
time = 0.05, size = 19, normalized size = 0.95

$$\frac{(a+x(b+cx))^{1+p}}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(a + b*x + c*x^2)^p,x]
[Out] (a + x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A]
time = 0.22, size = 21, normalized size = 1.05

method	result	size
gosper	$\frac{\left(c\,x^{2}+b x+a\right)^{1+p}}{1+p}$	21
derivativedivides	$\frac{\left(c\,x^{2}+b x+a\right)^{1+p}}{1+p}$	21
default	$\frac{\left(c\,x^{2}+b x+a\right)^{1+p}}{1+p}$	21
risch	$\frac{\left(c\,x^{2}+b x+a\right)\left(c\,x^{2}+b x+a\right)^{p}}{1+p}$	29
norman	$\frac{a\,\mathrm{e}^{p\ln\left(c\,x^{2}+b x+a\right)}}{1+p}+\frac{b x\,\mathrm{e}^{p\ln\left(c\,x^{2}+b x+a\right)}}{1+p}+\frac{c\,x^{2}\mathrm{e}^{p\ln\left(c\,x^{2}+b x+a\right)}}{1+p}$	69

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x+a)^p,x,method=_RETURNVERBOSE)
[Out] (c*x^2+b*x+a)^(1+p)/(1+p)
```

Maxima [A]
time = 0.28, size = 20, normalized size = 1.00

$$\frac{(cx^2+bx+a)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="maxima")
[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

Fricas [A]
time = 0.35, size = 28, normalized size = 1.40

$$\frac{(cx^2+bx+a)(cx^2+bx+a)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="fricas")
[Out] (c*x^2 + b*x + a)*(c*x^2 + b*x + a)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(15) = 30$.
time = 33.79, size = 104, normalized size = 5.20

$$\begin{cases} \frac{a(a+bx+cx^2)^p}{p+1} + \frac{bx(a+bx+cx^2)^p}{p+1} + \frac{cx^2(a+bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log\left(\frac{b}{2c} + x - \frac{\sqrt{-4ac+b^2}}{2c}\right) + \log\left(\frac{b}{2c} + x + \frac{\sqrt{-4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x+a)**p,x)
```

```
[Out] Piecewise((a*(a + b*x + c*x**2)**p/(p + 1) + b*x*(a + b*x + c*x**2)**p/(p +
 1) + c*x**2*(a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) + x -
sqrt(-4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(-4*a*c + b**2)/(2*c)),
True))
```

Giac [**A**]
time = 4.74, size = 20, normalized size = 1.00

$$\frac{(cx^2 + bx + a)^{p+1}}{p + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x+a)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x + a)^(p + 1)/(p + 1)
```

Mupad [**B**]
time = 2.04, size = 39, normalized size = 1.95

$$\left(\frac{a}{p+1} + \frac{b\,x}{p+1} + \frac{c\,x^2}{p+1}\right)\,{\left(c\,x^2 + b\,x + a\right)}^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(a + b*x + c*x^2)^p,x)
```

```
[Out] (a/(p + 1) + (b*x)/(p + 1) + (c*x^2)/(p + 1))*(a + b*x + c*x^2)^p
```

3.130 $\int x(b+2cx^2)\left(a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1+p)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1261, 643}

$$\frac{\left(a+bx^2+cx^4\right)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(a+bx^2+cx^4\right)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx, x, x^2\right)\\ &= \frac{\left(a+bx^2+cx^4\right)^{1+p}}{2(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 25, normalized size = 1.00

$$\frac{\left(a+bx^2+cx^4\right)^{1+p}}{2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Maple [A]
time = 0.02, size = 24, normalized size = 0.96

method	result	size
gosper	$\frac{(c\,x^4+b\,x^2+a)^{1+p}}{2+2p}$	24
risch	$\frac{(c\,x^4+b\,x^2+a)\,(c\,x^4+b\,x^2+a)^p}{2+2p}$	34
norman	$\frac{a\,e^{p\ln(c\,x^4+b\,x^2+a)}}{2+2p}+\frac{b\,x^2 e^{p\ln(c\,x^4+b\,x^2+a)}}{2+2p}+\frac{c\,x^4 e^{p\ln(c\,x^4+b\,x^2+a)}}{2+2p}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*x^4+b*x^2+a)^(1+p)/(1+p)
```

Maxima [A]
time = 0.32, size = 33, normalized size = 1.32

$$\frac{(cx^4+bx^2+a)(cx^4+bx^2+a)^p}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

Fricas [A]
time = 0.38, size = 33, normalized size = 1.32

$$\frac{(cx^4+bx^2+a)(cx^4+bx^2+a)^p}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="fricas")
```

```
[Out] 1/2*(c*x^4 + b*x^2 + a)*(c*x^4 + b*x^2 + a)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(19) = 38$.
time = 129.16, size = 201, normalized size = 8.04

$$\begin{cases} \frac{a(a+bx^2+cx^4)^p}{2p+2}+\frac{bx^2(a+bx^2+cx^4)^p}{2p+2}+\frac{cx^4(a+bx^2+cx^4)^p}{2p+2} & \text{for}\ p\neq -1 \\ \frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{-4ac+b^2}}{c}}}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2+a)**p,x)
```

```
[Out] Piecewise((a*(a + b*x**2 + c*x**4)**p/(2*p + 2) + b*x**2*(a + b*x**2 + c*x*
*4)**p/(2*p + 2) + c*x**4*(a + b*x**2 + c*x**4)**p/(2*p + 2), Ne(p, -1)), (
log(x - sqrt(2)*sqrt(-b/c - sqrt(-4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2)*s
qrt(-b/c - sqrt(-4*a*c + b**2)/c)/2)/2 + log(x - sqrt(2)*sqrt(-b/c + sqrt(-
4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2)*sqrt(-b/c + sqrt(-4*a*c + b**2)/c)/
2)/2, True))
```

Giac [A]
time = 3.59, size = 23, normalized size = 0.92

$$\frac{(cx^4 + bx^2 + a)^{p+1}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2+a)^p,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^4 + b*x^2 + a)^(p + 1)/(p + 1)
```

Mupad [B]
time = 2.09, size = 49, normalized size = 1.96

$$(c\,x^4 + b\,x^2 + a)^p\,\left(\frac{a}{2\,p+2} + \frac{b\,x^2}{2\,p+2} + \frac{c\,x^4}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] (a + b*x^2 + c*x^4)^p*(a/(2*p + 2) + (b*x^2)/(2*p + 2) + (c*x^4)/(2*p + 2))
```

3.131 $\int x^2(b+2cx^3)\left(a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=25

$$\frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*(c*x^6+b*x^3+a)^(1+p)/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1482, 643}

$$\frac{\left(a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}
\int x^2\left(b+2cx^3\right)\left(a+bx^3+cx^6\right)^p\,dx &= \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx,x,x^3\right)\\
&= \frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 25, normalized size = 1.00

$$\frac{\left(a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x]
[Out] (a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Maple [A]
time = 0.03, size = 24, normalized size = 0.96

method	result	size
gosper	$\frac{\left(c\,x^{6}+b\,x^{3}+a\right)^{1+p}}{3+3p}$	24
risch	$\frac{\left(c\,x^{6}+b\,x^{3}+a\right)\left(c\,x^{6}+b\,x^{3}+a\right)^{p}}{3+3p}$	34
norman	$\frac{a\,{\mathrm e}^{p\ln\left(c\,x^{6}+b\,x^{3}+a\right)}}{3+3p}+\frac{b\,x^{3}{\mathrm e}^{p\ln\left(c\,x^{6}+b\,x^{3}+a\right)}}{3+3p}+\frac{c\,x^{6}{\mathrm e}^{p\ln\left(c\,x^{6}+b\,x^{3}+a\right)}}{3+3p}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x,method=_RETURNVERBOSE)
[Out] 1/3*(c*x^6+b*x^3+a)^(1+p)/(1+p)
```

Maxima [A]
time = 0.31, size = 33, normalized size = 1.32

$$\frac{(cx^6+bx^3+a)(cx^6+bx^3+a)^p}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

Fricas [A]
time = 0.35, size = 33, normalized size = 1.32

$$\frac{(cx^6+bx^3+a)(cx^6+bx^3+a)^p}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^6 + b*x^3 + a)*(c*x^6 + b*x^3 + a)^p/(p + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3+a)**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.34, size = 23, normalized size = 0.92

$$\frac{(cx^6 + bx^3 + a)^{p+1}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3+a)^p,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^6 + b*x^3 + a)^(p + 1)/(p + 1)
```

Mupad [B]
time = 2.12, size = 49, normalized size = 1.96

$$(c\,x^6 + b\,x^3 + a)^p\,\left(\frac{a}{3\,p+3} + \frac{b\,x^3}{3\,p+3} + \frac{c\,x^6}{3\,p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(a + b*x^3 + c*x^6)^p,x)
```

```
[Out] (a + b*x^3 + c*x^6)^p*(a/(3*p + 3) + (b*x^3)/(3*p + 3) + (c*x^6)/(3*p + 3))
```

3.132 $\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{\left(a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

```
[Out] (a+b*x^n+c*x^(2*n))^(1+p)/n/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1482, 643}

$$\frac{\left(a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(a+bx^n+cx^{2n}\right)^p\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(a+bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\left(a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 26, normalized size = 0.96

$$\frac{(a+x^n(b+cx^n))^{1+p}}{n(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

Maple [**A**]
time = 0.04, size = 40, normalized size = 1.48

method	result	size
risch	$\frac{(a+b\,x^{n}+c\,x^{2n})\,(a+b\,x^{n}+c\,x^{2n})^{p}}{n(1+p)}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

```
[Out] (a+b*x^n+c*(x^n)^2)/n/(1+p)*(a+b*x^n+c*(x^n)^2)^p
```

Maxima [**A**]
time = 0.36, size = 39, normalized size = 1.44

$$\frac{(cx^{2\,n} + bx^n + a)(cx^{2\,n} + bx^n + a)^p}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*(p + 1))
```

Fricas [**A**]
time = 0.36, size = 38, normalized size = 1.41

$$\frac{(cx^{2\,n} + bx^n + a)(cx^{2\,n} + bx^n + a)^p}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] (c*x^(2*n) + b*x^n + a)*(c*x^(2*n) + b*x^n + a)^p/(n*p + n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.49, size = 27, normalized size = 1.00

$$\frac{(cx^{2n}+bx^n+a)^{p+1}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n + a)^(p + 1)/(n*(p + 1))
```

Mupad [B]
time = 2.57, size = 56, normalized size = 2.07

$$(a+b\,x^n+c\,x^{2\,n})^p\,\left(\frac{a}{n\,\left(p+1\right)}+\frac{b\,x^n}{n\,\left(p+1\right)}+\frac{c\,x^{2\,n}}{n\,\left(p+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] (a + b*x^n + c*x^(2*n))^p*(a/(n*(p + 1)) + (b*x^n)/(n*(p + 1)) + (c*x^(2*n)
)/(n*(p + 1)))
```

3.133 $\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx$

Optimal. Leaf size=22

$$\frac{\left(-a+bx+cx^2\right)^{1+p}}{1+p}$$

[Out] (c*x^2+b*x-a)^(1+p)/(1+p)

Rubi [A]
time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {643}

$$\frac{\left(-a+bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

[In] Int[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]

[Out] (-a + b*x + c*x^2)^(1 + p)/(1 + p)

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx=\frac{\left(-a+bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A]
time = 0.05, size = 21, normalized size = 0.95

$$\frac{(-a+x(b+cx))^{1+p}}{1+p}$$

Antiderivative was successfully verified.

[In] Integrate[(b + 2*c*x)*(-a + b*x + c*x^2)^p,x]

[Out] (-a + x*(b + c*x))^(1 + p)/(1 + p)

Maple [A]
time = 0.27, size = 23, normalized size = 1.05

method	result	size
gosper	$\frac{(c x^2+bx-a)^{1+p}}{1+p}$	23
derivativedivides	$\frac{(c x^2+bx-a)^{1+p}}{1+p}$	23
default	$\frac{(c x^2+bx-a)^{1+p}}{1+p}$	23
risch	$-\frac{(-c x^2-bx+a)(c x^2+bx-a)^p}{1+p}$	34
norman	$\frac{bx\,e^{p\ln(c x^2+bx-a)}}{1+p}+\frac{c x^2 e^{p\ln(c x^2+bx-a)}}{1+p}-\frac{a\,e^{p\ln(c x^2+bx-a)}}{1+p}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x-a)^p,x,method=_RETURNVERBOSE)
[Out] (c*x^2+b*x-a)^(1+p)/(1+p)
```

Maxima [A]
time = 0.27, size = 22, normalized size = 1.00

$$\frac{(cx^2+bx-a)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="maxima")
[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

Fricas [A]
time = 0.37, size = 32, normalized size = 1.45

$$\frac{(cx^2+bx-a)(cx^2+bx-a)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="fricas")
[Out] (c*x^2 + b*x - a)*(c*x^2 + b*x - a)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(15) = 30$.
time = 34.76, size = 104, normalized size = 4.73

$$\begin{cases} -\frac{a(-a+bx+cx^2)^p}{p+1}+\frac{bx(-a+bx+cx^2)^p}{p+1}+\frac{cx^2(-a+bx+cx^2)^p}{p+1} & \text{for } p \neq -1 \\ \log\left(\frac{b}{2c}+x-\frac{\sqrt{4ac+b^2}}{2c}\right)+\log\left(\frac{b}{2c}+x+\frac{\sqrt{4ac+b^2}}{2c}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x-a)**p,x)
```

```
[Out] Piecewise((-a*(-a + b*x + c*x**2)**p/(p + 1) + b*x*(-a + b*x + c*x**2)**p/(
p + 1) + c*x**2*(-a + b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (log(b/(2*c) +
x - sqrt(4*a*c + b**2)/(2*c)) + log(b/(2*c) + x + sqrt(4*a*c + b**2)/(2*c))
, True))
```

Giac [**A**]
time = 4.52, size = 22, normalized size = 1.00

$$\frac{(cx^2 + bx - a)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x-a)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x - a)^(p + 1)/(p + 1)
```

Mupad [**B**]
time = 2.05, size = 42, normalized size = 1.91

$$\left(\frac{b\,x}{p+1} - \frac{a}{p+1} + \frac{c\,x^2}{p+1}\right)\,\left(c\,x^2 + b\,x - a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b + 2*c*x)*(b*x - a + c*x^2)^p,x)
```

```
[Out] ((b*x)/(p + 1) - a/(p + 1) + (c*x^2)/(p + 1))*(b*x - a + c*x^2)^p
```

3.134 $\int x(b+2cx^2)\left(-a+bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{(-a+bx^2+cx^4)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*(c*x^4+b*x^2-a)^(1+p)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1261, 643}

$$\frac{(-a+bx^2+cx^4)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
```

```
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(b+2cx^2\right)\left(-a+bx^2+cx^4\right)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^2\right)\\ &= \frac{(-a+bx^2+cx^4)^{1+p}}{2(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 27, normalized size = 1.00

$$\frac{(-a+bx^2+cx^4)^{1+p}}{2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(-a + b*x^2 + c*x^4)^p,x]
[Out] (-a + b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Maple [A]
time = 0.02, size = 26, normalized size = 0.96

method	result	size
gosper	$\frac{\left(c\,x^{4}+b\,x^{2}-a\right)^{1+p}}{2+2p}$	26
risch	$-\frac{\left(-c\,x^{4}-b\,x^{2}+a\right)\left(c\,x^{4}+b\,x^{2}-a\right)^{p}}{2(1+p)}$	38
norman	$-\frac{a\,{\mathrm e}^{p\ln\left(c\,x^{4}+b\,x^{2}-a\right)}}{2(1+p)}+\frac{b\,x^{2}{\mathrm e}^{p\ln\left(c\,x^{4}+b\,x^{2}-a\right)}}{2+2p}+\frac{c\,x^{4}{\mathrm e}^{p\ln\left(c\,x^{4}+b\,x^{2}-a\right)}}{2+2p}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x,method=_RETURNVERBOSE)
[Out] 1/2*(c*x^4+b*x^2-a)^(1+p)/(1+p)
```

Maxima [A]
time = 0.31, size = 37, normalized size = 1.37

$$\frac{(cx^4+bx^2-a)(cx^4+bx^2-a)^p}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

Fricas [A]
time = 0.34, size = 37, normalized size = 1.37

$$\frac{(cx^4+bx^2-a)(cx^4+bx^2-a)^p}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + b*x^2 - a)*(c*x^4 + b*x^2 - a)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(19) = 38$.
time = 130.39, size = 201, normalized size = 7.44

$$\begin{cases} -\frac{a(-a+bx^2+cx^4)^p}{2p+2}+\frac{bx^2(-a+bx^2+cx^4)^p}{2p+2}+\frac{cx^4(-a+bx^2+cx^4)^p}{2p+2} & \text{for } p\neq -1 \\ \frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}-\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x-\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2}+\frac{\log\left(x+\frac{\sqrt{2}\sqrt{-\frac{b}{c}+\frac{\sqrt{4ac+b^2}}{c}}}{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2-a)**p,x)
```

```
[Out] Piecewise((-a*(-a + b*x**2 + c*x**4)**p/(2*p + 2) + b*x**2*(-a + b*x**2 + c
*x**4)**p/(2*p + 2) + c*x**4*(-a + b*x**2 + c*x**4)**p/(2*p + 2), Ne(p, -1)
), (log(x - sqrt(2)*sqrt(-b/c - sqrt(4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2
)*sqrt(-b/c - sqrt(4*a*c + b**2)/c)/2)/2 + log(x - sqrt(2)*sqrt(-b/c + sqrt
(4*a*c + b**2)/c)/2)/2 + log(x + sqrt(2)*sqrt(-b/c + sqrt(4*a*c + b**2)/c)/
2)/2, True))
```

Giac [A]
time = 4.21, size = 25, normalized size = 0.93

$$\frac{(cx^4 + bx^2 - a)^{p+1}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2-a)^p,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^4 + b*x^2 - a)^(p + 1)/(p + 1)
```

Mupad [B]
time = 2.05, size = 52, normalized size = 1.93

$$(c\,x^4 + b\,x^2 - a)^p\left(\frac{b\,x^2}{2\,p+2} - \frac{a}{2\,p+2} + \frac{c\,x^4}{2\,p+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 - a + c*x^4)^p,x)
```

```
[Out] (b*x^2 - a + c*x^4)^p*((b*x^2)/(2*p + 2) - a/(2*p + 2) + (c*x^4)/(2*p + 2))
```

3.135 $\int x^2(b+2cx^3)\left(-a+bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=27

$$\frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*(c*x^6+b*x^3-a)^(1+p)/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1482, 643}

$$\frac{\left(-a+bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
```

```
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(b+2cx^3\right)\left(-a+bx^3+cx^6\right)^p\,dx &= \frac{1}{3}\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^3\right)\\ &= \frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 27, normalized size = 1.00

$$\frac{\left(-a+bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(-a + b*x^3 + c*x^6)^p,x]
[Out] (-a + b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Maple [A]
time = 0.03, size = 26, normalized size = 0.96

method	result	size
gosper	$\frac{\left(c\,x^{6}+b\,x^{3}-a\right)^{1+p}}{3+3p}$	26
risch	$-\frac{\left(-c\,x^{6}-b\,x^{3}+a\right)\left(c\,x^{6}+b\,x^{3}-a\right)^{p}}{3(1+p)}$	38
norman	$-\frac{a\,{\mathrm e}^{p\ln\left(c\,x^{6}+b\,x^{3}-a\right)}}{3(1+p)}+\frac{b\,x^{3}{\mathrm e}^{p\ln\left(c\,x^{6}+b\,x^{3}-a\right)}}{3+3p}+\frac{c\,x^{6}{\mathrm e}^{p\ln\left(c\,x^{6}+b\,x^{3}-a\right)}}{3+3p}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x,method=_RETURNVERBOSE)
[Out] 1/3*(c*x^6+b*x^3-a)^(1+p)/(1+p)
```

Maxima [A]
time = 0.31, size = 37, normalized size = 1.37

$$\frac{(cx^6+bx^3-a)(cx^6+bx^3-a)^p}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="maxima")
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

Fricas [A]
time = 0.34, size = 37, normalized size = 1.37

$$\frac{(cx^6+bx^3-a)(cx^6+bx^3-a)^p}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="fricas")
[Out] 1/3*(c*x^6 + b*x^3 - a)*(c*x^6 + b*x^3 - a)^p/(p + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3-a)**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.85, size = 25, normalized size = 0.93

$$\frac{(cx^6 + bx^3 - a)^{p+1}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3-a)^p,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^6 + b*x^3 - a)^(p + 1)/(p + 1)
```

Mupad [B]
time = 2.08, size = 52, normalized size = 1.93

$$\left(c\,x^6 + b\,x^3 - a\right)^p\,\left(\frac{b\,x^3}{3\,p+3} - \frac{a}{3\,p+3} + \frac{c\,x^6}{3\,p+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 - a + c*x^6)^p,x)
```

```
[Out] (b*x^3 - a + c*x^6)^p*((b*x^3)/(3*p + 3) - a/(3*p + 3) + (c*x^6)/(3*p + 3))
```

3.136 $\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=29

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

```
[Out] (-a+b*x^n+c*x^(2*n))^(1+p)/n/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1482, 643}

$$\frac{\left(-a+bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (-a + b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(-a+bx^n+cx^{2n}\right)^p\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(-a+bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\left(-a+bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 28, normalized size = 0.97

$$\frac{\left(-a+x^n(b+cx^n)\right)^{1+p}}{n(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(-a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (-a + x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

Maple [A]
time = 0.05, size = 45, normalized size = 1.55

method	result	size
risch	$-\frac{(a-b x^n-c x^{2n})(-a+b x^n+c x^{2n})^p}{n(1+p)}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

```
[Out] -(-c*(x^n)^2-b*x^n+a)/n/(1+p)*(-a+b*x^n+c*(x^n)^2)^p
```

Maxima [A]
time = 0.34, size = 43, normalized size = 1.48

$$\frac{(cx^{2n}+bx^n-a)(cx^{2n}+bx^n-a)^p}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima"
)
```

```
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*(p + 1))
```

Fricas [A]
time = 0.37, size = 42, normalized size = 1.45

$$\frac{(cx^{2n}+bx^n-a)(cx^{2n}+bx^n-a)^p}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas"
)
```

```
[Out] (c*x^(2*n) + b*x^n - a)*(c*x^(2*n) + b*x^n - a)^p/(n*p + n)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(-a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 2.40, size = 29, normalized size = 1.00

$$\frac{(cx^{2n} + bx^n - a)^{p+1}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(-a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] (c*x^(2*n) + b*x^n - a)^(p + 1)/(n*(p + 1))
```

Mupad [B]
time = 2.54, size = 59, normalized size = 2.03

$$\left(\frac{b\,x^n}{n\,(p+1)} - \frac{a}{n\,(p+1)} + \frac{c\,x^{2\,n}}{n\,(p+1)}\right)\,\left(b\,x^n - a + c\,x^{2\,n}\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n - a + c*x^(2*n))^p,x)
```

```
[Out] ((b*x^n)/(n*(p + 1)) - a/(n*(p + 1)) + (c*x^(2*n))/(n*(p + 1)))*(b*x^n - a
+ c*x^(2*n))^p
```

3.137 $\int (b+2cx)\left(bx+cx^2\right)^p\,dx$

Optimal. Leaf size=19

$$\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

```
[Out] (c*x^2+b*x)^(1+p)/(1+p)
```

Rubi [A]
time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = $\{643\}$

$$\frac{\left(bx+cx^2\right)^{p+1}}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[(b + 2*c*x)*(b*x + c*x^2)^p,x]
[Out] (b*x + c*x^2)^(1 + p)/(1 + p)
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int (b+2cx)\left(bx+cx^2\right)^p\,dx=\frac{\left(bx+cx^2\right)^{1+p}}{1+p}$$

Mathematica [A]
time = 0.03, size = 17, normalized size = 0.89

$$\frac{(x(b+cx))^{1+p}}{1+p}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + 2*c*x)*(b*x + c*x^2)^p,x]
[Out] (x*(b + c*x))^(1 + p)/(1 + p)
```

Maple [A]
time = 0.17, size = 20, normalized size = 1.05

method	result	size
derivativedivides	$\frac{(c\,x^2+bx)^{1+p}}{1+p}$	20
default	$\frac{(c\,x^2+bx)^{1+p}}{1+p}$	20
risch	$\frac{x(cx+b)(x(cx+b))^p}{1+p}$	22
gosper	$\frac{x(cx+b)(c\,x^2+bx)^p}{1+p}$	24
norman	$\frac{bx\,\mathrm{e}^{p\ln(c\,x^2+bx)}}{1+p}+\frac{c\,x^2\mathrm{e}^{p\ln(c\,x^2+bx)}}{1+p}$	46

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*c*x+b)*(c*x^2+b*x)^p,x,method=_RETURNVERBOSE)
```

```
[Out] (c*x^2+b*x)^(1+p)/(1+p)
```

Maxima [A]
time = 0.28, size = 19, normalized size = 1.00

$$\frac{(cx^2+bx)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

Fricas [A]
time = 0.35, size = 26, normalized size = 1.37

$$\frac{(cx^2+bx)(cx^2+bx)^p}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

```
[Out] (c*x^2 + b*x)*(c*x^2 + b*x)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(14) = 28$.
time = 0.25, size = 46, normalized size = 2.42

$$\begin{cases} \frac{bx(bx+cx^2)^p}{p+1}+\frac{cx^2(bx+cx^2)^p}{p+1} & \text{for } p\neq -1 \\ \log(x)+\log\left(\frac{b}{c}+x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x**2+b*x)**p,x)
```

```
[Out] Piecewise((b*x*(b*x + c*x**2)**p/(p + 1) + c*x**2*(b*x + c*x**2)**p/(p + 1)
, Ne(p, -1)), (log(x) + log(b/c + x), True))
```

Giac [A]
time = 4.97, size = 19, normalized size = 1.00

$$\frac{(cx^2+bx)^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*c*x+b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

```
[Out] (c*x^2 + b*x)^(p + 1)/(p + 1)
```

Mupad [B]
time = 2.03, size = 23, normalized size = 1.21

$$\frac{x\,(c\,x^2+b\,x)^p\,(b+c\,x)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x + c*x^2)^p*(b + 2*c*x),x)
```

```
[Out] (x*(b*x + c*x^2)^p*(b + c*x))/(p + 1)
```

3.138 $\int x(b+2cx^2)\left(bx^2+cx^4\right)^p\,dx$

Optimal. Leaf size=24

$$\frac{(bx^2+cx^4)^{1+p}}{2(1+p)}$$

```
[Out] 1/2*(c*x^4+b*x^2)^(1+p)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {1602}

$$\frac{(bx^2+cx^4)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (b*x^2 + c*x^4)^(1 + p)/(2*(1 + p))
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\left(b+2cx^2\right)\left(bx^2+cx^4\right)^p\,dx=\frac{(bx^2+cx^4)^{1+p}}{2(1+p)}$$

Mathematica [A]
time = 0.07, size = 24, normalized size = 1.00

$$\frac{(x^2(b+cx^2))^{1+p}}{2(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x]
```

```
[Out] (x^2*(b + c*x^2))^(1 + p)/(2*(1 + p))
```

Maple [A]
time = 0.16, size = 31, normalized size = 1.29

method	result	size
gosper	$\frac{x^{2}\left(c\,x^{2}+b\right)\left(c\,x^{4}+b\,x^{2}\right)^{p}}{2+2p}$	31
risch	$\frac{x^{2}\left(c\,x^{2}+b\right)\left(x^{2}\left(c\,x^{2}+b\right)\right)^{p}}{2+2p}$	31
norman	$\frac{b\,x^{2}\mathrm{e}^{p\ln\left(c\,x^{4}+b\,x^{2}\right)}}{2+2p}+\frac{c\,x^{4}\mathrm{e}^{p\ln\left(c\,x^{4}+b\,x^{2}\right)}}{2+2p}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x,method=_RETURNVERBOSE)
[Out] 1/2*x^2*(c*x^2+b)/(1+p)*(c*x^4+b*x^2)^p
```

Maxima [A]
time = 0.31, size = 35, normalized size = 1.46

$$\frac{(cx^4+bx^2)e^{(p\log(cx^2+b)+2\,p\log(x))}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="maxima")
[Out] 1/2*(c*x^4 + b*x^2)*e^(p*log(c*x^2 + b) + 2*p*log(x))/(p + 1)
```

Fricas [A]
time = 0.34, size = 31, normalized size = 1.29

$$\frac{(cx^4+bx^2)(cx^4+bx^2)^p}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="fricas")
[Out] 1/2*(c*x^4 + b*x^2)*(c*x^4 + b*x^2)^p/(p + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(17) = 34.
time = 10.04, size = 75, normalized size = 3.12

$$\begin{cases} \frac{bx^2(bx^2+cx^4)^p}{2p+2}+\frac{cx^4(bx^2+cx^4)^p}{2p+2} & \text{for}\ p\neq -1 \\ \log(x)+\frac{\log\left(x-\sqrt{-\frac{b}{c}}\right)}{2}+\frac{\log\left(x+\sqrt{-\frac{b}{c}}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x**2+b)*(c*x**4+b*x**2)**p,x)
```

```
[Out] Piecewise((b*x**2*(b*x**2 + c*x**4)**p/(2*p + 2) + c*x**4*(b*x**2 + c*x**4)
**p/(2*p + 2), Ne(p, -1)), (log(x) + log(x - sqrt(-b/c))/2 + log(x + sqrt(-
b/c))/2, True))
```

Giac [A]
time = 3.91, size = 22, normalized size = 0.92

$$\frac{(cx^4+bx^2)^{p+1}}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*c*x^2+b)*(c*x^4+b*x^2)^p,x, algorithm="giac")
```

```
[Out] 1/2*(c*x^4 + b*x^2)^(p + 1)/(p + 1)
```

Mupad [B]
time = 2.07, size = 31, normalized size = 1.29

$$\frac{x^2\,(c\,x^2+b)\,\left(c\,x^4+b\,x^2\right)^p}{2\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b + 2*c*x^2)*(b*x^2 + c*x^4)^p,x)
```

```
[Out] (x^2*(b + c*x^2)*(b*x^2 + c*x^4)^p)/(2*(p + 1))
```

3.139 $\int x^2(b+2cx^3)\left(bx^3+cx^6\right)^p\,dx$

Optimal. Leaf size=24

$$\frac{\left(bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

```
[Out] 1/3*(c*x^6+b*x^3)^(1+p)/(1+p)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{1602\}$

$$\frac{\left(bx^3+cx^6\right)^{p+1}}{3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] (b*x^3 + c*x^6)^(1 + p)/(3*(1 + p))
```

Rule 1602

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(b+2cx^3\right)\left(bx^3+cx^6\right)^p\,dx=\frac{\left(bx^3+cx^6\right)^{1+p}}{3(1+p)}$$

Mathematica [A]
time = 0.07, size = 24, normalized size = 1.00

$$\frac{\left(x^3\left(b+cx^3\right)\right)^{1+p}}{3(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x]
```

```
[Out] (x^3*(b + c*x^3))^(1 + p)/(3*(1 + p))
```

Maple [A]
time = 0.15, size = 31, normalized size = 1.29

method	result	size
gosper	$\frac{(c\,x^3+b)x^3\,(c\,x^6+b\,x^3)^p}{3+3p}$	31
risch	$\frac{x^3\,(c\,x^3+b)\,(x^3\,(c\,x^3+b))^p}{3+3p}$	31
norman	$\frac{b\,x^3 \mathrm{e}^{p\ln\left(c\,x^6+b\,x^3\right)}}{3+3p}+\frac{c\,x^6 \mathrm{e}^{p\ln\left(c\,x^6+b\,x^3\right)}}{3+3p}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(c*x^3+b)*x^3/(1+p)*(c*x^6+b*x^3)^p
```

Maxima [A]
time = 0.30, size = 35, normalized size = 1.46

$$\frac{(cx^6+bx^3)e^{(p\log(cx^3+b)+3\,p\log(x))}}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="maxima")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*e^(p*log(c*x^3 + b) + 3*p*log(x))/(p + 1)
```

Fricas [A]
time = 0.37, size = 31, normalized size = 1.29

$$\frac{(cx^6+bx^3)(cx^6+bx^3)^p}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="fricas")
```

```
[Out] 1/3*(c*x^6 + b*x^3)*(c*x^6 + b*x^3)^p/(p + 1)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*c*x**3+b)*(c*x**6+b*x**3)**p,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.62, size = 22, normalized size = 0.92

$$\frac{(cx^6 + bx^3)^{p+1}}{3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*c*x^3+b)*(c*x^6+b*x^3)^p,x, algorithm="giac")
```

```
[Out] 1/3*(c*x^6 + b*x^3)^(p + 1)/(p + 1)
```

Mupad [B]
time = 2.07, size = 31, normalized size = 1.29

$$\frac{x^3 (c\,x^3 + b)\,(c\,x^6 + b\,x^3)^p}{3\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b + 2*c*x^3)*(b*x^3 + c*x^6)^p,x)
```

```
[Out] (x^3*(b + c*x^3)*(b*x^3 + c*x^6)^p)/(3*(p + 1))
```

3.140 $\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx$

Optimal. Leaf size=26

$$\frac{\left(bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}$$

```
[Out] (b*x^n+c*x^(2*n))^(1+p)/n/(1+p)
```

Rubi [A]

time = 0.05, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2059, 643}

$$\frac{\left(bx^n+cx^{2n}\right)^{p+1}}{n(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (b*x^n + c*x^(2*n))^(1 + p)/(n*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 2059

```
Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_))^(p_)*((c_) + (d_.)*(x_)
^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*
(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; F
reeQ[{a, b, c, d, j, k, m, n, p, q}, x] &&  !IntegerQ[p] && NeQ[k, j] && In
tegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m +
1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n}(b+2cx^n)\left(bx^n+cx^{2n}\right)^p\,dx &= \frac{\text{Subst}\left(\int (b+2cx)\left(bx+cx^2\right)^p\,dx,x,x^n\right)}{n}\\ &= \frac{\left(bx^n+cx^{2n}\right)^{1+p}}{n(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 24, normalized size = 0.92

$$\frac{(x^n(b+cx^n))^{1+p}}{n(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + n)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x^n*(b + c*x^n))^(1 + p)/(n*(1 + p))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.22, size = 155, normalized size = 5.96

method	result
risch	$\frac{x^n(b+c\,x^n)\mathrm{e}^{\frac{p\left(-i\pi\mathrm{csgn}(ix^n(b+c\,x^n))^3+i\pi\mathrm{csgn}(ix^n(b+c\,x^n))^2\mathrm{csgn}(ix^n)+i\pi\mathrm{csgn}(ix^n(b+c\,x^n))^2\mathrm{csgn}(i(b+c\,x^n))-i\pi\,\mathrm{csgn}(ix^n(b+c\,x^n))\mathrm{csgn}(ix}{2}}}{n(1+p)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x,method=_RETURNVERBOSE)
```

```
[Out] x^n*(b+c*x^n)/n/(1+p)*exp(1/2*p*(-I*Pi*csgn(I*x^n*(b+c*x^n))^3+I*Pi*csgn(I*
x^n*(b+c*x^n))^2*csgn(I*x^n)+I*Pi*csgn(I*x^n*(b+c*x^n))^2*csgn(I*(b+c*x^n))
-I*Pi*csgn(I*x^n*(b+c*x^n))*csgn(I*x^n)*csgn(I*(b+c*x^n))+2*ln(x^n)+2*ln(b+
c*x^n)))
```

Maxima [A]
time = 0.35, size = 40, normalized size = 1.54

$$\frac{(cx^{2n}+bx^n)e^{(p\log(cx^n+b)+p\log(x^n))}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] (c*x^(2*n) + b*x^n)*e^(p*log(c*x^n + b) + p*log(x^n))/(n*(p + 1))
```

Fricas [A]
time = 0.36, size = 36, normalized size = 1.38

$$\frac{(cx^{2n}+bx^n)(cx^{2n}+bx^n)^p}{np+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] (c*x^(2*n) + b*x^n)*(c*x^(2*n) + b*x^n)^p/(n*p + n)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*(b+2*c*x**n)*(b*x**n+c*x**(2*n))**p,x)
[Out] Timed out
```

Giac [**A**]
time = 3.69, size = 26, normalized size = 1.00

$$\frac{(cx^{2n} + bx^n)^{p+1}}{n(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*(b+2*c*x^n)*(b*x^n+c*x^(2*n))^p,x, algorithm="giac")
[Out] (c*x^(2*n) + b*x^n)^(p + 1)/(n*(p + 1))
```

Mupad [**B**]
time = 2.13, size = 34, normalized size = 1.31

$$\frac{x^n\,\left(b+c\,x^n\right)\,{\left(b\,x^n+c\,x^{2\,n}\right)}^p}{n\,\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(n - 1)*(b + 2*c*x^n)*(b*x^n + c*x^(2*n))^p,x)
[Out] (x^n*(b + c*x^n)*(b*x^n + c*x^(2*n))^p)/(n*(p + 1))
```

3.141 $\int \frac{(fx)^m(d+ex^n)}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=196

$$\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+}{n}\right.}{\left(b+\sqrt{b^2-4ac}\right)f}$$

```
[Out] (f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/
2)))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))+(f*
x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.19, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1574, 371}

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^
2 - 4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 +
m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c])])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int \frac{(fx)^m\,(d+ex^n)}{a+bx^n+cx^{2n}}\,dx = \int \left(\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^m}{b-\sqrt{b^2-4ac}+2cx^n}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\right)dx$$

$$= \left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{b+\sqrt{b^2-4ac}+2cx^n}\,dx+\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(}{b-\sqrt{b^2}}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{n};\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)}+\frac{\left(e-\frac{2cd-b}{\sqrt{b^2-4}}\right.}{}$$

Mathematica [A]
time = 0.64, size = 318, normalized size = 1.62

$$\frac{2^{-\frac{1+m+n}{n}}x(fx)^m\left(2^{\frac{1+m+n}{n}}\sqrt{b^2-4ac}\,d-\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\left(\frac{cx^n}{b-\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{1+m}{n}}{}_2F_1\left(-\frac{1+m}{n},-\frac{1+m}{n};1-\frac{1+m}{n};\frac{b-\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}+2cx^n}\right)-\left(-bd+\sqrt{b^2-4ac}\,d+2ae\right)\left(\frac{cx^n}{b+\sqrt{b^2-4ac}+2cx^n}\right)^{-\frac{1+m}{n}}{}_2F_1\left(-\frac{1+m}{n},-\frac{1+m}{n};1-\frac{1+m}{n};\frac{b+\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}+2cx^n}\right)\right)}{a\sqrt{b^2-4ac}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (x*(f*x)^m*(2^((1 + m + n)/n)*Sqrt[b^2 - 4*a*c]*d - ((b*d + Sqrt[b^2 - 4*a*
c]*d - 2*a*e)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n,
(b - Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b - S
qrt[b^2 - 4*a*c] + 2*c*x^n))^((1 + m)/n) - ((-(b*d) + Sqrt[b^2 - 4*a*c]*d +
 2*a*e)*Hypergeometric2F1[-((1 + m)/n), -((1 + m)/n), 1 - (1 + m)/n, (b + S
qrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)])/((c*x^n)/(b + Sqrt[b^
2 - 4*a*c] + 2*c*x^n))^((1 + m)/n)))/(2^((1 + m + n)/n)*a*Sqrt[b^2 - 4*a*c]
*(1 + m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(d+e\,x^n)}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d + ex^n)}{a + bx^n + cx^{2n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**n)/(a + b*x**n + c*x**(2*n)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f\,x)^m\,(d + e\,x^n)}{a + b\,x^n + c\,x^{2\,n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n)), x)
```

3.142 $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx$

Optimal. Leaf size=374

$$\frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)}-\frac{c\left((bd-2ae)(1+m-n)-\frac{4acd(1+m-2n)-b^2d(1+m-n)+2aben}{\sqrt{b^2-4ac}}\right)(f}{a\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right.}$$

```
[Out] (f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/f/n/(a+
b*x^n+c*x^(2*n))-c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/
(b-(-4*a*c+b^2)^(1/2)))*((-2*a*e+b*d)*(1+m-n)+(-4*a*c*d*(1+m-2*n)+b^2*d*(1+
m-n)-2*a*b*e*n)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/f/(1+m)/n/(b-(-4*a*c+b^2
)^(1/2))-c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a
*c+b^2)^(1/2)))*((-2*a*e+b*d)*(1+m-n)+(4*a*c*d*(1+m-2*n)-b^2*d*(1+m-n)+2*a*
b*e*n)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)/f/(1+m)/n/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.88, antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1572, 1574, 371}

$$-\frac{c(fx)^{m+1}\left((m-n+1)(bd-2ae)-\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}\right)\,_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(\frac{2aben+4acd(m-2n+1)+b^2(-d)(m-n+1)}{\sqrt{b^2-4ac}}+(m-n+1)(bd-2ae)\right)\,_2F_1\left(1,\frac{m+1}{n};\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)n\left(b^2-4ac\right)\left(\sqrt{b^2-4ac}+b\right)}+\frac{(fx)^{m+1}\left(cx^n(bd-2ae)-abe-2acd+b^2d\right)}{afn\left(b^2-4ac\right)\left(a+bx^n+cx^{2n}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(a*(b^2 - 4
*a*c)*f*n*(a + b*x^n + c*x^(2*n))) - (c*((b*d - 2*a*e)*(1 + m - n) - (4*a*c
*d*(1 + m - 2*n) - b^2*d*(1 + m - n) + 2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^
(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)*n) - (c
*((b*d - 2*a*e)*(1 + m - n) + (4*a*c*d*(1 + m - 2*n) - b^2*d*(1 + m - n) +
2*a*b*e*n)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n,
 (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(a*(b^2 - 4*a*c)*(b +
Sqrt[b^2 - 4*a*c])*f*(1 + m)*n)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1572

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m +
 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*
x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^2}\,dx = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\int \frac{(fx)^m(-abe(1+m)-2acd(1+m-2n)+b^2}{a+bx^n+}}{a\left(b^2-\right.}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\int\left(\frac{\left(c(bd-2ae)(1+m-n)+\frac{c\left(b^2d-4acd+b\right.}{}\right.}{b-\sqrt{b^2-}}\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{\left(c\left((bd-2ae)(1+m-n)-\frac{4a}{}\right.\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)} - \frac{c\left((bd-2ae)(1+m-n)-\frac{4acd}{}\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5363 vs. 2(374) = 748.
time = 6.66, size = 5363, normalized size = 14.34

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{m} (d + e\, x^{n})}{(a + b\, x^{n} + c\, x^{2n})^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="maxima")
```

```
[Out] ((b^2*d*f^m - 2*a*c*d*f^m - a*b*f^m*e)*x*x^m + (b*c*d*f^m - 2*a*c*f^m*e)*x*
e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*c*n + (a*b^2*c*n - 4*a^2*c^2*n)
*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n) - integrate(((b^2*d*f^m*(m - n + 1)
 - 2*a*c*d*f^m*(m - 2*n + 1) - a*b*f^m*(m + 1)*e)*x^m + (b*c*d*f^m*(m - n +
 1) - 2*a*c*f^m*(m - n + 1)*e)*e^(m*log(x) + n*log(x)))/(a^2*b^2*n - 4*a^3*
c*n + (a*b^2*c*n - 4*a^2*c^2*n)*x^(2*n) + (a*b^3*n - 4*a^2*b*c*n)*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)*(f*x)^m/(c^2*x^(4*n) + b^2*x^(2*n) + 2*a*b*x^n + a^2 +
 2*(b*c*x^n + a*c)*x^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^2,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^m\,(d+e\,x^n)}{(a+b\,x^n+c\,x^{2\,n})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2,x)
```

```
[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^2, x)
```

3.143 $\int \frac{(fx)^m(d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx$

Optimal. Leaf size=816

$$\frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2}+\frac{(fx)^{1+m}\left(\left(b^2-2ac\right)\left(abe(1+m)+2acd(1+m-4n)-b^2d(1\right.\right.}{}$$

```
[Out] 1/2*(f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^n)/a/(-4*a*c+b^2)/f/n
/(a+b*x^n+c*x^(2*n))^2+1/2*(f*x)^(1+m)*((-2*a*c+b^2)*(a*b*e*(1+m)+2*a*c*d*(
1+m-4*n)-b^2*d*(1+m-2*n))+a*b*c*(-2*a*e+b*d)*(1+m-3*n)+c*(a*b^2*e*(1+m)+2*a
*b*c*d*(2+2*m-7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1+m-2*n))*x^n)/a^2/(-4*a*c+b^
2)^2/f/n^2/(a+b*x^n+c*x^(2*n))-1/2*c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1
+m+n)/n],-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))*((a*b^2*e*(1+m)+2*a*b*c*d*(2+2*m-
7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1+m-2*n))*(1+m-n)+(a*b^3*e*(1+m)*(1+m-n)-4*
a^2*b*c*e*(1+m^2+m*(2-n)-n-3*n^2)-b^4*d*(1+m^2+m*(2-3*n)-3*n+2*n^2)+6*a*b^2
*c*d*(1+m^2+m*(2-4*n)-4*n+3*n^2)-8*a^2*c^2*d*(1+m^2+m*(2-6*n)-6*n+8*n^2))/(
-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2/f/(1+m)/n^2/(b-(-4*a*c+b^2)^(1/2))-1/
2*c*(f*x)^(1+m)*hypergeom([1, (1+m)/n],[(1+m+n)/n],-2*c*x^n/(b+(-4*a*c+b^2)
^(1/2)))*((a*b^2*e*(1+m)+2*a*b*c*d*(2+2*m-7*n)-4*a^2*c*e*(1+m-3*n)-b^3*d*(1
+m-2*n))*(1+m-n)+(-a*b^3*e*(1+m)*(1+m-n)+4*a^2*b*c*e*(1+m^2+m*(2-n)-n-3*n^2
)+b^4*d*(1+m^2+m*(2-3*n)-3*n+2*n^2)-6*a*b^2*c*d*(1+m^2+m*(2-4*n)-4*n+3*n^2)
+8*a^2*c^2*d*(1+m^2+m*(2-6*n)-6*n+8*n^2))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b
^2)^2/f/(1+m)/n^2/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 3.52, antiderivative size = 816, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1572, 1574, 371}

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^n))/(2*a*(b^2 -
 4*a*c)*f*n*(a + b*x^n + c*x^(2*n))^2) + ((f*x)^(1 + m)*((b^2 - 2*a*c)*(a*b
*e*(1 + m) + 2*a*c*d*(1 + m - 4*n) - b^2*d*(1 + m - 2*n)) + a*b*c*(b*d - 2*
a*e)*(1 + m - 3*n) + c*(a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) - 4*a^2
*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*x^n))/(2*a^2*(b^2 - 4*a*c)^2*f*n^
2*(a + b*x^n + c*x^(2*n))) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*
n) - 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) + (a*b^3*e*
(1 + m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d
```

```
*(1 + m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n)
 - 4*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b
^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (
-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b - Sqrt[b^2 -
4*a*c])*f*(1 + m)*n^2) - (c*((a*b^2*e*(1 + m) + 2*a*b*c*d*(2 + 2*m - 7*n) -
 4*a^2*c*e*(1 + m - 3*n) - b^3*d*(1 + m - 2*n))*(1 + m - n) - (a*b^3*e*(1 +
 m)*(1 + m - n) - 4*a^2*b*c*e*(1 + m^2 + m*(2 - n) - n - 3*n^2) - b^4*d*(1
+ m^2 + m*(2 - 3*n) - 3*n + 2*n^2) + 6*a*b^2*c*d*(1 + m^2 + m*(2 - 4*n) - 4
*n + 3*n^2) - 8*a^2*c^2*d*(1 + m^2 + m*(2 - 6*n) - 6*n + 8*n^2))/Sqrt[b^2 -
 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/n, (1 + m + n)/n, (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/(2*a^2*(b^2 - 4*a*c)^2*(b + Sqrt[b^2 - 4*a*
c])*f*(1 + m)*n^2)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1572

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))*((a_) + (b_.)*(x_)^(n_) + (
c_.)*(x_)^(n2_))^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^n + c*x^
(2*n))^(p + 1)*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^n)/(a*f*n*(p +
 1)*(b^2 - 4*a*c))), x] + Dist[1/(a*n*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(
a + b*x^n + c*x^(2*n))^(p + 1)*Simp[d*(b^2*(m + n*(p + 1) + 1) - 2*a*c*(m +
 2*n*(p + 1) + 1)) - a*b*e*(m + 1) + (m + n*(2*p + 3) + 1)*(b*d - 2*a*e)*c*
x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && EqQ[n2, 2*n] && Ne
Q[b^2 - 4*a*c, 0] && ILtQ[p + 1, 0]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^n)}{(a+bx^n+cx^{2n})^3}\,dx = \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} - \frac{\int \frac{(fx)^m(-abe(1+m)-2acd(1+m-4n)+b^2d(}{(a+bx^n+c}}{2a\left(b^2-\right.}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)(abe(1+m)\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)(abe(1+m)\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)(abe(1+m)\right.}{}$$

$$= \frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^n\right)}{2a\left(b^2-4ac\right)fn\left(a+bx^n+cx^{2n}\right)^2} + \frac{(fx)^{1+m}\left(\left(b^2-2ac\right)(abe(1+m)\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 20515 vs. $2(816) = 1632$.
time = 7.54, size = 20515, normalized size = 25.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x]
[Out] Result too large to show
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(d+e\,x^n\right)}{\left(a+b\,x^n+c\,x^{2n}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
[Out] int((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((a^2*b^2*c*d*f^m*(5*m - 21*n + 5) - a*b^4*d*f^m*(m - 3*n + 1) - 4*a^3*
c^2*d*f^m*(m - 6*n + 1) - (2*a^3*b*c*f^m*(2*m - 5*n + 2) - a^2*b^3*f^m*(m -
 n + 1))*e)*x*x^m + (2*a*b*c^3*d*f^m*(2*m - 7*n + 2) - b^3*c^2*d*f^m*(m - 2
*n + 1) - (4*a^2*c^3*f^m*(m - 3*n + 1) - a*b^2*c^2*f^m*(m + 1))*e)*x*e^(m*l
og(x) + 3*n*log(x)) + (a*b^2*c^2*d*f^m*(9*m - 29*n + 9) - 2*b^4*c*d*f^m*(m
- 2*n + 1) - 4*a^2*c^3*d*f^m*(m - 4*n + 1) - 2*(a^2*b*c^2*f^m*(4*m - 9*n +
4) - a*b^3*c*f^m*(m + 1))*e)*x*e^(m*log(x) + 2*n*log(x)) - (b^5*d*f^m*(m -
2*n + 1) - 4*a*b^3*c*d*f^m*(m - 3*n + 1) + 2*a^2*b*c^2*d*f^m*n + (a^2*b^2*c
*f^m*(3*m - 4*n + 3) + 4*a^3*c^2*f^m*(m - 5*n + 1) - a*b^4*f^m*(m + 1))*e)*
x*e^(m*log(x) + n*log(x)))/(a^4*b^4*n^2 - 8*a^5*b^2*c*n^2 + 16*a^6*c^2*n^2
+ (a^2*b^4*c^2*n^2 - 8*a^3*b^2*c^3*n^2 + 16*a^4*c^4*n^2)*x^(4*n) + 2*(a^2*b
^5*c*n^2 - 8*a^3*b^3*c^2*n^2 + 16*a^4*b*c^3*n^2)*x^(3*n) + (a^2*b^6*n^2 - 6
*a^3*b^4*c*n^2 + 32*a^5*c^3*n^2)*x^(2*n) + 2*(a^3*b^5*n^2 - 8*a^4*b^3*c*n^2
 + 16*a^5*b*c^2*n^2)*x^n) + integrate(1/2*(((m^2 - m*(3*n - 2) + 2*n^2 - 3*
n + 1)*b^4*d*f^m - (5*m^2 - m*(21*n - 10) + 16*n^2 - 21*n + 5)*a*b^2*c*d*f^
m + 4*(m^2 - 2*m*(3*n - 1) + 8*n^2 - 6*n + 1)*a^2*c^2*d*f^m - ((m^2 - m*(n
- 2) - n + 1)*a*b^3*f^m - 2*(2*m^2 - m*(5*n - 4) - 5*n + 2)*a^2*b*c*f^m)*e)
*x^m + ((m^2 - m*(3*n - 2) + 2*n^2 - 3*n + 1)*b^3*c*d*f^m - 2*(2*m^2 - m*(9
*n - 4) + 7*n^2 - 9*n + 2)*a*b*c^2*d*f^m - ((m^2 - m*(n - 2) - n + 1)*a*b^2
*c*f^m - 4*(m^2 - 2*m*(2*n - 1) + 3*n^2 - 4*n + 1)*a^2*c^2*f^m)*e)*e^(m*log
(x) + n*log(x)))/(a^3*b^4*n^2 - 8*a^4*b^2*c*n^2 + 16*a^5*c^2*n^2 + (a^2*b^4
*c*n^2 - 8*a^3*b^2*c^2*n^2 + 16*a^4*c^3*n^2)*x^(2*n) + (a^2*b^5*n^2 - 8*a^3
*b^3*c*n^2 + 16*a^4*b*c^2*n^2)*x^n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)*(f*x)^m/(c^3*x^(6*n) + b^3*x^(3*n) + 3*a*b^2*x^(2*n) +
 3*a^2*b*x^n + a^3 + 3*(b*c^2*x^n + a*c^2)*x^(4*n) + 3*(b^2*c*x^(2*n) + 2*a
*b*c*x^n + a^2*c)*x^(2*n)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)/(a+b*x**n+c*x**(2*n))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)/(a+b*x^n+c*x^(2*n))^3,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)*(f*x)^m/(c*x^(2*n) + b*x^n + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^m\,(d+e\,x^n)}{(a+b\,x^n+c\,x^{2\,n})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3,x)
```

```
[Out] int(((f*x)^m*(d + e*x^n))/(a + b*x^n + c*x^(2*n))^3, x)
```

3.144 $\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}\,x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}\,dx^{4/3}}\,dx$

Optimal. Leaf size=47

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}\,d^{2/3}}$$

[Out] -3*ln(c^(2/3)-c^(1/3)*d^(1/3)*x^(1/3)+d^(2/3)*x^(2/3))/c^(1/3)/d^(2/3)

Rubi [A]
time = 0.04, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$, Rules used = {1608, 1482, 642}

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}\,d^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x + c^(1/3)*d*x^(4/3)),x]

[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2/3))

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1482

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + b*
x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && E
qQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 1608

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rubi steps

$$\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{c\sqrt[3]{d}\,x^{2/3}-c^{2/3}d^{2/3}x+\sqrt[3]{c}\,dx^{4/3}}\,dx=\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\sqrt[3]{x}}{\left(c\sqrt[3]{d}-c^{2/3}d^{2/3}\sqrt[3]{x}+\sqrt[3]{c}\,dx^{2/3}\right)x^{2/3}}\,dx$$
$$=3\mathrm{Subst}\left(\int \frac{\sqrt[3]{c}-2\sqrt[3]{d}\,x}{c\sqrt[3]{d}-c^{2/3}d^{2/3}x+\sqrt[3]{c}\,dx^2}\,dx,x,\sqrt[3]{x}\right)$$
$$=-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}\,d^{2/3}}$$

Mathematica [A]
time = 0.05, size = 47, normalized size = 1.00

$$-\frac{3\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}\sqrt[3]{x}+d^{2/3}x^{2/3}\right)}{\sqrt[3]{c}\,d^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3
)*x + c^(1/3)*d*x^(4/3)),x]
```

```
[Out] (-3*Log[c^(2/3) - c^(1/3)*d^(1/3)*x^(1/3) + d^(2/3)*x^(2/3)])/(c^(1/3)*d^(2
/3))
```

Maple [A]
time = 0.01, size = 36, normalized size = 0.77

method	result	size
derivativedivides	$-\frac{3\ln\left(c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}-c^{\frac{1}{3}}dx^{\frac{2}{3}}-cd^{\frac{1}{3}}\right)}{d^{\frac{2}{3}}c^{\frac{1}{3}}}$	36
default	$-\frac{3\ln\left(c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}-c^{\frac{1}{3}}dx^{\frac{2}{3}}-cd^{\frac{1}{3}}\right)}{d^{\frac{2}{3}}c^{\frac{1}{3}}}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+c^(1/3
)*d*x^(4/3)),x,method=_RETURNVERBOSE)
```

```
[Out] -3/d^(2/3)/c^(1/3)*ln(c^(2/3)*d^(2/3)*x^(1/3)-c^(1/3)*d*x^(2/3)-c*d^(1/3))
```

Maxima [A]
time = 0.28, size = 34, normalized size = 0.72

$$-\frac{3\,\log\left(c^{\frac{1}{3}}dx^{\frac{2}{3}}-c^{\frac{2}{3}}d^{\frac{2}{3}}x^{\frac{1}{3}}+cd^{\frac{1}{3}}\right)}{c^{\frac{1}{3}}d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="maxima")
```

```
[Out] -3*log(c^(1/3)*d*x^(2/3) - c^(2/3)*d^(2/3)*x^(1/3) + c*d^(1/3))/(c^(1/3)*d^
(2/3))
```

Fricas [A]
time = 0.32, size = 33, normalized size = 0.70

$$-\frac{3 \log \left(dx^{\frac{2}{3}} - c^{\frac{1}{3}} d^{\frac{2}{3}} x^{\frac{1}{3}} + c^{\frac{2}{3}} d^{\frac{1}{3}}\right)}{c^{\frac{1}{3}} d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="fricas")
```

```
[Out] -3*log(d*x^(2/3) - c^(1/3)*d^(2/3)*x^(1/3) + c^(2/3)*d^(1/3))/(c^(1/3)*d^(2
/3))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(44) = 88$.
time = 2.15, size = 119, normalized size = 2.53

$$-\frac{3 \log \left(-\frac{\sqrt[3]{c}}{2 \sqrt[3]{d}} + \sqrt[3]{x} - \frac{\sqrt{3} \sqrt{-c^{\frac{4}{3}} d^{\frac{4}{3}}}}{2 \sqrt[3]{c}\, d}\right)}{\sqrt[3]{c}\, d^{\frac{2}{3}}} - \frac{3 \log \left(-\frac{\sqrt[3]{c}}{2 \sqrt[3]{d}} + \sqrt[3]{x} + \frac{\sqrt{3} \sqrt{-c^{\frac{4}{3}} d^{\frac{4}{3}}}}{2 \sqrt[3]{c}\, d}\right)}{\sqrt[3]{c}\, d^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c**(1/3)-2*d**(1/3)*x**(1/3))/(c*d**(1/3)*x**(2/3)-c**(2/3)*d**(
2/3)*x+c**(1/3)*d*x**(4/3)),x)
```

```
[Out] -3*log(-c**(1/3)/(2*d**(1/3)) + x**(1/3) - sqrt(3)*sqrt(-c**(4/3)*d**(4/3))
/(2*c**(1/3)*d))/(c**(1/3)*d**(2/3)) - 3*log(-c**(1/3)/(2*d**(1/3)) + x**(1
/3) + sqrt(3)*sqrt(-c**(4/3)*d**(4/3))/(2*c**(1/3)*d))/(c**(1/3)*d**(2/3))
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c^(1/3)-2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3)-c^(2/3)*d^(2/3)*x+
c^(1/3)*d*x^(4/3)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to roun
ding error%%%{%%{[%%%{%%{[%%%{1,[1]%%%},0]:[1,0,0,%%%{-1,[1]%%%}]%%},[1]%%%
},0]:[
```

Mupad [B]
time = 2.46, size = 31, normalized size = 0.66

$$-\frac{3\,\ln\left(x^{2/3}+\frac{c^{2/3}}{d^{2/3}}-\frac{c^{1/3}\,x^{1/3}}{d^{1/3}}\right)}{c^{1/3}\,d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c^(1/3) - 2*d^(1/3)*x^(1/3))/(c*d^(1/3)*x^(2/3) - c^(2/3)*d^(2/3)*x +
c^(1/3)*d*x^(4/3)),x)
```

```
[Out] -(3*log(x^(2/3) + c^(2/3)/d^(2/3) - (c^(1/3)*x^(1/3))/d^(1/3)))/(c^(1/3)*d^
(2/3))
```

3.145 $\int \frac{(fx)^m(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=245

$$\frac{2c(fx)^{1+m}\left(d+ex^n\right)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1+m}{n};1,-q;\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)}-\frac{2c(fx)^{1+m}\left(d+ex^n\right)^q\left(1\right.}{\sqrt{b}}$$

```
[Out] 2*c*(f*x)^(1+m)*(d+e*x^n)^q*AppellF1((1+m)/n,1,-q,(1+m+n)/n,-2*c*x^n/(b-(-4
*a*c+b^2)^(1/2)),-e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(b-(-4*a*c+b^2)^(1/2))/(
-4*a*c+b^2)^(1/2)-2*c*(f*x)^(1+m)*(d+e*x^n)^q*AppellF1((1+m)/n,1,-q,(1+m+n)
/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/f/(1+m)/((1+e*x^n/d)^q)/(-4*a*
c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.22, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1570, 525, 524}

$$\frac{2c(fx)^{m+1}\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}\left(d+ex^n\right)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{m+1}{n};1,-q;\frac{m+n+1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n,
(-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b -
Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^n)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e
*x^n)^q*AppellF1[(1 + m)/n, 1, -q, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2
- 4*a*c]), -((e*x^n)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 +
 m)*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{(fx)^m(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(fx)^m(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right.}{\sqrt{b^2-}}$$

$$= \frac{2c(fx)^{1+m}(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1+m}{n};1,-q;\frac{1+m+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)}$$

Mathematica [**F**]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [**F**]
time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((f*x)^m*(x^n*e + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((f*x)^m*(x^n*e + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((f*x)^m*(x^n*e + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^m\,(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
[Out] int(((f*x)^m*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

3.146 $\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=210

$$-\frac{2cx^3(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)}-\frac{2cx^3(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{3}{n};1\right.}{3\left(b^2-4ac+b\right.}$$

```
[Out] -2/3*c*x^3*(d+e*x^n)^q*AppellF1(3/n,1,-q,(3+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(
1/2)),-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2/3*c*x^3*
(d+e*x^n)^q*AppellF1(3/n,1,-q,(3+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^
n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.18, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1570, 525, 524}

$$-\frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}-\frac{2cx^3(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{n+3}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2*c*x^n)/(b - Sqr
t[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x^3*(d + e*x^n)^q*AppellF1[3/n, 1, -q, (3 + n)/n, (-2
*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/(3*(b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{x^2(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x^2(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$
$$= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x^2\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)}{\sqrt{b^2-4}}$$
$$= -\frac{2cx^3(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{3}{n};1,-q;\frac{3+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{3\left(b^2-4ac-b\sqrt{b^2-4ac}\right)} - \frac{2cx^3(d+}{}$$

Mathematica [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^2(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] Integrate[(x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((x^n*e + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((x^n*e + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((x^n*e + d)^q*x^2/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(d+e\,x^n\right)}^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
[Out] int((x^2*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

3.147 $\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=206

$$-\frac{cx^2(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{cx^2(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{2}{n};1,\right.}{b^2-4ac+b}$$

```
[Out] -c*x^2*(d+e*x^n)^q*AppellF1(2/n,1,-q,(2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)
),-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-c*x^2*(d+e*x^n
)^q*AppellF1(2/n,1,-q,(2+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/((1
+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.13, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1570, 525, 524}

$$-\frac{cx^2(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{cx^2(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{n+2}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] -((c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x^n)/(b - Sqrt
[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e
*x^n)/d)^q)) - (c*x^2*(d + e*x^n)^q*AppellF1[2/n, 1, -q, (2 + n)/n, (-2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*
a*c])*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{x(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{x(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{x\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int}{\sqrt{b^2-4a}}$$

$$= -\frac{cx^2(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{2}{n};1,-q;\frac{2+n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{cx^2(d+ex}{}$$

Mathematica [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[(x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
[Out] int(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^q*x/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(d+e\,x^n\right)}^q}{a+b\,x^n+c\,x^{2\,n}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int((x*(d + e*x^n)^q)/(a + b*x^n + c*x^(2*n)), x)
```

3.148 $\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$

Optimal. Leaf size=194

$$-\frac{2cx(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{2cx(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1}{n};1\right.}{b^2-4ac+}$$

```
[Out] -2*c*x*(d+e*x^n)^q*AppellF1(1/n,1,-q,1+1/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),
-e*x^n/d)/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*(d+e*x^n)^
q*AppellF1(1/n,1,-q,1+1/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/((1+e*x
^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.10, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1442, 441, 440}

$$-\frac{2cx(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
```

```
[Out] (-2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 +
 (e*x^n)/d)^q) - (2*c*x*(d + e*x^n)^q*AppellF1[n^(-1), 1, -q, 1 + n^(-1), (
-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^
2 - 4*a*c])*(1 + (e*x^n)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1442

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^
n)^q/(b - r + 2*c*x^n), x], x] - Dist[2*(c/r), Int[(d + e*x^n)^q/(b + r + 2
*c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n, q}, x] && EqQ[n2, 2*n] && NeQ[
b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx = \frac{(2c)\int \frac{(d+ex^n)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{b+\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^n}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)}{\sqrt{b^2-4}}$$

$$= -\frac{2cx(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(\frac{1}{n};1,-q;1+\frac{1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx(d+}{}$$

Mathematica [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^n)^q}{a+bx^n+cx^{2n}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x]
[Out] Integrate[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
[Out] int((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^q/(c*x^(2*n) + b*x^n + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(d+e\,x^n)^q}{a+b\,x^n+c\,x^{2\,n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),x)
```

```
[Out] int((d + e*x^n)^q/(a + b*x^n + c*x^(2*n)), x)
```

3.149 $\int \frac{(d+ex^n)^q}{x\left(a+bx^n+cx^{2n}\right)}\,dx$

Optimal. Leaf size=263

$$\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^n)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)n(1+q)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex}{a\left(2cd-\right.}$$

```
[Out] -(d+e*x^n)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^n/d)/a/d/n/(1+q)+c*(d+e*x^n
)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(d+e*x^n)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/
2))))*(1+b/(-4*a*c+b^2)^(1/2))/a/n/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+c
*(d+e*x^n)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(d+e*x^n)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/a/n/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))
```

Rubi [A]
time = 0.53, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1488, 974, 67, 844, 70}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^n)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^n+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{an(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(d+ex^n)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{ex^n}{d}+1\right)}{adn(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d
- (b - Sqrt[b^2 - 4*a*c])*e)*n*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*n*(1
 + q)) - ((d + e*x^n)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^n
)/d])/(a*d*n*(1 + q))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1488

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*((d_) + (
e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)
/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c
, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx = \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x(a+bx+cx^2)}\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax} + \frac{(-b-cx)(d+ex)^q}{a(a+bx+cx^2)}\right)\,dx, x, x^n\right)}{n}$$

$$= \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx, x, x^n\right)}{an} + \frac{\text{Subst}\left(\int \frac{(-b-cx)(d+ex)^q}{a+bx+cx^2}\,dx, x, x^n\right)}{an}$$

$$= -\frac{(d+ex^n)^{1+q}\ {}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{adn(1+q)} + \frac{\text{Subst}\left(\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+}{b-\sqrt{b^2-4ac}+2cx}\right.\right.}{}$$

$$= -\frac{(d+ex^n)^{1+q}\ {}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{adn(1+q)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\right.}{an}$$

$$= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^n)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^n)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)n(1+q)} +$$

Mathematica [A]
time = 0.57, size = 218, normalized size = 0.83

$$\frac{(d+ex^n)^{1+q}\left(\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^n)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^n)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e} - \frac{{}_2F_1\left(1,1+q;2+q;1+\frac{ex^n}{d}\right)}{d}\right)}{an(1+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x]
[Out] ((d + e*x^n)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^n))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric
2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^n))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2 + q,
1 + (e*x^n)/d]/d))/(a*n*(1 + q))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^q}{x\,(a+b\,x^n+c\,x^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)^q/(c*x*x^(2*n) + b*x*x^n + a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^n)^q}{x\,(a+bx^n+cx^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x/(a+b*x**n+c*x**(2*n)),x)
```

```
[Out] Integral((d + e*x**n)**q/(x*(a + b*x**n + c*x**(2*n))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)^q/((c*x^(2*n) + b*x^n + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^q}{x\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))),x)
[Out] int((d + e*x^n)^q/(x*(a + b*x^n + c*x^(2*n))), x)
```

3.150 $\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=212

$$\frac{2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x}+\frac{2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(-\frac{1}{n};1,\ldots\right)}{\left(b^2-4ac+b\sqrt{\ldots}\right.}$$

```
[Out] 2*c*(d+e*x^n)^q*AppellF1(-1/n,1,-q,(-1+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2))
,-e*x^n/d)/x/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+2*c*(d+e*x^n)
^q*AppellF1(-1/n,1,-q,(-1+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/x/
((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.17, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1570, 525, 524}

$$\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{2c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n), (-2*c*x^n)/(b - S
qrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x*(1
 + (e*x^n)/d)^q) + (2*c*(d + e*x^n)^q*AppellF1[-n^(-1), 1, -q, -((1 - n)/n)
, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt
[b^2 - 4*a*c])*x*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx &= \frac{(2c)\int \frac{(d+ex^n)^q}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{x^2\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}\\ &= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^2\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q\left(1+\right.\right.}{}\\ &= \frac{2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(-\frac{1}{n};1,-q;-\frac{1-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x} + \frac{2c(d}{}\end{aligned}$$

Mathematica [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^n)^q}{x^2(a+bx^n+cx^{2n})}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x]
[Out] Integrate[(d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^q}{x^2(a+b\,x^n+c\,x^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((x^n*e + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((x^n*e + d)^q/(c*x^2*x^(2*n) + b*x^2*x^n + a*x^2), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**2/(a+b*x**n+c*x**(2*n)),x)
[Out] Exception raised: HeuristicGCDFailed >> no luck
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^2/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((x^n*e + d)^q/((c*x^(2*n) + b*x^n + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^q}{x^2\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))),x)
[Out] int((d + e*x^n)^q/(x^2*(a + b*x^n + c*x^(2*n))), x)
```

3.151 $\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx$

Optimal. Leaf size=210

$$\frac{c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2}+\frac{c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(-\frac{2}{n};1,\right.}{\left(b^2-4ac+b\sqrt{\right.}}$$

```
[Out] c*(d+e*x^n)^q*AppellF1(-2/n,1,-q,(-2+n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-
e*x^n/d)/x^2/((1+e*x^n/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))+c*(d+e*x^n)^q
*AppellF1(-2/n,1,-q,(-2+n)/n,-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)),-e*x^n/d)/x^2/
((1+e*x^n/d)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1570, 525, 524}

$$\frac{c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(-b\sqrt{b^2-4ac}-4ac+b^2\right)}+\frac{c(d+ex^n)^q\left(\frac{ex^n}{d}+1\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{x^2\left(b\sqrt{b^2-4ac}-4ac+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
```

```
[Out] (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*x^2*(1 +
(e*x^n)/d)^q) + (c*(d + e*x^n)^q*AppellF1[-2/n, 1, -q, -((2 - n)/n), (-2*c*
x^n)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^n)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4
*a*c])*x^2*(1 + (e*x^n)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
```

```
NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1570

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^(n_))^(q_))/((a_) + (c_.)*(x_)^(
n2_.) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[
2*(c/r), Int[(f*x)^m*((d + e*x^n)^q/(b - r + 2*c*x^n)), x], x] - Dist[2*(c/
r), Int[(f*x)^m*((d + e*x^n)^q/(b + r + 2*c*x^n)), x], x]] /; FreeQ[{a, b,
c, d, e, f, m, n, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx = \frac{(2c)\int \frac{(d+ex^n)^q}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^n)^q}{x^3\left(b+\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^n}{d}\right)^q}{x^3\left(b-\sqrt{b^2-4ac}+2cx^n\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^n)^q\left(1+\right.\right.}{}$$

$$= \frac{c(d+ex^n)^q\left(1+\frac{ex^n}{d}\right)^{-q}F_1\left(-\frac{2}{n};1,-q;-\frac{2-n}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{ex^n}{d}\right)}{\left(b^2-4ac-b\sqrt{b^2-4ac}\right)x^2} + \frac{c(d+e}{}$$

Mathematica [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^n)^q}{x^3(a+bx^n+cx^{2n})}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x]
[Out] Integrate[(d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(d+e\,x^n)^q}{x^3\,(a+b\,x^n+c\,x^{2n})}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
```

```
[Out] int((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="maxima")
[Out] integrate((x^n*e + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="fricas")
[Out] integral((x^n*e + d)^q/(c*x^3*x^(2*n) + b*x^3*x^n + a*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x**n)**q/x**3/(a+b*x**n+c*x**(2*n)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x^n)^q/x^3/(a+b*x^n+c*x^(2*n)),x, algorithm="giac")
[Out] integrate((x^n*e + d)^q/((c*x^(2*n) + b*x^n + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d+e\,x^n)^q}{x^3\,\left(a+b\,x^n+c\,x^{2\,n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))),x)
[Out] int((d + e*x^n)^q/(x^3*(a + b*x^n + c*x^(2*n))), x)
```

3.152 $\int (fx)^m (d+ex^n)^2 (a+bx^n+cx^{2n})^p \, dx$

Optimal. Leaf size=498

$$\frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{}{b-\sqrt{}}\right.}{f(1+m)}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c
*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+2*d
*e*x^(1+n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+n)/n,-p,-p,(1+m+2*n)
/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+m+n)
/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))
^p)+e^2*x^(1+2*n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+2*n)/n,-p,-p,
(1+m+3*n)/n,-2*c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2))
)/(1+m+2*n)/((1+2*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b
^2)^(1/2)))^p)
```

Rubi [A]

time = 0.40, antiderivative size = 498, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1574, 1399, 524, 20}

$$\frac{d^2(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}+\frac{2dex^{n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{m+n+1}{n};-p,-p;\frac{m+2n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{m+n+1}+\frac{e^2x^{2n+1}(fx)^m\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}(a+bx^n+cx^{2n})^p F_1\left(\frac{m+2n+1}{n};-p,-p;\frac{m+3n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{m+2n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (d^2*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1
 + m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 -
 4*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x
^n)/(b + Sqrt[b^2 - 4*a*c]))^p) + (2*d*e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x
^(2*n))^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 +
(2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])
)^p) + (e^2*x^(1 + 2*n)*(f*x)^m*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m +
 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c
*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + 2*n)*(1 + (2*c*x^n)/(b - Sqrt[b^2
 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
```

```
IntegerQ[m + n]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int (fx)^m (d+ex^n)^2 \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(d^2(fx)^m \left(a+bx^n+cx^{2n}\right)^p + 2dex^n(fx)^m \left(a+bx^n+cx^{2n}\right)$$

$$= d^2 \int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx + (2de) \int x^n(fx)^m \left(a+bx^n\right.$$

$$= \left(2dex^{-m}(fx)^m\right) \int x^{m+n}\left(a+bx^n+cx^{2n}\right)^p \, dx + \left(e^2x^{-m}(fx)^m\right.$$

$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a-}{}$$

$$= \frac{d^2(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a-}{}$$

Mathematica [A]
time = 0.80, size = 391, normalized size = 0.79

$$\frac{x(fx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(d^2(1+m^2+3n+2n^2+m(2+3n))F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^n\left(2d(1+m+2n)F_1\left(\frac{1+m+n}{n};-p,-p;\frac{1+m+2n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+e(1+m+n)x^nF_1\left(\frac{1+m+2n}{n};-p,-p;\frac{1+m+3n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)\right)}{(1+m)(1+m+n)(1+m+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d^2*(1 + m^2 + 3*n + 2*n^2 + m*(2 + 3*n
))*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^n*(2*d*(1 + m + 2*
n)*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b + Sqrt[b^
2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m + n)*x^n*Appell
F1[(1 + m + 2*n)/n, -p, -p, (1 + m + 3*n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])))/((1 + m)*(1 + m + n)*(1 + m +
2*n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b + Sq
rt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int (fx)^m\left(d+e\,x^n\right)^2\left(a+b\,x^n+c\,x^{2n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)^2*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((2*d*x^n*e + d^2 + x^(2*n)*e^2)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m,
 x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)**2*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)^2*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Simplification assuming sageVARx near
 0Simplification assuming sageVARf near 0Simplification assuming sageVARx n
ear 0S
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,(d+e\,x^n)^2\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d + e*x^n)^2*(a + b*x^n + c*x^(2*n))^p, x)
```

3.153 $\int (fx)^m (d+ex^n) (a+bx^n+cx^{2n})^p \, dx$

Optimal. Leaf size=323

$$\frac{d(fx)^{1+m} \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+bx^n+cx^{2n})^p F_1\left(\frac{1+m}{n}; -p, -p; \frac{1+m+n}{n}; -\frac{}{b-\sqrt{b}}\right.}{f(1+m)}$$

```
[Out] d*(f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x
^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*c*
x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)+e*x^(
1+n)*(f*x)^m*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m+n)/n,-p,-p,(1+m+2*n)/n,-2*
c*x^n/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/(1+m+n)/((1+2
*c*x^n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.24, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1574, 1399, 524, 20}

$$\frac{d(fx)^{m+1} \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p} (a+bx^n+cx^{2n})^p F_1\left(\frac{m+1}{n}; -p, -p; \frac{m+n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)} + \frac{ex^{n+1}(fx)^m \left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p} \left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p} (a+bx^n+cx^{2n})^p F_1\left(\frac{m+n+1}{n}; -p, -p; \frac{m+2n+1}{n}; -\frac{2cx^n}{b-\sqrt{b^2-4ac}}, -\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{m+n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (d*(f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 +
 m + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n
)/(b + Sqrt[b^2 - 4*a*c]))^p) + (e*x^(1 + n)*(f*x)^m*(a + b*x^n + c*x^(2*n)
)^p*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n)/n, (-2*c*x^n)/(b - Sqrt[b
^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c])])/((1 + m + n)*(1 + (2*c*x
^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !
IntegerQ[m + n]
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
```

```
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rule 1574

```
Int[((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.)*(
(d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d
+ e*x^n)^q*(a + b*x^n + c*x^(2*n))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ
[q, 0])
```

Rubi steps

$$\int (fx)^m \left(d+ex^n\right) \left(a+bx^n+cx^{2n}\right)^p \, dx = \int \left(d(fx)^m \left(a+bx^n+cx^{2n}\right)^p + ex^n (fx)^m \left(a+bx^n+cx^{2n}\right)^p\right) d$$

$$= d\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx + e\int x^n (fx)^m \left(a+bx^n+cx^{2n}\right.$$

$$= \left(ex^{-m}(fx)^m\right)\int x^{m+n}\left(a+bx^n+cx^{2n}\right)^p \, dx + \left(d\left(1+\frac{2c}{b-\sqrt{b^2}}\right.\right.$$

$$= \frac{d(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+}{}$$

$$= \frac{d(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+}{}$$

Mathematica [A]
time = 0.49, size = 273, normalized size = 0.85

$$\frac{x(fx)^m\left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}(a+x^n(b+cx^n))^p\left(d(1+m+n)F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^nF_1\left(\frac{1+m+n}{n};-p,-p;\frac{1+m+2n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)\right)}{(1+m)(1+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*(d*(1 + m + n)*AppellF1[(1 + m)/n, -p, -
p, (1 + m + n)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[
b^2 - 4*a*c])] + e*(1 + m)*x^n*AppellF1[(1 + m + n)/n, -p, -p, (1 + m + 2*n
)/n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])
]))/((1 + m)*(1 + m + n)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 -
 4*a*c]))^p*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (fx)^{m} \left(d+e\,x^{n}\right) \left(a+b\,x^{n}+c\,x^{2n}\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((x^n*e + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((x^n*e + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(d+e*x**n)*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(d+e*x^n)*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((x^n*e + d)*(c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(d+e\,x^n\right)\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(d + e*x^n)*(a + b*x^n + c*x^(2*n))^p, x)
```

3.154 $\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx$

Optimal. Leaf size=158

$$\frac{(fx)^{1+m}\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p}\left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2c}{b-\sqrt{b^2}}\right.}{f(1+m)}$$

```
[Out] (f*x)^(1+m)*(a+b*x^n+c*x^(2*n))^p*AppellF1((1+m)/n,-p,-p,(1+m+n)/n,-2*c*x^n
/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))/f/(1+m)/((1+2*c*x^
n/(b-(-4*a*c+b^2)^(1/2)))^p)/((1+2*c*x^n/(b+(-4*a*c+b^2)^(1/2)))^p)
```

Rubi [A]
time = 0.09, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1399, 524}

$$\frac{(fx)^{m+1}\left(\frac{2cx^n}{b-\sqrt{b^2-4ac}}+1\right)^{-p}\left(\frac{2cx^n}{\sqrt{b^2-4ac}+b}+1\right)^{-p}\left(a+bx^n+cx^{2n}\right)^p F_1\left(\frac{m+1}{n};-p,-p;\frac{m+n+1}{n};-\frac{2cx^n}{b-\sqrt{b^2-4ac}},-\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*x^n + c*x^(2*n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m
 + n)/n, (-2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^n)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*(1 + (2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*(1 + (2*c*x^n)/
(b + Sqrt[b^2 - 4*a*c]))^p)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1399

```
Int[((d_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x
_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n + c*x^(2*n))^FracPart[p]/((1 + 2*
c*(x^n/(b + Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^n/(b - Rt[b^2 - 4
*a*c, 2])))^FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^n/(b + Sqrt[b^2 - 4*a*c]
)))^p*(1 + 2*c*(x^n/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c,
d, m, n, p}, x] && EqQ[n2, 2*n]
```

Rubi steps

$$\int (fx)^m \left(a+bx^n+cx^{2n}\right)^p \, dx = \left(\left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+bx^n+cx^{2n}\right)^p \right)$$

$$= \frac{(fx)^{1+m} \left(1+\frac{2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(1+\frac{2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} \left(a+bx^n+cx^{2n}\right)}{f(1+m)}$$

Mathematica [A]
time = 0.22, size = 181, normalized size = 1.15

$$\frac{x(fx)^m \left(\frac{b-\sqrt{b^2-4ac}+2cx^n}{b-\sqrt{b^2-4ac}}\right)^{-p} \left(\frac{b+\sqrt{b^2-4ac}+2cx^n}{b+\sqrt{b^2-4ac}}\right)^{-p} (a+x^n(b+cx^n))^p F_1\left(\frac{1+m}{n};-p,-p;\frac{1+m+n}{n};-\frac{2cx^n}{b+\sqrt{b^2-4ac}},\frac{2cx^n}{-b+\sqrt{b^2-4ac}}\right)}{1+m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(a + b*x^n + c*x^(2*n))^p,x]
```

```
[Out] (x*(f*x)^m*(a + x^n*(b + c*x^n))^p*AppellF1[(1 + m)/n, -p, -p, (1 + m + n)/
n, (-2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^n)/(-b + Sqrt[b^2 - 4*a*c])])
/((1 + m)*((b - Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b - Sqrt[b^2 - 4*a*c]))^p*((b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^n)/(b + Sqrt[b^2 - 4*a*c]))^p)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int (fx)^m \left(a+b\,x^n+c\,x^{2n}\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (f\,x)^m\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a + b*x^n + c*x^(2*n))^p,x)
```

```
[Out] int((f*x)^m*(a + b*x^n + c*x^(2*n))^p, x)
```

3.155 $\int \frac{(fx)^m(a+bx^n+cx^{2n})^p}{d+ex^n} \, dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n} \, dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n} \, dx = \int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n} \, dx$$

Mathematica [A]
time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+bx^n+cx^{2n}\right)^p}{d+ex^n} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x]
[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,x^n+c\,x^{2n}\right)^p}{d+e\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="maxima")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(x^n*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="fricas")
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(x^n*e + d), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n),x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n),x, algorithm="giac")
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(x^n*e + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f\,x)^m\,\left(a+b\,x^n+c\,x^{2\,n}\right)^p}{d+e\,x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n),x)
[Out] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n), x)
```

3.156 $\int \frac{(fx)^m(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx = \int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Mathematica [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+bx^n+cx^{2n})^p}{(d+ex^n)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x]
```

Maple [A]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\,x^n+c\,x^{2n})^p}{(d+e\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

```
[Out] int((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(x^n*e + d)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="fricas")
```

```
[Out] integral((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(2*d*x^n*e + d^2 + x^(2*n)*e^2),
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*x**n+c*x**(2*n))**p/(d+e*x**n)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*x^n+c*x^(2*n))^p/(d+e*x^n)^2,x, algorithm="giac")
```

```
[Out] integrate((c*x^(2*n) + b*x^n + a)^p*(f*x)^m/(x^n*e + d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f\,x)^m\,(a+b\,x^n+c\,x^{2\,n})^p}{(d+e\,x^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2,x)
[Out] int(((f*x)^m*(a + b*x^n + c*x^(2*n))^p)/(d + e*x^n)^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                        Small rewrite of logic in main function to make it*)
(*                        match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
   #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

   return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(4,m1)    #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)    #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)    #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```